Date:

Q:2: ①

Direct Stiffness Method:

In direct Stiffness method
First of all, kinematic indeterminacy of the structure is determined.
Kinematic indeterminacy is to number of unknown joints displacement of the structure.
In stiffness method, unknown joints displacements are determined and remaining unknown quantities (Support reactions and Member End Actions) in the structure are functionally dependent on the displacements.
For computing joint displacements, the structure is made restrained (i.e K.I=0) under external loadings and member End actions are calculated for fixed end actions corresponding to unknown joint displacements i.e (ADL).
Then stiffness matrix is formed and coefficients are calculated under the action of unit displacement corresponding to unknown joint displacements.
If no of unknown joint displacements are 'd', then order of

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Stiffness matrix will be 'dxd'.

Coefficients are calculated under the action of unit displacement corresponding to unknown joint displacements.
If no of unknown joint displacements are 'd', then order of

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Stiffness matrix will be 'dxd'.
After computing coefficients of stiffness matrix under the action of unit displacements, the following equation is used to calculate unknown joint displacements.

$$[D] = [S]^{-1}\left\{[AD] - [ADL]\right\}$$

Where

D = unknown joint displacements

$[S]^{-1}$ = Inverse of Stiffness Matrix

$[AD]$ = External Concentrated Action corresponding unknown joint displacement.

$[ADL]$ = Member End Actions in restrained structure under external loading.

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in restrained structure
under external loading.

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Once, displacements are found, other
unknown structural quantities are
functionally dependent on computed
displacements, and hence can be
calculated by method of superposition
for the given structure under external
loading and under the action of
unit displacements.

①

(a) Nodes

①

Q No 4:

In Flexibility method, unknowns to be calculated are Joint or support reactions.
Unknown reactions are made redundant and the structure is made statically determinate and analyzed for end actions.
After that, unit forces corresponding to unknown reactions are applied on redundant structure from which flexibility matrix is formed and calculated. Order of flexibility matrix is equal no of unknown or redundant actions.
This method is also called Force method.
In stiffness method, unknowns to be determined are joint displacements and for calculating stiffness matrix unit displacements are applied.
Order of stiffness matrix is equal to the no of unknown joint displacements i.e Kinematic Indeterminacy.

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displacements i.e Kinematic Indeterminacy.

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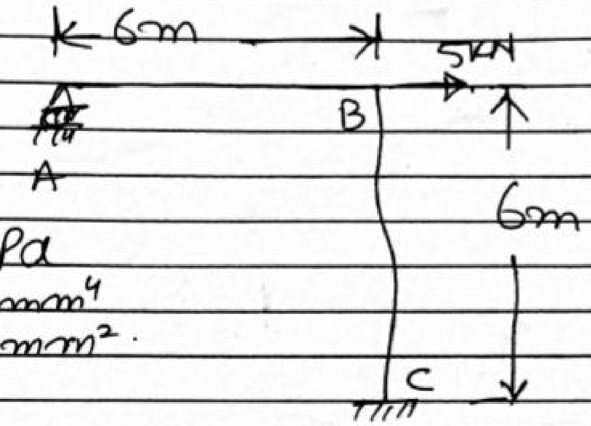
Q.No.1:



$E = 200$ GPa
$I = 60 \times 10^6\ mm^4$
$A = 600\ mm^2$.

Deflection and Rotation @ 'B' = ?
Reactions @ Supports = ?
S.F.D and B.M.D = ?

$\Rightarrow EI = 200 \times 10^9 * \frac{60 * 10^6}{(1000)^4} = 12 \times 10^6\ N\text{-}m^2 = 12000\ kN\text{-}m^2$

$EA = 200 \times 10^9 * \frac{600}{(1000)^2} = 12 \times 10^6\ N$. $= 12000\ kN$

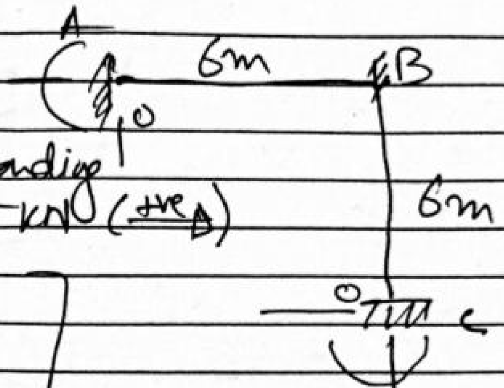
Now Restrained Structure under existing loading is

Darsi Notes

②

Darsi Notes

Now Restrained Structure under existing loading is



End Action Corresponding Joint 'B' is 5 kN (+ve)

i.e

$$[AD] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$

→ External Action Corresponding horizontal displacement @ B.

End Action on restrained Structure is

$$ADL = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Q_A, Q_B & Δ_A, Δ_B to be determined.

Given structure under unit rotation at (A) making remaining structure restrained i.e

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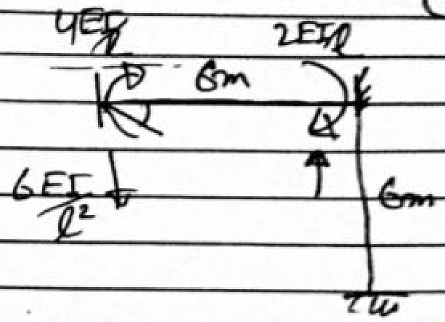
$\frac{4EI}{l} = \frac{4 \times 12000}{6}$

$= 8000$ kN-m

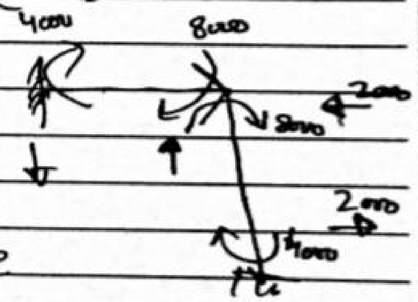


$2EI/l = 4000$ kN-m.

$6EI/l^2 = \frac{6 \times 12000}{6 \times 6} = 2000$ kN

$S_{11} = 8000$, $S_{21} = 4000$, $S_{31} = 0$, $S_{41} = 0$

Now unit +ve rotation @ B i.e



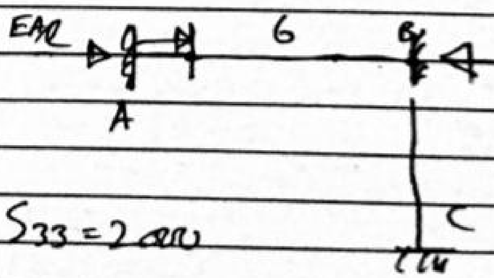
$S_{12} = 4000$, $S_{22} = 16000$, $S_{32} = 0$

$S_{42} = -2000$

Unit +ve translation @ (A) i.e

$\frac{EA}{l} = \frac{12000}{6}(1)$

$= 2000$ kN



So

$S_{13} = 0$, $S_{23} = 0$, $S_{33} = 2000$

$S_{43} = -2000$

$S_{43} = -2000$

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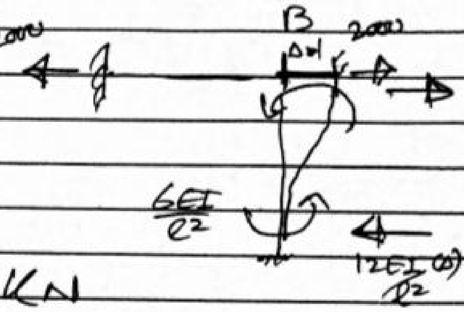
Now unit +ve translation @ B i.e

$\frac{6EI(1)}{\ell^2} =$

$= \frac{6 \times 12000}{6 \times 6} = 2000$ kN-m



$\frac{12 \times 12000}{6 \times 6 \times 6} = 666.67$ kN

$S_{14} = 0$, $S_{24} = -2000$, $S_{34} = -2000$
$S_{44} = 2666.67$

So, Stiffness matrix is

$$[S]_{4\times4} = \begin{bmatrix} 8000 & 4000 & 0 & 0 \\ 4000 & 16000 & 0 & -2000 \\ 0 & 0 & 2000 & -2000 \\ 0 & -2000 & -2000 & 2666.67 \end{bmatrix}$$

Now unknown joint displacements can be found from the following equation i.e

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$$\begin{bmatrix} \theta_A \\ \theta_B \\ \Delta_A \\ \Delta_B \end{bmatrix} = [S]^{-1}_{4\times4} \left\{ [AD] - [ADL] \right\}$$

here [S] calculated on previous page

$$[AD] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \text{ and } [ADL] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

putting values and solving for unknown displacements, we get.

$$\begin{bmatrix} \theta_A \\ \theta_B \\ \Delta_A \\ \Delta_B \end{bmatrix} = \begin{bmatrix} -0.001 \\ 0.002 \\ 0.013 \\ 0.013 \end{bmatrix}$$ (unknown displacements in structure)

Now, Reactions at ~~sup~~ supports can be calculated as

$$[R] = [R_L] + [S][D]$$

(6)

$[R] = [R_L] + [S][D]$

where

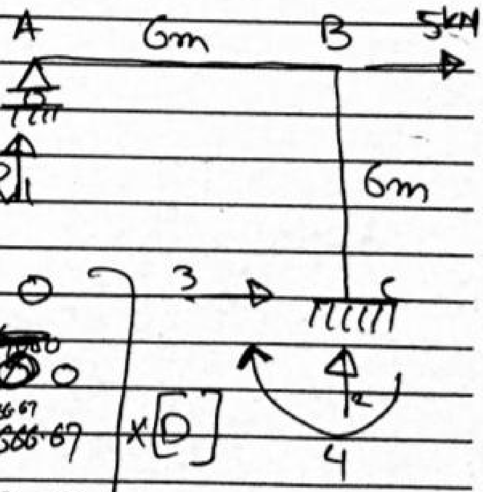
R = Reactions @ Supports

R_L = Reactions on restrained structure due to external loading

[S] = Corresponding Stiffness values

[D] = Displacement Matrix.

$$\begin{bmatrix} R_1 \\ 2 \\ 3 \\ M \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2000 & -2000 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2000 & 0 & -666.67 \\ 0 & 4000 & 0 & -2000 \end{bmatrix} \times [D]$$



so $R_1 = -2\,kN$, $R_2 = 0$, $R_3 = -4.7\,kN$

$M = -18\ kN\text{-}m$.

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$$\begin{bmatrix} 0 & 4000 & 0 & -2000 \end{bmatrix}$$

So $R_1 = -2\,kN$, $R_2 = 0$, $R_3 = -4.7\,kN$

$M = -18\ kN\text{-}m.$

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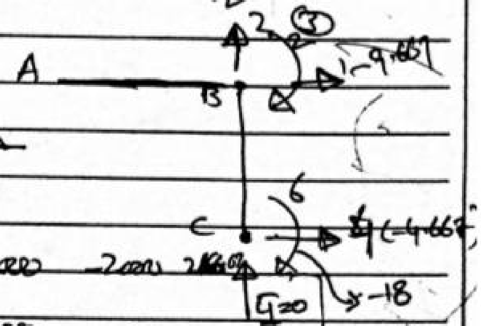
Now Member End Action in ~~Struct~~ at Joint 'B' and 'C' is ~~Structure~~



$$A_{RL} = \begin{bmatrix} +5 \\ 0 \\ 0 \\ 0 \\ -5 \\ -30 \end{bmatrix} + \begin{bmatrix} 0 & -2000 & -2000 & 2000 \\ 2000 & +2000 & 0 & 0 \\ 4000 & +16000 & 0 & -2000 \\ 0 & 2000 & 0 & -666.67 \\ 0 & 0 & 0 & 0 \\ 0 & 4000 & 0 & -2000 \end{bmatrix} \times [D]$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4.667 \\ 2 \\ 2 \\ -4.667 \\ 0 \\ -18 \end{bmatrix} \begin{matrix} 4.667 \\ 2 \\ 2 \\ -4.667 \\ 0 \\ -18 \end{matrix}$$

End Actions of Structure

4	-4.667	-4.667 Structure
5	0	0
6	-18	-18

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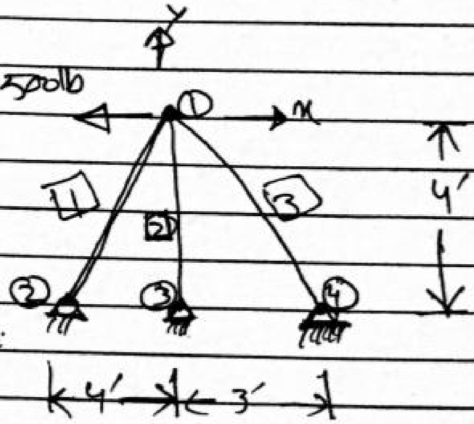
Q.No3:



Horizontal displacement at joint ① = ?

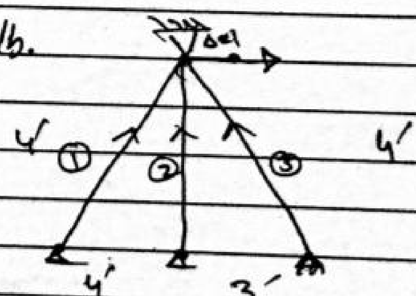
Force in member [2] = ?

$A = 0.75\ in^2$, $E = 29\times10^3$ ksi

Δ_x & Δ_y = ?

Applying unit +ve translation along x and y-axis i.e

$\frac{EA}{ft} = 29\times10^3 \times 0.75 = 21750$ lb.



Member ① : $l = 5.65$ ft.

Member ② : $l = 4$ ft.

Member ③ : $l = 5$ ft.

Member ① : $\frac{EA}{L} = 320.8$ lb/in

Member ② : 453.125 lb/in

Member ③ : 362.5 lb/in.