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Mid term Examination

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(i) The order of matrix A is mxp & order of B is pxn. Then the order of matrix AB is?

A1 (i) order of matrix AB = mxn

Q1 (ii) The number of non-zero rows in an Echdon form?

A1 (ii) One

Q1 iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is singular matrix then a = ?

Sol

$$|A| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$(a \times 1) - (2 \times 4) = 0$$
$$a - 8 = 0$$
$$a = 8$$

Q1 (iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = 2i(-i) - i \times i$$

$$= -2i - i^2$$

$$= -2(-1) - (-1)$$

$$= 2 + 1 = 3$$

Q1 (v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

A1 (v) Scalar Matrix

Q1 (vi) Solve $\frac{dy}{dx} + 2xy = y$?

Sol

$$\frac{dy}{dx} + 2xy = y.$$

Separating variables

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{y} = (1 - 2x)\,dx$$

$$\Rightarrow \int \frac{1}{y} \cdot dy = \int 1\,dx - \int 2x\,dx$$

$$\Rightarrow \ln y = x - \frac{2x^2}{2} + C$$

$$= x^2 + C \quad \text{Ans}$$

Q1 Page 3

Q1 vii The order & degree of differential equation.

order = 1
Degree = 3

Q1 viii The order & degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

order = 2
Degree = 1

Q1 PG-4

Q1 ix The D.E $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0)=5$

A1 ix Sol

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$\int 2\,dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + C$$

$$y = \frac{2x^2}{2\times 2} + \frac{3x}{2} - \frac{yx^3}{3\times 2} + C$$

$$y = \frac{x^2}{2} + \frac{3}{2}x - \frac{x^3y}{6} + C$$

put $x=0$, $y=5$

$$5 = 0 + 0 - 0 + C$$

$C=5$

then $y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$

Ans

Q1 (a) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

A1 (a) Sol

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= bc^2 - cb^2 - a(c^2-b^2) + a^2(c-b)$$

$$= bc.(c-b) - a(c-b) + a^2(c-b)$$

$$= (c-b)(bc - a + a^2) \quad \text{Ans}$$

Q2 (a)

Q Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Sol

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by Row 1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

Taking common abc

$$abc\,(bc^2 - b^2c - ac^3 + a^2c + ab^2 - a^2b)$$

$$= abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right]$$ Ans

Q2 Part B

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristick equal to $|A - \lambda I| = 0$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now taking determinant

$$|A - \lambda I| = 0$$

Q2(b) Page 2

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow (2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \times$$

$$\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 1 \times \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow \text{eq, B}$$

Now

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow (3-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - \times(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} +\!\!+$$

$$-1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

Q2
(b)

$$= (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1\left[(-1)(2-\lambda) - (-1)(-1)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= -\lambda^3 + 8\lambda^2 - 18\lambda + 8 \rightarrow \text{eq (a)}$$

Now

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= -\lambda^2 + 6\lambda - 8 \rightarrow \text{eq (b)}$$

$$\Rightarrow \quad -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$-\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \quad \rightarrow \text{eq } C$$

put a, b, c in eq B

$$\Rightarrow (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right] - \lambda^2+6\lambda-8-\lambda^2$$
$$+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda$$
$$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda = 0$$

Q2(b)

By synthatic division we get

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16) = 0$$

$$(\lambda) = (0)$$

$$\lambda - 2 = 0 \Rightarrow \lambda = 2$$

$$\lambda^2 - 8\lambda + 16 = 0$$

By factorization Method

$$\lambda^2 - 8\lambda + 16 = 0$$
$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda-4) - 4(\lambda-4) = 0$$
$$(\lambda-4)(\lambda-4)$$

$$\lambda = 4 \quad , \lambda = 4$$

$$\lambda_1 = 0$$
$$\lambda_2 = 2$$
$$\lambda_3 = 4$$
$$\lambda_4 = 4$$

Ans

Haris Ali Shah 7530

Q. NO 3 page :- 1

Q) The rate of change in the form of differential eq is given by

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

Find the General solution at $x = 2$, $y = 6$

Sol

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

÷ing b.s by $2xy\,dx$

$$\frac{(x^2+3y^2)\,dx}{2xy\,dx} = \frac{2xy\,dy}{2xy\,dx}$$

$$\frac{(x^2+3y^2)\,dx}{2xy\,dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow \text{eq 1}$$

Let $y = vx$, $dy = v\,dx + x\,dy$

"Dividing dy by dx

Q3 Page 2

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow \text{eq } 2$$

put eq 2 in eq 1

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + \frac{3vx}{x}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

"x ing" & "÷ing" b.s by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying b.s by $\frac{dx}{dv}$ we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Q3 Page 3

xing b.s by $\frac{v}{x(1+v^2)}$ we get

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Taking "$\int$" on b.s

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + C$$

$$\ln|1+v^2| = \ln x + \ln c$$

Taking "e" on b.s

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

Now putting $v = y/x$

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

Q3 Page 4

$$x^2 + y^2 = x^3 c \rightarrow \text{eq } (3)$$

put $x = 2$ & $y = 6$ in eq (3)

$$4 + 36 = 8c$$
$$c = \frac{40}{8}$$
$$c = 5 \rightarrow \text{put it in eq 3}$$

then,

$$x^2 + y^2 = x^3 \times 5$$
$$y^2 = 5x^3 - x^2$$
$$y^2 = x^2(5x - 1)$$

Taking square root on b.s

$$\sqrt{y^2} = \sqrt{x^2(5x-1)}$$

then $y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

$$y = \pm x\sqrt{5x-1}$$