Name:- Sarwat Ali
ID No:- 16041
Subject:- liner Algebra
Program:- BSCS
Final assignment
Semester 2

Question:-1

Consider the following vector R^3:

$$v_1 = \begin{pmatrix} ID1 \\ ID2 \\ ID3 \end{pmatrix}, v_2 = \begin{pmatrix} ID2 \\ ID3 \\ ID4 \end{pmatrix}, v_3 \begin{pmatrix} ID3 \\ ID4 \\ ID5 \end{pmatrix}$$

Solve the System and find if these vector are Linearly independent?

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$$V_1 = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix} + V_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} + V_3 \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 0 \\ 6 & 0 & 4 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now we apply Row operation

Step 1

we consider augment Matrix.

$$= \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 6 & 0 & 4 & 0 \\ 0 & 4 & 1 & 0 \end{array}\right]$$

Now first of all we find the leading one.

$$R_2 \leftrightarrow R_1 \left(\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 6 & 0 & 4 & 6 \end{array}\right)$$

$$R_3 - 6R_1 = \left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & -36 & 4 & 0 \end{array}\right]$$

$$R_3 / 4 \left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & -9 & 1 & 0 \end{array}\right]$$

$$R_3 / 9 \left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 1 & -1/9 & 0 \end{array}\right]$$

$$R_1 - 6R_3 \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 1 & -1/9 & 0 \end{array}\right]$$

$$R_2 - 3R_3 \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 4/3 & 0 \\ 0 & 1 & -1/9 & 0 \end{array}\right]$$

$$R_3 - R_2 \left[\begin{array}{ccc} 1 & 0 & 2/3 \\ 0 & 1 & 4/3 \\ 0 & 0 & -13/9 \end{array}\right]$$

$$R_3 \times -\frac{9}{3} \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 4/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$a_1 + 2/3\ a_3 = 0 \Rightarrow -2/3$

$a_2 + 4/3\ a_3 = 0 \Rightarrow -5/8$

$a_3 = 0$

$a(-2/3, -5/8, 0)$ Ans

Question:- 2

Part (a)

Total cost Per unit of product x = material Per unit of x + labour Per unit of x + overhead

Putting the values

Rs = 450 + 250 + 150

= Rs 850

total cost Per unit of Product y = material Per unit of y + labour Per unit of y + overhead

Per unit of y
Rs 400 + 350 + 150
Rs 900
Find the total cost
Production vector is given
as $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Price of total units of
Product x = no of unit
of x × price per unit
of x
Price of total unit of
Product of total unit is
x = 1000 × 250
Rs 850000
Price of total unit
of product y = no. of
unit of y × Price unit
of product y
~~total~~ Co Price of total
unit of
y = 500 × 900
total cost = Price of

Product x

$= 85000 \times 45000$

$= 1300000$ Ans

b) Explain the linear transformation properities with help of above is an example.

- $T(U+V) = T(U) + T(V)$

Solution:

$$T(U+V) = T(U) + T(V)$$

$$\vec{U} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \vec{V} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$T\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 + b_1 - (a_2 + b_2) \\ a_1 + b_1 + (a_2 + b_2) \\ 2a_1 + 2b_1 \end{bmatrix} = T(U+V)$$

$$T(U) + T(V)$$

$$T\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + T\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a_1 \end{bmatrix} + \begin{bmatrix} b_1 - b_2 \\ b_1 + b_2 \\ 2b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 + b_1 - (a_2 + b_2) \\ a_1 + b_1 + (a_2 + b_2) \\ 2a_1 + 2b_1 \end{bmatrix} = T(U) + T(V)$$

proved
$T(u+v) = T(u) + T(v)$

- $T(cu) = cT(u)$

If T is a linear transformation and $\vec{u}$ and $\vec{v}$ are vector and c is a scalar, then the following hold.

$$\vec{u} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$T\left(c\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} c \cdot a_1 \\ c \cdot a_2 \end{bmatrix}\right)$$

$$\begin{bmatrix} ca_1 - ca_2 \\ ca_1 + ca_2 \\ 2 \cdot c \cdot a_1 \end{bmatrix}$$

$$c \cdot T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right)$$

$$c \cdot \begin{bmatrix} (a_1 - a_2) \\ (a_1 + a_2) \\ (2a_1) \end{bmatrix}$$

$$= \begin{bmatrix} c(a_1 - a_2) \\ c(a_1 + a_2) \\ c(2a_1) \end{bmatrix} = c \cdot T(\vec{u})$$

Proved
$T(cu) = cT(u)$

Question:-3

What are the four main things we need to define for a vector space?
which of the following is a vector space over R?
For those that are not vector spaces, modify one part of the definition to make it into vector space
a. V = {2X2 matrix entires in R}, usual matrix addition and $k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & b \\ kc & d \end{bmatrix}$ for $k \in R$

Note that V is not closed under addition for a, b, c, d $\in R$ we have $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix}$ but

$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} \in V$ and

$K \in R$

we have

$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \oplus \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} =$

$\begin{pmatrix} 1 & a+c \\ & \end{pmatrix} \in R$

$k + \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix} = \begin{pmatrix} 1 & ka \\ kb & 1 \end{pmatrix} =$

$\in V$ Answer.

b) V = Polynomials with complex coefficients of degree ≤ 3, with usual addition and scalar multiplication of polynomials.

we conclude that V is not a vector space. with the given operation

The set V of all

Matrices of the form $\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}$ where $a, b \in R$ over R with addition and Scalar multiplication defination By

$$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+c \\ b+d & 1 \end{pmatrix}$$

$$k \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & ka \\ kb & 1 \end{pmatrix}$$ we claim that

V is indeed a vector space with the given operations Note first that V is closed under the addition and Scalar multiplication for $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ So this is addition and Scalar multiplication of polynomials.

Question:- 4

let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2x2 matrix

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$M = [(1)(1) - (0)(0)$

$M = 1$

For inverse $M^{-1} = \frac{1}{\text{Deter}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$M^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Ans.

b) write down all 2x2 bit matrixes with determinant 1. (Remember bit are either 0 or 1 and 1+1 = 0 in bits)?

let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(A) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

det $B = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$\det(B) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$ Answer!

c) write down 2x2 bit matrices with determinant 0:

let $C = \begin{pmatrix} 1 & 1+1 \\ 1 & 1 \end{pmatrix}$

$\det(C) \begin{vmatrix} 1 & 1+1 \\ 1 & 1 \end{vmatrix}$

$\begin{vmatrix} 1 & 1+1 \\ 1 & 1 \end{vmatrix}$

$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$|1-0| = 0$ Answer

d) Compute de A for below 3x3 matrix

$$A = \begin{bmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{bmatrix}$$

Solution:-

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$$A \begin{pmatrix} 1 & 1 & 1 \\ 6 & 0 & 6 \\ 4 & 1 & 1 \end{pmatrix}$$

$$|A| = 1\begin{vmatrix} 0 & 6 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 6 & 6 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 6 & 6 \\ 4 & 1 \end{vmatrix}$$

=

$1(0-6)-1(6-24)+1(6-0)$

$1(-6)-1(-8)+1(6)$

$-6+18+6$

18 Answer.