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(1)

QNo#1:→

Euler equation.

(1) $x^2 y''' + 2x^2 y' + 2y - 10x + 10/x$.

Solution:-

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2xD^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2 D + 2)y = 10x + 10x^{-1} \quad \text{——— (i)}$$

let $x = e^t \Rightarrow t = \ln x$.

$$xD = D$$

$$x^2D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2)$$

Subtituting into eq(i).

$$\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10^x + 10x^{-1}.$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^3 + 2)y = 10e^t + 10/e^t$$

using synthetic division.

	1	-1	0	2
-1		-1	2	-2
	1	-2	2	$\boxed{0}$

$$\Delta^3 - 2\Delta + 2 = 0$$

Now using quadratic formula.

$a = 1,\ b = -2,\ c = 2$

(P-T-O) ⇒

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{4-8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$= \frac{2 + 2i}{2}$$

$$= \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$D = 1 \pm i$$

since roots are complex:

$$y_c = e^{-x}(c_1 \cos t + c_2 \sin t).$$

now Particular integration.

$$yp = \frac{1}{D^3 - D^3 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^t$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{5\cancel{10}e^t}{\cancel{2}} + \frac{5\cancel{10}e^{-t}}{\cancel{2}}$$

$$= 5e^t + 5e^t$$

(P-T-O)

$= 5e^t + 5e^t$

$y_P = 5e^t + 5e^t$

General solution:→

$y = y_c + y_P$

$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$

Put $e^t = x$ and $t = \ln x$.

$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$

QNo#2:→ $x^3 \frac{d^3y}{dx} + 4x^2 \frac{d^2y}{dx} - 5x \frac{dy}{dx} - 15y = x^4$.

Solution:→ let $d/dx = D$

$x^3 D^3 y + 4x^2 D^2 y - 5x Dy - 15y = x^4$.

$(x^3 D^3 + 4x^2 D^2 - 5xD - 15)y = x^4$.

let $x = e^t \Rightarrow t = \ln x$

$xD = D$.

$x^2 D^2 = D(D-1) = D^2 - D$.

$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

Now substituting:→

$(x^3 D^3 + 4x^2 D^2 - 5xD - 15)y = x^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$

$(D^3 + D^2 - 7D - 15)y = e^{4t}$

(P-T-O) ⇒

Synthetic division :->

$$\begin{array}{c|cccc} & 1 & +1 & -7 & -15 \\ 5 & & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 & \underline{10} \end{array}$$

$$s^2 + 4s + 5 = 0$$

Quadratic Formula.

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$s = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$y_c = e^{s_0}(c_1 \cos t + c_2 \sin t).$$

For $y_p = ?$

$$y_p = \frac{1}{s^3 + s^2 - 7s - 15} \cdot e^{4t}$$

$$= \frac{1}{(4)^3 + (4)^2 - 7(4) - 15} \cdot e^{4t}$$

$$= \frac{1}{64 + 16 - 28 - 15} \cdot e^{4t}$$

$\Rightarrow$

(P - T - O)

$$y_p = \frac{1}{80-43} \cdot e^{4t}$$

$$y_p = \frac{1}{37} \cdot e^{4t}$$

Hence. $y = y_c + y_p$.

$$y = (c_1 \cos t + c_2 \sin t) + \tfrac{1}{37} e^{4t}$$

again put $t = \ln x$ and $x = \ln x$.

$$y = e^{3y}\left(c_1 \cos \ln x + c_2 \sin \log x\right) + \tfrac{1}{37} e^{4x}$$

Ans

QNo3: $\rightarrow$ $x^2 + y'' + 2xy' - 6y = 10x^2$.

Solution: $y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2.$$

$$\left(x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6\right) y = 10x^2.$$

Put $xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$.

$x = e^t$ and $\log x = t$

$$(\Delta^2 - \Delta + 2\Delta - 6) y = 10 e^{2t}$$

The characteristic equation.

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$$\Rightarrow \Delta(\Delta + 3) - 2(\Delta + 3) = 0$$

$$\Rightarrow (\Delta + 3)(\Delta - 2) = 0$$

$$\Delta + 3 = 0, \quad \Delta - 2 = 0$$

(P.T.O) $\Rightarrow$

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$s = 2, \quad s = -3$

Since roots are real and distinct,

For $y_c = ?$

$$y_c = c_1 e^{-3t} + c_2 e^{2t}$$

For $y_p = ?$

$$y_p = \frac{1}{s^2 - s - 6} \cdot 10^{2t}.$$

$$= \frac{10}{s^2 - s - 6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \text{ fails}$$

Now $10 \frac{1}{d/ds(s^2 + s - 6} \cdot e^{2t}$

$$\Rightarrow 10 \frac{t}{2s+1} e^{2t}$$

$$10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$y_p = 2t e^{2t}.$$

General Solution.

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2t e^{2t}$$

$$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2 \text{ — (B)}$$

Put $y(1) = 1$: i.e $x = 1, y = 1$ in (B)

$$1 = c_1 (1)^{-3} + c_2 (1)^2 + 2\log(1)$$

$$1 = c_1 + c_2 \longrightarrow \text{(C)}$$

Now differentiate eq (B) w.r.t x.

$$y_1 = -3c_1 x^{-4} + 2c_2 x + \tfrac{2}{x}(x^2) + 4x\log x.$$

Now put $y_1(1) = -6$ i.e $y' = -6$ and $x = -6$.

(P.T.O)

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$-6 = -3c_1 + 2c_2 + 2$

$\Rightarrow -6 = -3c_1 + 2c_2 + 2$

$-6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2$ —— (1)

xing eq (1) with (2) and -ing from (17).

$$2c_1 + \cancel{2c_2} = 2$$
$$\mp 3c_1 \pm \cancel{2c_2} = \mp 8$$

$$5c_1 = 10$$

$c_1 = \frac{\cancel{10}^2}{\cancel{5}_1}$ $\boxed{c_1 = 2}$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8 + 6$

$2c_2 = -2$

$c_2 = -\cancel{2}/\cancel{2}\ 1$

$\boxed{c_2 = -1}$

Now put the value of c_1 and c_2 in eq (B).

$y = 2x^{-3} - x^2 + 2\ln x\, x(x^2)$

$\boxed{y = \frac{2}{x^3} - x^2 + 2x^2 \log x}$ ← Ans

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Q No # 4 → $x^2y'' + 7xy' + 5y = x^5$ (8)

$y(10) = 2$ and $y'(1) = 2$.

Solution:→

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5 \quad \text{—(A)}$$

Put $xD = \Delta \Rightarrow x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$.

$x = e^t \Rightarrow \log x = t$ in eq (A)

$\Rightarrow (\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$

$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}$

By Quadratic Formula.

$$\Delta = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2} \Rightarrow \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{6 \pm \sqrt{4^2}}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$\Delta = -3 \pm 2$ since roots are real and distinct.

$y_c = c_1e^{5t} + c_2e^{t}$

For $y_p = ?$

$$y_p = \frac{1}{\Delta^2 + 6\Delta + 5} \cdot e^{5t}$$

$$= \frac{1}{(5)^2 + 6(5) + 5} \cdot e^{5t}$$

(P—T—O) ⇒

$= \frac{1}{60} e^{5t}$

Now General Solution is

$y = y_c + y_p$

$y = c_1 e^{5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}$.

$y = c_1 x^{-5} + c_2 x^{-1} + \frac{1}{60} x^5 \longrightarrow (B)$

$x = 0$ put in this equation.

No in eq (B) $e^0 = 1$,

Put $y(0) = 2$ i-e $y = 2$ and $x = 2$.

$2 = c_1 (2)^{-5} + c_2 (2)^{-1} + \frac{1}{60} (2)^5$.

$2 = -32 c_1 - 2 c_2 + \frac{1}{\cancel{60}_{15}} (\cancel{32}^{8})$

$2 = -32 c_1 - 2 c_2 + \frac{8}{15}$

$2 - \frac{8}{15} = -32 c_1 - 2 c_2$.

$\frac{22}{15} = -32 c_1 - 2 c_2 \longrightarrow (C)$

Now differentiate eq (B) w.r.t (x).

$y' = -5 c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4 \longrightarrow$

Put $y'(1) = 2$ i-e $y' = 2$ and $x = 2$ in above equation.

$2 = -5 c_1 (-64) - c_2 (4) + \frac{1}{12} (16)$.

$2 = 320 c_1 + 4 c_2 + 4/3$

$\Rightarrow 2 - 4/3 = 320 c_1 + 4 c_2$

$\Rightarrow 2/3 = 320 c_1 - 4 c_2 \longrightarrow (D)$.

xing eq (C) with 2 and then -ing eq (C) from (D).

$-\frac{44}{15} = 64 c_1 + 4 c_2$.

$-\frac{44}{15} = 64 c_1 + 4 c_1$.

$\Rightarrow$

P-T-O

$$+\frac{2}{3} = +320c_1 + 4c_2$$

$$\frac{34}{15} = -256c_1$$

$$c_1 = \frac{34}{15} \times 256$$

$$\boxed{c_1 = 580}$$

Put the value of c_1 in eq (c)

$$\Rightarrow \frac{22}{15} = -32(580) - 2c_2$$

$$\Rightarrow \frac{22}{15} + 18560 = -2c_2$$

$$\Rightarrow \frac{18561}{-2} = c_2$$

$$\boxed{-9280 = c_2}$$

Now put the value of c_1 and c_2 in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$$

Ans

QNo#5:→ $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$.

solution:→ $(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$.

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right)y = x^2.$$

$$\left[(x+1)^2 D^2 - 3(x+1)D + 4\right]y = x^2 \longrightarrow (A)$$

Put $(x+1)D = D \Rightarrow (x+1)^2 D^2 = D(D-1) = D^2 - D$

$x = e^t$ in eq (A).

$$\Rightarrow [D^2 - D - 3D + 4]y = e^{2t}$$

$$\Rightarrow [D^2 - 4D + 4)y = e^{2t}$$

$\Rightarrow$

(P-T-O)

$(D^2 - 4D + 4)x = e^{2t}$.

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Ye we find the roots.

$D^2 - 4D + 4 = 0$

$D^2 - 2D - 2D + 4 = 0$

$D(D-2) - 2(D-2) = 0$

$D - 2 = 0, \; D = 2.$

$D - 2 = 0, \; D = 2.$

So the roots are real repeated.

The General Solution

$y = (c_1 + c_2 x)^{2x}$

$y = (c_1 + c_3 x)^{2x}$

For $yp = ?$

$(2)^2 - 4(2) + 4 \Rightarrow 0$

$yp = \frac{1}{D^2 - 4D + 4}$

$yp = \frac{2}{2D - 4} e^{2t}$

if we put "2"

$2D - 4 \Rightarrow 2(2) - 4 = 0$

we take again derivative

$yp = 2/2 \cdot e^{2t}$

$$\boxed{y = (c_1 + c_2 x)^{2t} + e^{2t}}$$

Ans

General Solution

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