-: COURSE DETAILS :-

Course Tittle :- linear Algebra

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Mudule :- 1st

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QNO.1:- Part (A)

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify the $(3,2)$ entry of AB.

Solution:- Identify $(3,2) = ?$

$Row_3(A) \cdot Column_2(B)$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

$\Rightarrow (0)(4) + (1)(-1) + (-2)(2)$

$0 - 1 - 4$

$\Rightarrow \boxed{-5}$

Ans.

Q No. 1:- Part (B)

Lable the quadratic polynomial that interpolate the points (1, 3), (2, 4), (3, 4).

Sol:-

As

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y^3$$

Now $(x_1 y) = (1, 3)$, $(x_2, y_2) = (2, 4)$

and $(x_3, y_3) = (3, 7)$

put in the above solution

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 7$$

$$A_b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ \\ R_3 - 9R_1 \end{array}$$

$$\approx \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

Sol:- $a_2 + a_1 + a_0 = 3 \rightarrow (1)$

$-2a_1 - 3a_0 = -8 \rightarrow (2)$

$a_0 = 4 \rightarrow (3)$

So $\boxed{a_0 = 4}$ put in eq (2)

$-2a_1 - 3(4) = -8$

$-2a_1 - 12 = -8$

$-2a_1 = -8 + 12$

$-2a_1 = 4$

$a_1 = \frac{4}{-2}$

$\boxed{a_1 = -2}$ → put a_1 and a_0 in eq (1)

$a_2 + (-2) + (4) = 3$

$a_2 - 2 + 4 = 3$

$a_2 + 2 = 3$

$a_2 = \frac{3}{+2}$

$\boxed{a_2 = 1.5}$ Ans.

QNo. 2 Part (A)

If A and B are $n \times n$ matrices where $|A| = 2$ and $|B| = -3$, calculate $|A^{-1} B^{T}|$.

Solution:-

$|A^{-1} B^{t}|$

$|A^{-1}| \, |B^{t}|$ $\rightarrow |B^{t}| = |B|$

$\frac{1}{|A|} |B|$

So:- $|A^{-1} B^{t}| = \frac{1}{|A|} |B|$

$= \frac{1}{2} \cdot 3$

$= \boxed{\frac{3}{2}}$ Ans.

Q No. 2 Part (B) :-

(B) Estimate the linear system of equation

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3.$$

Sol:-
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] \frac{R_2}{-3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & -\frac{13}{2} & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & -8/13 \end{array}\right] R_3 \times \frac{2}{-13}$$

$x + y + 2z = 1 \rightarrow (1)$

$y + \frac{1}{3} z = 2 \rightarrow (2)$

$z = \frac{-8}{13} \rightarrow (3)$

Now put e.q (3) in (2)

$y + \frac{1}{3} \times \left(-\frac{8}{13}\right) = 2$

$y - \frac{8}{39} = 2$

$y = 2 + \frac{8}{39}$

$y = \frac{78+8}{39} = \frac{86}{39}$

Now put value of y in (1)

$x + \frac{86}{39} + 2\left(\frac{-8}{13}\right) = 1$

$x + \frac{86}{39} - \frac{16}{13} = 1$

$x + \frac{38}{39} = 1$

$x = 1 - \frac{38}{39}$

$\boxed{x = \frac{1}{39}}$ Ans.

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Q No. 3

Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Solution:-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

Expand by Row$_1$

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} - (-2)\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + (1)\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & +2 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$\Rightarrow 3(-18-0) + 2(-15-2) + 1(0-6)$

$\Rightarrow 3(-18) + 2(17) + 1(-6)$

$\Rightarrow -54 - 34 - 6$

$\Rightarrow -94$

$$|A| = -94$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = +17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}.$$

so $A^{-1} = \frac{1}{|A|} \times \text{adj of } A$

$$A^{-1} = \frac{1}{-94} \begin{vmatrix} 18 & 6 & 10 \\ 17 & 10 & 1 \\ 6 & 2 & -28 \end{vmatrix}$$