Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: ____________ **Total Marks:** 30

Student Details

Name: ____________ **Student ID:** ____________

Q1.

(a) Consider the following analog signal

Marks 5
CLO 1

$$x_a(t) = 3cos100\pi t + 4sin200\pi t$$

i. Determine the minimum sampling rate required to avoid aliasing.
ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.
iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

(b) Consider a discrete time signal which is given by

Marks 5
CLO 1

$$x(n) = \begin{cases} 0.5^n , & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$.

i. Draw the sampled signal.
ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .
iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.

Q2.

(a) Determine the response of the system to the following input signal with given impulse response

Marks 5
CLO 2

$$x[n] = \left\{ 2, \underset{\uparrow}{1}, -2, 3, -4 \right\} \quad , h[n] = \left\{ \underset{\uparrow}{3}, 1, 2, 1, 4 \right\}$$

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^n, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Subject: DSP Abdul Aziz 13741 (1)

Q.(1)(i) $x(t) = 3\cos\pi t + 4\sin 200\pi t$.

Minimum sampling rate?

$3\cos 100\pi t \Rightarrow W = 100\pi$

$2\pi F_1 = 100\pi$

$F_1 = \frac{100\pi}{2\pi} = 50Hz$

$4\sin 200\pi t \Rightarrow W = 200\pi$

$2\pi F_2 = 200\pi$

$F_2 = \frac{200\pi}{2\pi} = 100Hz$

As $f_2 = 100Hz$ so $f_{max} = 100Hz$

minimum Nyquist rate $\Rightarrow$ $F_s = 2F_m$

$F_s = 2 \times 100$

$F_s = 200Hz$

$\Rightarrow$ (ii) $F_s = 100Hz$

$F_s = 2F_{max}$

$100Hz = 2F_{max}$

$$\boxed{F_{max} = 50Hz}$$

As $t = nT = n/F_s$ (by sampling)

DSP Abdul Aziz 13741

As $x(t) = 3\cos 100\pi t + 4\sin 200\pi t$.

$$x(n) = 3\cos\frac{100\pi n}{100} + 4\sin\frac{200n\pi}{100}$$

$$x(n) = 3\cos n\pi + 4\sin 2n\pi$$

As $F_{max} = 50Hz$

Also we know

$$F_0 = F_K - KF_s$$

Put $K=1$

$$F_0 = F_1 - F_s$$

$$F_0 = 50 - 50 = 0Hz$$

Put $K=2$

$$F_0 = 100 - 2\times 50$$
$$F_0 = 100 - 100$$
$$F_0 = 0Hz$$

As both frequencies are not more than folding maximum frequencies. Thus there will be no effect of sampling over nearly generated dist.

(iii) ⇒ As we obtain

$3\cos n\pi + 4\sin n\pi$

Converting to time domain

$3\cos n\pi \frac{100}{100} + 4\sin \frac{2n\pi}{100} \times 100 \quad \because F_s = 100$

$3\cos 100\pi t + 4\sin 200\pi t \quad \because \frac{n}{F_s} = t$

Thus we have two frequency components in above which is 50Hz and 100Hz upon which sampling was done.

$x(t) = 3\cos 100\pi t + 4\sin 200\pi t.$

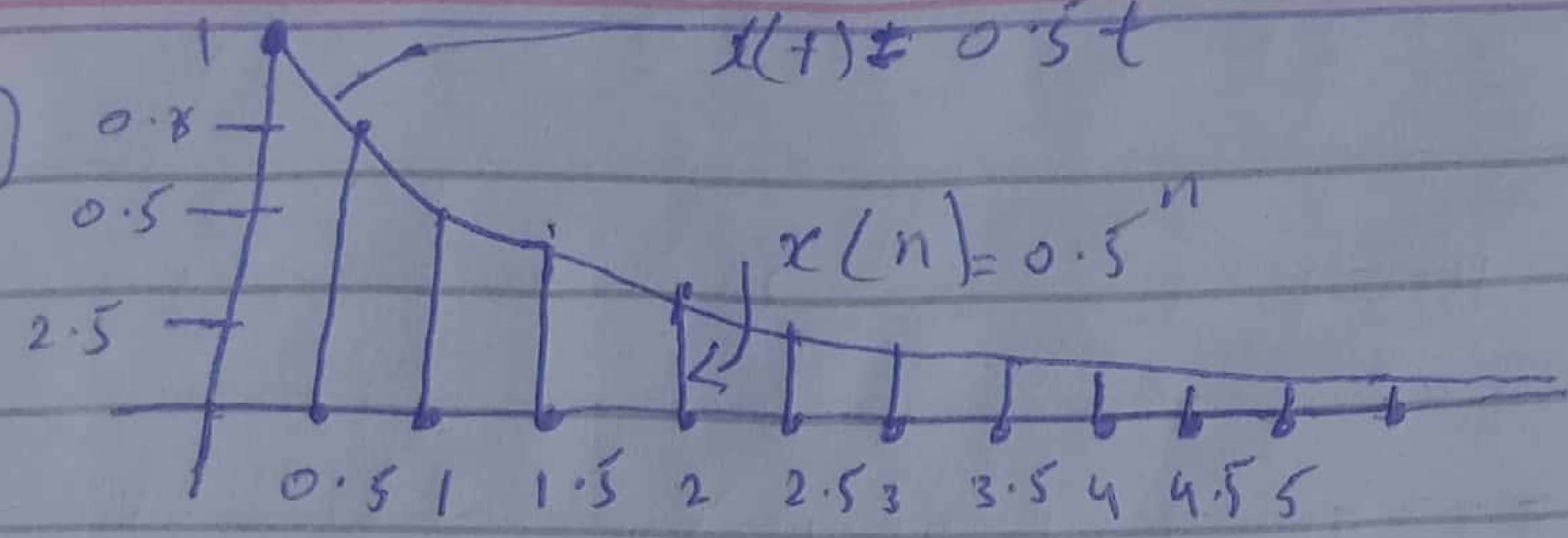
Which is exactly same as original one. (As signal has not changed by sampling).

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n \ngeq 0 \end{cases}$$

$F_s = 2 Hz$

$T_s = \frac{1}{2} = 0.5 \text{ sec}$

(i)



(ii) here $n = 3$ bits per sample

$L = 2^3$ (L is quantization level)

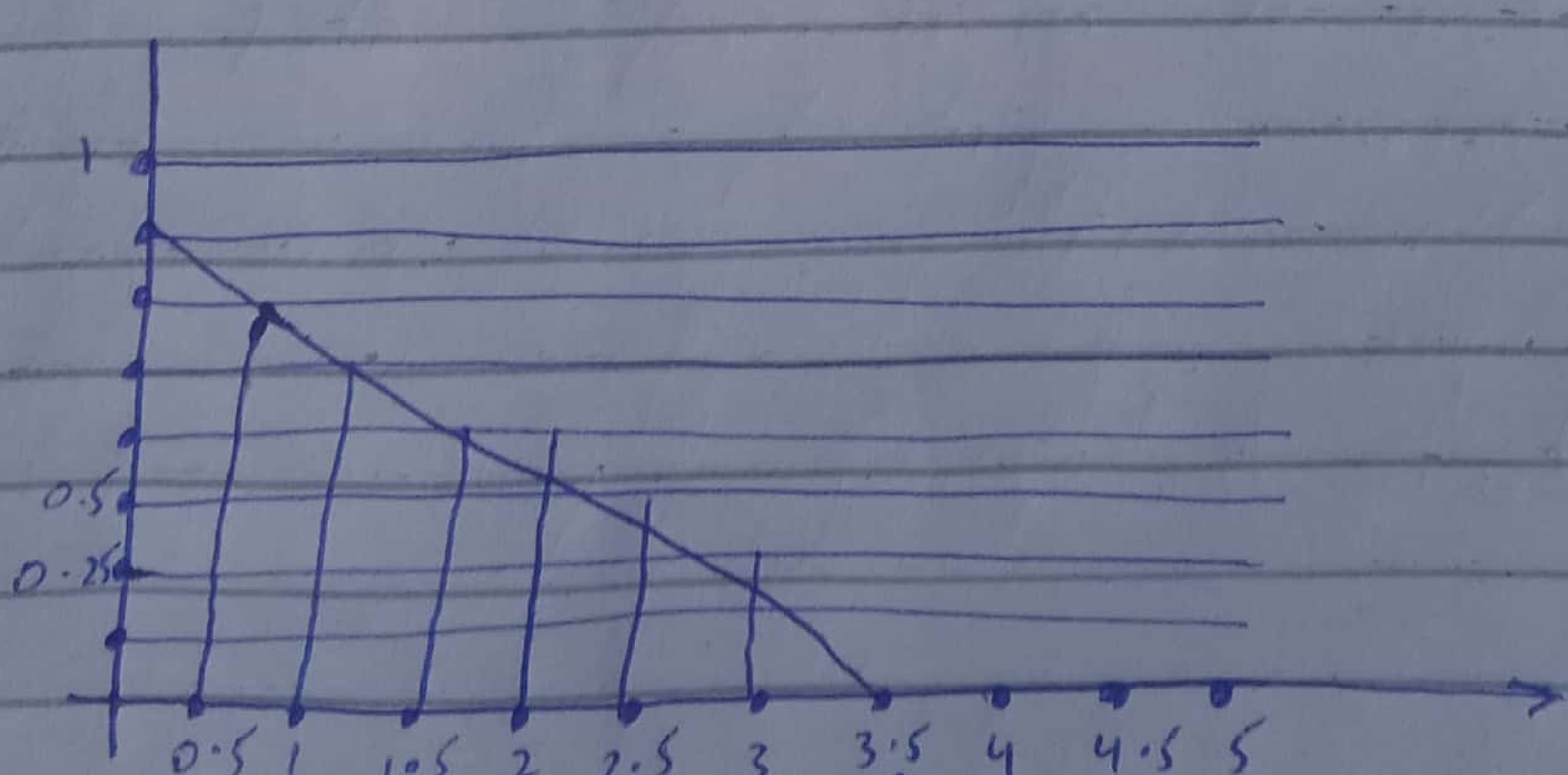
$L = 2^3 = 8$

$\boxed{L = 8 \text{ levels}}$

Quantization resolution / steps size Δ is

$$\Delta = \frac{x_{max} - x_{min}}{L - 1}$$

$$\Delta = \frac{1 - 0}{8 - 1} = \frac{1}{7} = 0.142$$



$x_N(x) = Q[x(n)]$

iii) ⇒ Remaining Part.

n	x(n)	$X_2(n)$	Rounding	error
	1			
0	0.5	1.0	1.0	0.0
1	0.25	0.5	0.5	+0.05
2	0.015625	0.2	0.3	-0.015625
3	0.0625	0.0	0.0	+0.0375
4	0.03155	0.0	0.0	+0.3125
5	0.15625	0.0	0.0	-0.015625
6	0.0078125	0.0	0.0	-0.0078125
7	0.00390625	0.0	0.0	-0.00390625
8	0.001953125	0.0	0.0	-0.001953125
9	0.0009765625	0.0	0.0	-0.0009765625
10	0.707	0.0	0.0	-0.007
0.5	0.3535	0.7	0.7	0.0465
1.5	0.17677	0.3	0.4	0.02323
2.5	0.08838	0.1	0.2	0.01162
3.5		0.0	0.1	-0.4419
4.5	0.04419	0.0	00	

and so on.

DSP Abdul Aziz 13741

Q2 $\Rightarrow$ $x[n] = \{2, \cancel{7}, -2, 3, -4\}$

$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 1, 4\}$

$$\sum_{K=-1}^{4} n(k)$$

$$y[n] = \sum_{K=-1}^{4} n(k)\, h(n-k)$$

$y[0] = \{x(-1) \times h(-(-1))\} + \{x(0)\, h(-1)\} + \{n(1)\, h(0)\}$

* $y[0] = 2 \times 1 = 2 + 0 + (-6) + 3 + (-8) + 0$

~~$y[1] = x(-1) \times h(2)$~~

$$y[1] = \sum_{K=-1}^{4} \{n(k)\, h(1-k)\}$$

$y[1] = x(-1) \times h(2) + x(0) \times h(1) + \cdots$

$= 4 + 1 + (-6)$

* $y[1] = -1$

$y[+2] = 2 + 2 + (-2) + 9 = 11$

$y[-1] = 6 + 0 + = 6$

$y[-2] = 0$

Abdul Aziz 13741

7

$y[3] = 8 + 1(-4) + 3 + (-12) = -4$

$y[4] = 0 + 4 + (2) + 6 - 4 = 4.$

$y[5] = -8 + 3 + 8 = 3$

$y[6] = 12 - 4 = 8$

$y[7] = -16.$

So

$$y[n] = \{6, -\underset{\uparrow}{9}, -1, 11, -4, 4, 3, 8, -16\}.$$

Q.3(i) ⟹ As $X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$

$$= \sum_{-\infty}^{\infty} \left(\tfrac{1}{4}\right)^{n} u(n)$$

$$= \sum_{-\infty}^{1} \left(\tfrac{1}{3}\right)^{-n} z^{-n} + \sum_{0}^{\infty} \left(\tfrac{1}{4}\right)^{n} z^{-n}$$

$$= \sum_{1}^{\infty} \left(\tfrac{1}{3} z\right)^{1/\infty} + \sum_{0}^{\infty} \left(\tfrac{1}{4}\right) z^{-1})^{m}$$

$$= \frac{1}{1 - \frac{1}{3}z} - 1 + \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$= \frac{\left(1 - \frac{1}{4}z^{-1}\right) - \left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right) + \left(1 - \frac{1}{3}z\right)}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

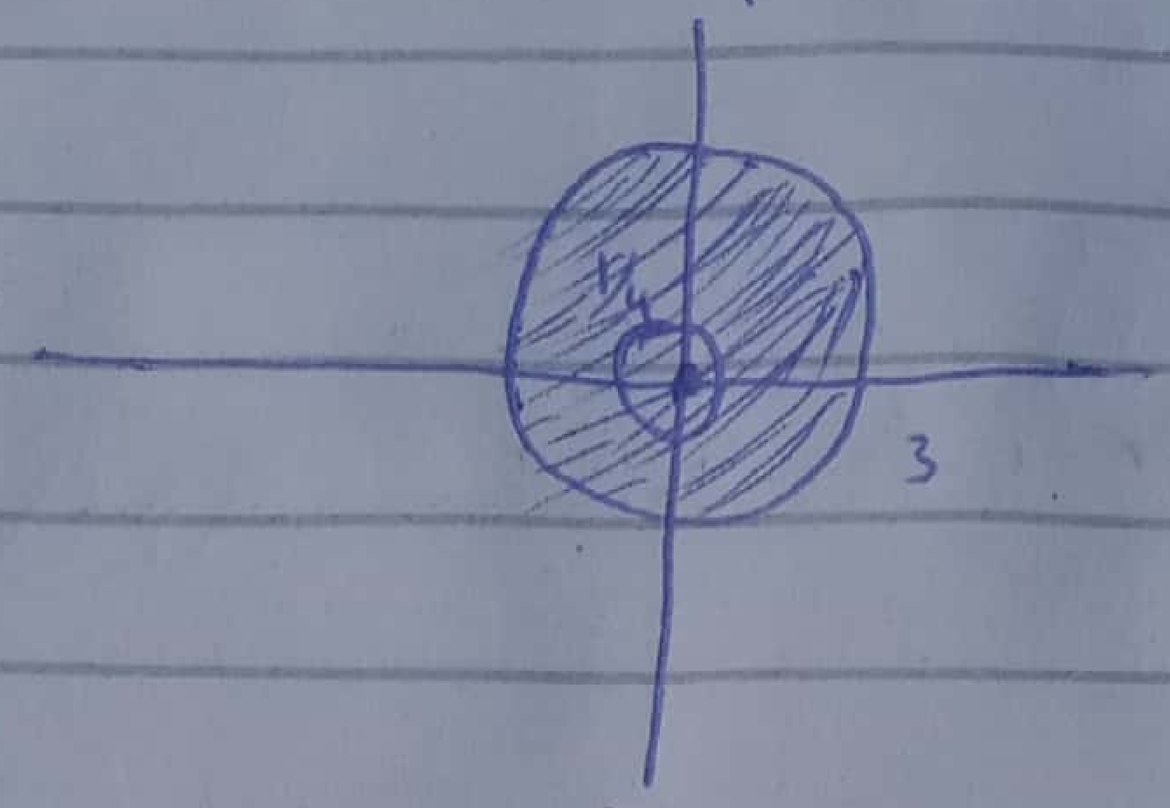
$$= \frac{1 - \frac{1}{4}z^{-1} - \left(1 - \frac{1}{4}z^{-1} - \frac{1}{3}z + \frac{1}{12}\right) + 1 - \frac{1}{3}z}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

~~$X(z) = \frac{11}{12}$~~

$$X(z) = \frac{11/12}{\left(1 - \frac{1}{3}z\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

So

ROC is $\frac{1}{4} < |z| < 3$



(ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n & n > 0 \\ 0 & \text{elsewhere} \end{cases}$

$$X(z) = \sum_{0}^{\infty} \left(\tfrac{1}{2}\right)^n z^{-n} - \sum_{0}^{\infty} 3^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^n - \sum_{n=0}^{\infty} \left(3z^{-1}\right)^n$$

$$X(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{(1 - 3z^{-1})\,(1 - \frac{1}{2} z^{-1})}{(1 - \frac{1}{2} z^{-1})\,(1 - 3z^{-1})}$$

$$= \frac{z^{-1}\left(\frac{1}{2} - 3\right)}{''}$$

$$X(z) = \frac{-5z^{-1}/2}{(1 - \frac{1}{2} z^{-1})\,(1 - 3z^{-1})}$$

So ROC is $|z| = 3$, $|z| > \frac{1}{2}$.

Overall ROC is $|z| > \frac{1}{2}$