Department of Electrical Engineering

Final – Assignment Spring 2020 Date: 23/06/2020

Course Details

Course Title: Communication System **Module:**

Instructor: Dr Shahid Katif **Total Marks:** 50

Student Details

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Note: Attempt all of the following questions.

Q1.	(a)	Consider the signal g(t), a periodic train of rectangular pulses of duration 0.25 seconds and period of 1 second. This even signal is described analytically over one period as: $$g(t) = \begin{cases} 2, & 0 \le t \le 1/8 \\ 0, & 1/8 < t < 7/8 \\ 2, & 7/8 \le t \le 1 \end{cases}$$ (a) Using the complex exponential Fourier series coefficients, determine the amplitude spectrum and the power spectrum of g(t). (b) Determine what portion of the power lies within the main lobe and also find the frequency W, where W is an integer, so about 96% of the power lies in the frequency range [–W, W].	Marks 5 CLO 2
	(b)	A signal is Fourier transformable if it satisfies the Dirichlet's conditions. What are these conditions for Fourier Transform Pair?	Marks 5 CLO 2
Q2.	(a)	Fourier Transform for Periodic Signals in a strict mathematical sense does not exist, as periodic signals are not energy signals. Consider the periodic signal g(t) with period T0. Define the periodic signal g(t) using the generating function p(t), where p(t) equals g(t) over one single period and is zero elsewhere.	Marks 5 CLO 2
	(b)	Determine the Fourier transform of g(t) = sinc(t)?	Marks 5 CLO 2
Q3.	(a)	"The bandwidth of a signal reflects a range of positive frequencies with significant spectral content". Keeping this statement in view classify atleast four types of bandwidths, considering $B=f_2-f_1$, where $f_2 \ge f_1 \ge 0$.	Marks 5 CLO 2
	(b)	The impulse response of an LTI system is h(t)=u(t)-u(t-2). Determine the output signal y(t) provided that the input signal is x(t)=u(t)-u(t-3).	Marks 5 CLO 2
Q4.	(a)	"Convolution is an input-output relationship in time domain". Denotes the convolution operation. Write and prove equation for the convolution integral function y(t) is response to convolution input x(t)and impulse response h(t).	Marks 5 CLO 2
	(b)		Marks 5

		The frequency response of an LTI system is $H(f)=\frac{1}{2+j2\pi f}$ Determine the output signal in the time domain provided that the input signal is $x(t)=e^{-t}u(t)$	CLO 2
Q5.	(a)	Differentiate between distortion less transmission and non-linear distortion.	Marks 5 CLO 2
	(b)	Differentiate between low-pass filter, high-pass filter, band-pass filter and band-stop filter	Marks 5 CLO 2

Q1.a. Consider the signal g(t), a periodic train of rectangular pulses of duration 0.25 seconds and period of 1 second. This even signal is described analytically over one period as:

$$g(t)=\begin{cases}2, & 0\leq t\leq 1/8\\ 0, & 1/8<t<7/8\\ 2, & 7/8\leq t\leq 1\end{cases}$$

(c) Using the complex exponential Fourier series coefficients, determine the amplitude spectrum and the power spectrum of g(t).
Determine what portion of the power lies within the main lobe and also find the frequency W, where W is an integer, so about 96% of the power lies in the frequency range [–W, W].

Sol:-

Q1 a.

Sol

a) The amplitude spectrum is thus $|c_n| = |0.25 \operatorname{sinc}(\frac{n}{4})|$.

we thus get the power spectrum as follows-

$$P(f) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{16}\right) \operatorname{sinc}^2\left(\frac{n}{4}\right) \delta(f - nf_0)$$

b) Using table 3.1 the total average power is than as follow.

$$P = \left(\frac{1}{T_0}\right) \int_{-T_0/2}^{T_0/2} |g(t)|^2 dt = \int_{-1/8}^{1/8} 4\, dt = 1.$$

The power content in terms of the numbers of power components. There are nine power components in the frequency range [-4, 4], whose sum is thus :-

$$\sum_{n=-4}^{4} (1/16) \operatorname{sinc}^2(n/4) = 0.904$$ (ie., over 90% of total power in lobe)

By considering a wider range than the frequency range [-4, 4], the portion of power can be beyond 90%. To this end, we now need to solve

$$\sum_{n=-W}^{W} \left(\frac{1}{16}\right) \operatorname{sinc}^2\left(\frac{n}{4}\right) \geq 0.96$$

for W, where W is an integer. For W = 13 Hz, more than 96% of the total power lies in the frequency range [-13, 13] [i.e., a total of 27 power components].

<u>Q1.b.</u> A signal is Fourier transformable if it satisfies the Dirichlet's conditions. What are these conditions for Fourier Transform Pair?

A signal is Fourier transformable if it satisfies the Dirichlet's conditions. These conditions, which are sufficient but not strictly necessary, are as follows:
i) The signal is single-valued, with a finite number of maxima and minima and a finite number of discontinuities in any finite time interval. ii) The signal is absolutely integrable over the entire time, i.e.

$$\int_{-\infty}^{\infty} |g(t)| dt < \infty$$

Note that a sinc function does not satisfy Dirichlet's condition, as it is not absolutely integrable, yet it possesses the Fourier transform. Also, certain signals do not have Fourier transforms in the ordinary sense, but their Fourier transforms can be obtained in the limit. The physical existence of a signal (i.e., a signal with finite energy) is a sufficient condition for the existence of its Fourier transform. We can now define the Fourier transform of a signal as follows:

$$G(f) = F[g(t)] = \int_{-\infty}^{\infty} e^{-j2\pi f t} dt$$

Where F denotes the linear operator of the Fourier transform, j= $\sqrt{-1}$, and the variable t denotes time measured in seconds (s), and the variable f denotes frequency measured in Hertz (Hz). The inverse Fourier transform, through which the original signal in the time domain can be recovered, is as follows:

$$G(t) = F^{-1}[G(f)] = \int_{-\infty}^{\infty} G(f)^{j2\pi f t} df$$

Where F^{-1}denotes the linear operator of the inverse Fourier transform. It is important to note that G (f) is the spectral density per unit bandwidth, but in practice it is customarily called the spectrum of g (t) rather than spectral density of g (t). We call G (f), the Fourier spectrum of g (t), as a lowercase letter denotes a time function and an uppercase letter denotes a function in frequency. A signal is uniquely defined by either its time-domain representation or its frequency domain representation; a change in one results in a change in the other. A shorthand notation for the Fourier-transform pair is as follows:
G (t)←>G (f)

Q2.a. Fourier Transform for Periodic Signals in a strict mathematical sense does not exist, as periodic signals are not energy signals. Consider the periodic signal g(t) with period T0. Define the periodic signal g(t) using the generating function p(t), where p(t) equals g(t) over one single period and is zero elsewhere.

The Fourier transform of a periodic signal, in a strict mathematical sense, does not exist, as periodic signals are not energy signals. In a limiting sense, Fourier transform can be defined for complex exponentials. As such, the Fourier transform of a periodic signal can be obtained through the Fourier transform of its complex exponential Fourier series term by term. To this effect, all periodic signals have a common feature in that their Fourier transforms consist of delta functions. Thus, the delta (impulse) function, which does not exist physically and is not defined explicitly, can provide a unified method of describing periodic signals in the frequency domain.

Consider the periodic signal g (t) with period T0. We can then define the periodic signal g(t) using the generating function p(t), where p(t) equals g(t) over one single period and is zero elsewhere, as shown by:

$$g(t) = \sum_{n=0}^{\infty} p(t - mTo)$$

On the other hand, this periodic signal can be r represented in terms of the complex exponential Fourier series:

$$g(t) = \sum_{n=-\infty}^{\infty} \left(c_n \exp(\frac{j2\pi nt}{To}) \right)$$

$$c_n = (\frac{1}{To}) \int_{-t_{o/2}}^{t_{o/2}} g(t) \exp\,(\frac{j2\pi nt}{T_o})dt = f_o \int_{-x}^{x} P(t) \exp(-j2\pi n f_o t)\, dt = f_o P(nf_o)$$

Where Cn is the complex coefficient, P (f) is the Fourier transform of p(t), and $f_{o=}1/T_o$. As the RHS of indicates, the samples of the Fourier transform of the generating function at multiples of the fundamental frequency f_o and the complex exponential Fourier series coefficients of the periodic function are linearly related. Substituting and combining the result with yields the following:

$$g(t) = \sum_{m=-\infty}^{\infty} p(t - mT_o = f_o \sum_{-\infty}^{\infty} P(nf_{o)} \exp\,(j2\pi n f_o t)$$

Where it is known as Poisson's sum formula. Noting the Fourier transform of an exponential function is a delta function, the Fourier transform of the periodic signal g (t) is then as follows:

$$g(t) = \sum_{-\infty}^{\infty} p(t - mTo) \leftrightarrow G(t) = f_o \sum_{-\infty}^{\infty} P(nf_o)\delta(f - nf_o)$$

This highlights the fact that the Fourier transform of a periodic signalconsistsofdeltafunctionsoccurringatintegermultiplesofthefundamental frequency, including the origin, and each delta function is scaled by a factor equal to the corresponding value of P (nf_o). Note that the non-periodic generating function p(t), constituting one period of g(t), has a continuous spectrum, but the periodic g(t) has a discrete spectrum. In other words, periodicity in the time domain results in a discrete spectrum defined at integer multiple of the fundamental frequency.

Note that for an infinite sequence of uniformly spaced delta functions, we have p (t) =$\delta(t)$

And we thus have P (f) =1. We can have the following interesting relation:

$$\sum_{m=-\infty}^{\infty} \delta(t - mT_o) \leftrightarrow f_o \sum_{n=-\infty}^{\infty} \delta(f - nf_o)$$

Note that if $T_{o=1(i.e., f_{o=1})}$ then an infinite sequence of uniformly spaced delta functions in the time domain is its own Fourier transform.

Q.2.b Determine the Fourier transform of g (t) = sinc (t)?

We know we have the following transform pair:

$\left(u\left(t + \frac{1}{2}\right) - \left(t - \frac{1}{2}\right)\right) \leftrightarrow sinc$(f)

By using the duality property of the Fourier transform, we get the following

Sinc (t) $\leftrightarrow \left(u\left(-f + \frac{1}{2}\right) - u\left(-f - \frac{1}{2}\right)\right)$

Since $\left(u\left(-f + \frac{1}{2}\right) - u\left(-f - \frac{1}{2}\right)\right)$ is an even function, we then have the following:

Sinc (t) $\leftrightarrow \left(u\left(f + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)\right)$

Q.3.a."The bandwidth of a signal reflects a range of positive frequencies with significant spectral content". Keeping this statement in view classify at least four types of bandwidths, considering $B=f_2 - f_1$, where $f_2 \geq f_1 \geq 0$.

Bandwidth is a very important measure of performance in digital communication systems .Never the less, the term bandwidth is usually used loosely .It is thus critical to provide an accurate definition and a quantitative description of band width. The definitions of bandwidth for signals also apply to systems. The bandwidth of a signal reflects a range of positive frequencies with significant spectral content. All practical signals are time-limited and their spectra thus extend to infinity. It is therefore not clear as to how to determine what part of the spectrum constitutes a significant amount of energy or power of the signal. Clearly, it is difficult to have a universally accepted definition of bandwidth, as signals and their applications vary significantly. However, there are many definitions that are commonly used.

It is important to note that shifting the spectral content of a low pass signal by a sufficiently large frequency—through a process known as modulation—to produce its corresponding bandpass signal has the effect of increasing the bandwidth of the signal. The term bandwidth, denoted by B, may be defined, as

The difference (in Hz) between two nominal frequencies, f_1 $and f_2$,i.e., B=f_2 - f_1 ,where $f_2 \geq f_1 \geq 0$ and f_2 and f_1 are determined by one of the following definitions of bandwidth.

Absolute Bandwidth

Absolute bandwidth provides a theoretical definition. Assuming the spectrum is zero beyond$f_2 \geq f_1 \geq 0$, we have. B=f_2-f_1. Absolute bandwidth can be applied to frequency-limited signals and ideal lowpass and bandpass filters. No absolute bandwidth can be defined for high-pass filters, As.,$f_2 \rightarrow \infty$ and consequently B$\rightarrow \infty$. In essence, for all realizable signals and filters, the absolute bandwidth is infinite.

3-dB (or Half-Power) Bandwidth

The3-dB (or half-power) band width is one of the widely-used definitions. Assuming the maximum (peak) value of the magnitude spectrum occurs at a frequency inside the band $[f_1, f_2]$, we have B=f_2-f_1 , where the magnitude spectrum at any frequency inside the band falls no lower than $1/\sqrt{2}$ times the peak value. The signal power at f_1 $and f_2$ is thus

3 $(\cong -20 log_{10}(1/\sqrt{2})$ dB lower than the peak signal power. However, this definition becomes ambiguous when the magnitude spectrum has multiple peaks. With this definition, the bandwidth can be easily read from a plot of magnitude spectrum. However, it may not be quite representative when the magnitude spectrum has slowly decreasing tails

Fractional-Power Bandwidth

The occupied bandwidth, as adopted by FCC, defines a band off frequencies with 99% of the signal power, where 0.5% of the signal power is above the upper-frequency limit and 0.5% of the signal power is below the lower-frequency limit. This definition is primarily focused on passband signals and filters. However, for low pass signals and filters, the definition may be modified to include 1% of the signal power above the upper frequency limit (f_2), as the signal power below the lower-frequency limit ($f_1 = 0$) is zero.

Null-to-Null (Zero-Crossing) Bandwidth

The null-to-null (zero-crossing) bandwidth is a commonly-used definition. For band pass signals, when the magnitude spectrum has a main lobe (the lobe with the peak value) bounded by nulls (the frequencies $f_1\ and f_2$ at which the magnitude spectrum is zero), we have B=f_2-f_1, as the main lobes centered on the frequency f_c= (f_2+f_1)/2. For low pass signals, we have $f_1 = 0$ (i.e., only one half of the width of the main spectral lobe is the bandwidth). Note that the null-to-null bandwidth can be easily read from a plot of magnitude spectrum.

Q.3.b The impulse response of an LTI system is h(t)=u(t)-u(t-2). Determine the output signal y(t) provided that the input signal is x(t)=u(t)-u(t-3).

Solution

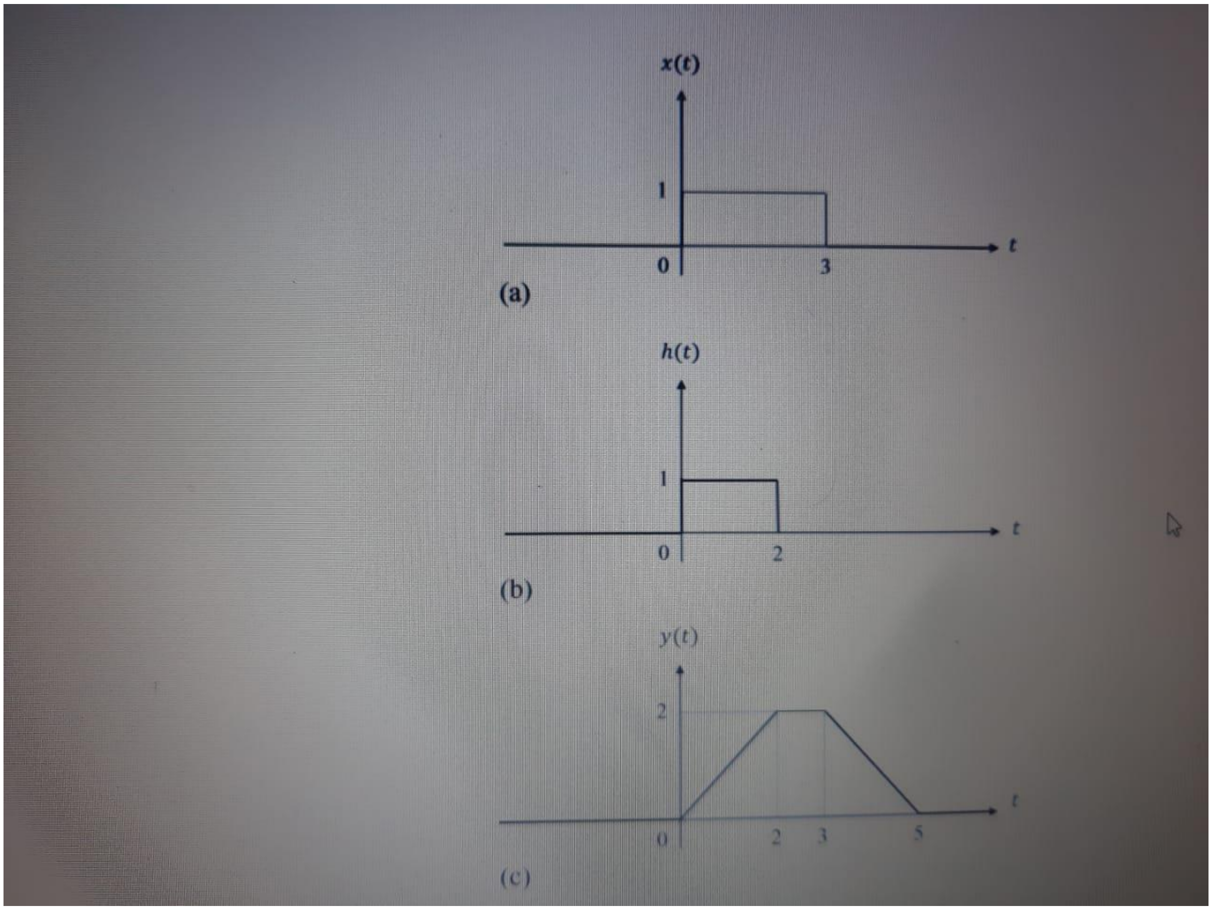
After substituting h(t) and x(t)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) \quad d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau)u(t-\tau) \quad d\tau - \int_{-\infty}^{\infty} u(\tau)u(t-2-\tau) \quad d\tau - \int_{-\infty}^{\infty} u(\tau-3)u(t-\tau) \quad d\tau + \int_{-\infty}^{\infty} u(\tau-3)u(t-2-\tau) \quad d\tau$$

= t u(t)-(t-2) u(t-2) – (t-3) u(t-3) + (t-5) u(t-5),

X[t] h[t] and y[t] are all shown below


x(t)
1
0
3
t
(a)
h(t)
1
0
2
t
(b)
y(t)
2
0
2
3
5
t
(c)

Q.4.a.Convolution is an input-output relationship in time domain". Denotes the convolution operation. Write and prove equation for the convolution integral function y(t) is response to convolution input x(t)and impulse response h(t).

Convolution is the input-output relationship in the time domain. By convolution, the output y (t) in an LTI system can be derived from the input x (t) and the impulse response h (t). It can be shown that the output y (t) can be derived as follows:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) \quad d\tau = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau) \quad d\tau = h(t) * x(t)$$

Where * denotes the convolution operation. Equation (3.72) is called the convolution integral, and shows that y (t), which is the response to x (t), is the convolution of the input x (t) and the impulse response h (t). Note that y (t) is nonzero for the interval that is the sum of the intervals during which x (t) and h (t) are nonzero. In other words, if x (t) is limited to the time interval [a, b] and h (t) is limited to the time interval [c, d], y (t) is then limited to the time interval [a+c, b+d]. Reflects the fact that the present value of the output signal is a weighted integral over the past history of the input signal, weighted according to the impulse response of the system. In a way, the impulse response h (t) acts as a memory function for the system. For a causal LTI system, there can be no output prior to the time t=0. Therefore, the lower limit of the integration can be changed to zero. Foran LTI system, the impulse response h (t) contains all the information needed, and thus completely characterizes the system

Q.4.b. The frequency response of an LTI system is

$H(f) = \frac{1}{2+j2\pi f}$

$x(t) = e^{-t}u(t)$ **Determine the output signal in the time domain provided that the input signal is**

Solution

The Fourier transform of the input signal is as follows:

$$\text{X(f)} = \frac{1}{1 + j2\pi f}$$

We then determine the Fourier transform off) the output signal:

Y (f) = X (f) H (f) =$(\frac{1}{2+j2\pi f})\ (\frac{1}{1+j2\pi f}) = \frac{1}{(2+j2\pi f)(1+j2\pi f)} = \frac{1}{1+j2\pi f} - \frac{1}{2+j2\pi f}$

The inverse Fourier transform is as follows

Y(t)=$e^{-t}u(t) - e^{-2t}u(t) = (e^{-t} - e^{-2t})u(t)$

Q.5.a.Differentiate between distortion less transmission and non-linear distortion.

Distortionless Transmission:-

It is of paramount interest that in a communication channel the output signal be an exact replica of the input signal; after all, that is the ultimate goal in signal transmission. It is therefore important to determine the characteristics of a communication system that allows no distortion. In a distortion less transmission, the input and output signals in the time domain have identical shapes, except for a possible change of amplitude and a constant delay. In other words, for the input signal x (t) transmitted through a distortion less channel, the output signal y (t) is defined by:

$$Y(t) = kx(t - t_d)$$

Where the constant $k < 1$ reflects the transmission attenuation, and the constant $td > 0$ accounts for the transmission delay, as a transmission medium always introduces an attenuation, no matter how small, and a delay, no matter how short. By applying the Fourier transform. We get the following

$$y(f) = kX(f)ke^{-j2\pi f t_d}$$

The transfer function of a distortion less channel H (f) is then defined as follows:

$$\text{H(f)} = \frac{Y(f)}{X(f)} = ke^{-j2\pi f t_d}$$

It indicates that in order to achieve distortion less transmission through a channel, the magnitude response of the channel must be a constant and the phase response must be a linear function of frequency that passes through the origin. In other words, the following two requirements must be satisfied over the frequency band of interest (the band of frequencies that the spectrum of the transmitted signal exists):

[H(f)=k]

$$< H(f) = \beta(f) = -2\pi f t_d$$

When the magnitude response of the channel [H(f)] not constant over the frequency band of interest, we have magnitude distortion (i.e., the frequency components of the input signal experience different amounts of attenuation, or possibly gain). Also, when the phase response of the channel $\beta\ (f)$ is not linear with respect to the frequency inside the band of interest, we have phase distortion (i.e., the components of different frequencies undergo different amounts of delay). Interestingly, the human ear is insensitive to phase distortion, but relatively sensitive to amplitude distortion. However, the human eye is more sensitive to time delay, rather than amplitude distortion.

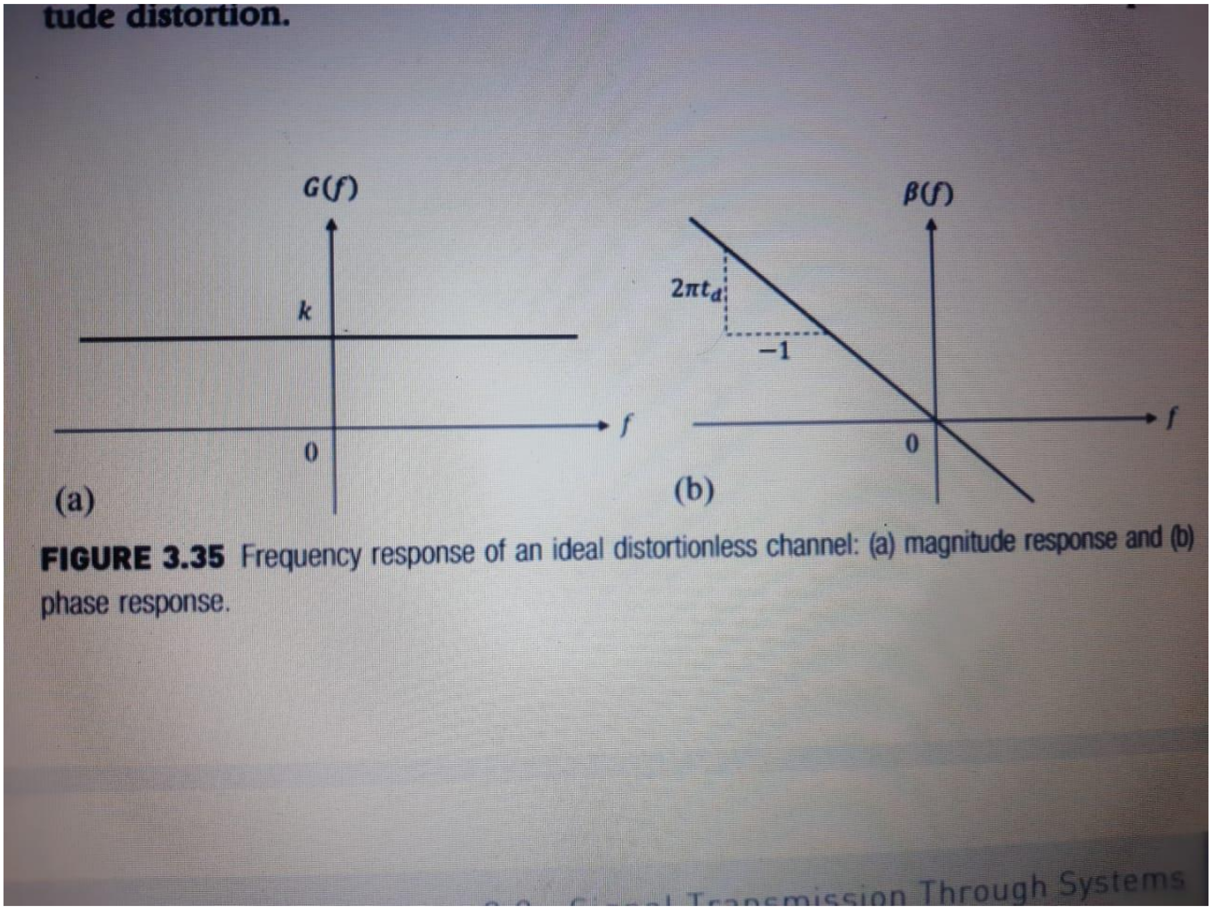


FIGURE 3.35 Frequency response of an ideal distortionless channel: (a) magnitude response and (b) phase response.

Non liner Distortion:-

A nonlinear system cannot be described by a transfer function, as a change in the input signal may not directly produce a corresponding change in the output signal. We assume here the system is memoryless in the sense that the output $y\ (t)$ depends only on the input $x\ (t)$ at time t. To evaluate the nonlinear distortion, the common procedure is to approximate the input-output relation, also known as the transfer characteristics, by a power series of the input $x\ (t)$:

Y(t)=$a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots$

Assuming $X(f)$ is the Fourier transform of $x(t)$, the Fourier transform becomes as follows:

Y(f)=$a_1 X(f) + a_2 X(f) * X(f) + a_3 X(f) * X(f) * X(f) + \cdots$

Where * denotes the convolution operation. Assuming x (t) is band-limited to WHz, x2 (t)is band-limited to 2WHz, x3(t) is band-limited to 3WHz, and soon and so forth .The nonlinearities have thus created new output frequency components that are not present in the input. With appropriate filtering, these out-of-band frequency components ([f] ≥W) can be suppressed. However, the second-, third-, and the higher-order nonlinearities all produce undesirable in-band frequency components ($|f| \leq W$). Since these frequency components, which lie in the frequency band of interest, cannot be removed, we have nonlinear distortion.

Q.5.b. Differentiate between low-pass filter, high-pass filter, band-pass filter and band-stop filter

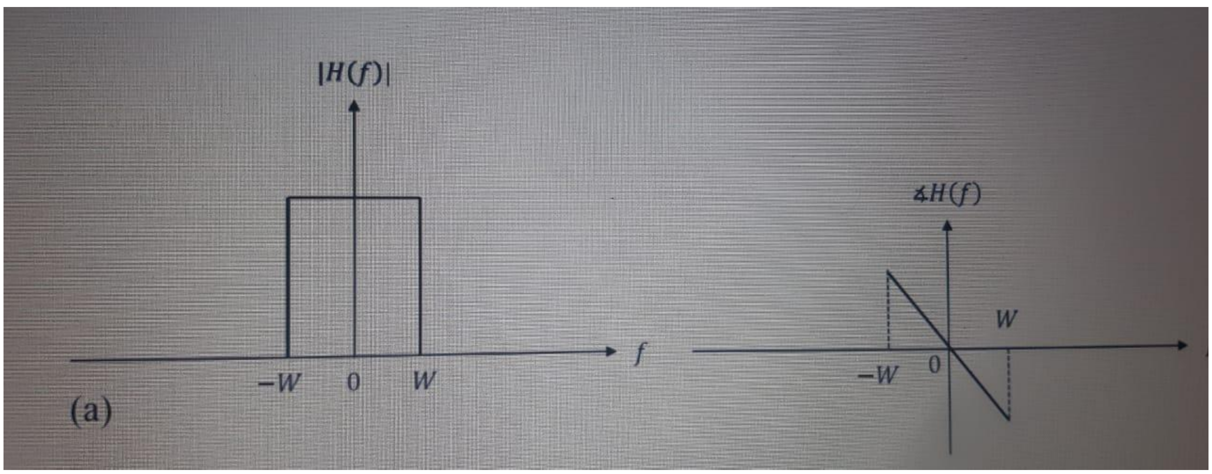
An ideal filter exactly passes signals at certain sets of frequencies and completely rejects the rest

The most common types of filters are

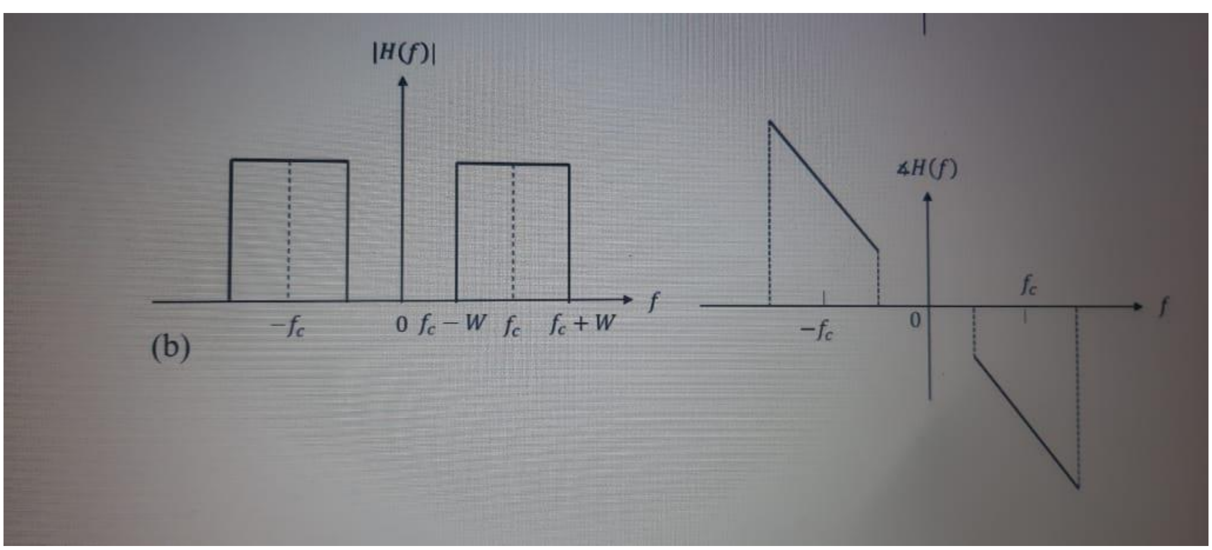
1. Low pass filter
2. High pass filter
3. Band pass filter
4. Band stop filter

Figure

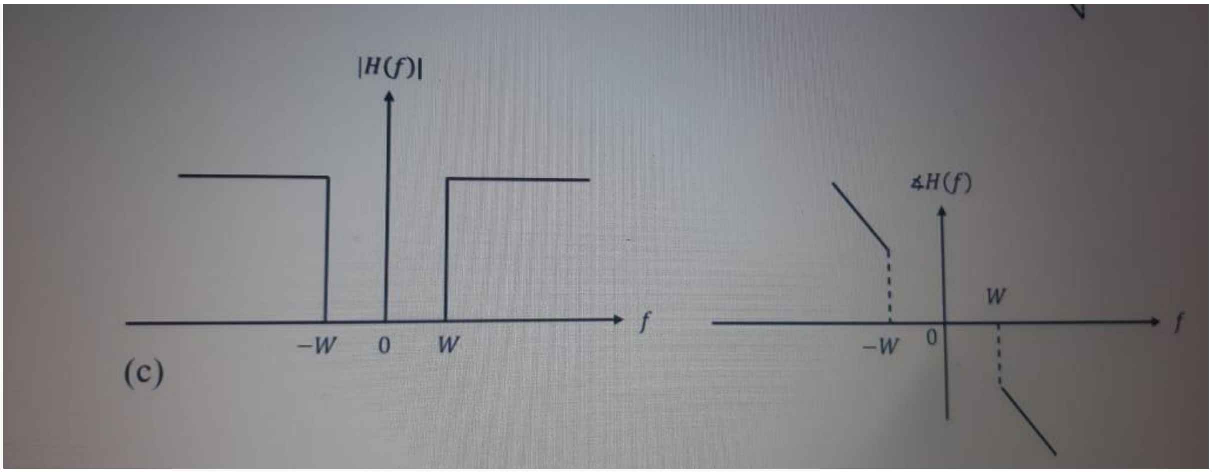
1. Low pass filter



2. High pass Filter



3. Band pass filter



4. Band stop filter