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Subject # Applied Mathematic II

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Q No 1 Ans(a) $\int x^2.e^x\,dx$

Sol: $\int x^2 e^x\,dx$

Now by integration by parts, we get

let; $u = x^2$

$$\frac{du}{dx} = 2x$$

and $v = e^x$

$$\frac{dv}{dx} = e^x$$

As; $\Rightarrow \int u\,\frac{dv}{dx}\,dx = uv - \int v\,\frac{du}{dx}\,dx$

and;

Now; $\Rightarrow \int x^2.e^x\,dx = x^2e^x - 2\int xe^x\,dx$

$$= x^2e^x - 2\int xe^x\,dx \longrightarrow (i)$$

let $u = x$; $\frac{dv}{dx} = e^x$

$\frac{du}{dx} = 1$; $v = e^x$

Now; $\int xe^x = xe^x - \int e^x(1)\,dx$

$$= xe^x - e^x + C \longrightarrow (ii)$$

Now put eq (ii) in (i)

$$\Rightarrow \int x^2 \cdot e^x \, dx = x^2 e^x - 2(xe^x - e^x + C)$$

$$\Rightarrow \boxed{\int x^2 \cdot e^x \, dx = x^2 \cdot e^x - 2xe^x + 2e^x - C}$$ Ans:-

Q)
No 1

Ans: (b)

$\int (1+3t)t^3\,dt$

Sol:. $\int (1+3t).t^3\,dt = \int (t^3+3t^4)$

$\Rightarrow \int t^3 dt + \int 3t^4 dt$

$= \frac{t^4}{4} + \frac{3t^{4+1}}{4+1} + c$

$= \frac{t^4}{4} + \frac{3t^5}{5} + c$

Now check Through differention i.e.

$\Rightarrow \frac{d}{dt}\left[\frac{t^4}{4} + \frac{3t^5}{5} + c\right] = \frac{1}{\cancel{4}} \times \cancel{4}t^{4-1} + \frac{3}{\cancel{5}} \times \cancel{5}t^{5-1} - 0$

$= t^3 + 3t^4$

$= (1+3t)t^3$

Ans:.(C) $\int (e^x - e^3)\,dx$

Sol:. $\int (e^x - e^3)\,dx$

$\Rightarrow \int (e^x - e^3)\,dx = \int e^x dx - \int e^3 dx$

$\Rightarrow \int (e^x - e^3)\,dx = e^x - \frac{e^3}{3} + c$

$\Rightarrow \boxed{\int (e^x - e^3)\,dx = \left(e^x - \frac{1}{3}e^3 + c\right)}$ Answer

Q2 Find the taylor Series for $f(x) = e^{-6x}$ about $x_0 = -4$

Ans:-

Solution:- As $\Rightarrow f(x) = e^{-6x}$; $\Rightarrow f(-4) = e^{24}$

$\Rightarrow f'(x) = -6e^{-6x}$; $\Rightarrow f'(-4) = -6e^{24}$

$\Rightarrow f''(x) = 36e^{-6x}$; $\Rightarrow f''(-4) = 36e^{24}$

$\Rightarrow f'''(x) = -216e^{-6x}$; $\Rightarrow f'''(-4) = -216e^{24}$

$\Rightarrow f^{iv}(x) = 1296e^{-6x}$; $\Rightarrow f^{iv}(-4) = 1296e^{24}$

and So. on.

So by Taylor's infinite Expansion at $x_0 = -4$

$$\Rightarrow f(x) = x_0 + (x-x_0)f'(x) + \frac{(x-x_0)^2}{2!}f''(x) + \frac{(x-x_0)^3}{3!}f'''(x)$$
$$+ \frac{(x-x_0)^4}{4!}f^{iv}(x) + \cdots\cdots$$

Now ;

$$\Rightarrow e^{-6x} = (e^{24}) + (x+4)(-6e^{-6x}) + \frac{(x-4)^2}{2!}(36e^{-6x}) +$$
$$\frac{(x-4)^3}{3!}(-216e^{-6x}) + \frac{(x-4)^4}{4!}(1296e^{-6x}) + \cdots$$

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$$\Rightarrow e^{-6x} = e^{24} - (x+4)\,6e^{24} + \frac{(x-4)^2}{\cancel{2}\times 1}\,\overset{18}{\cancel{36}}\,e^{24} - \frac{(x-4)^3}{\cancel{3}\times\cancel{2}\times 1}\,\overset{36}{\cancel{\overset{72}{\cancel{216}}}}\,e^{24} + \frac{(x-4)^4}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}\,\overset{54}{\cancel{\overset{108}{\cancel{\overset{324}{\cancel{1296}}}}}}\,e^{24} + \cdots$$

$$\boxed{\Rightarrow e^{-6x} = e^{24} - (x+4)\,6e^{24} + (x-4)^2\,18\,e^{24} - (x-4)^3\,36\,e^{24} + (x-4)^4\,54\,e^{24} + \cdots}$$

Answer

Q.No 3 (a) $F(y) = x \cdot \sin x$

Sol:- $F(y) = x \cdot \sin x$

Now; differentiate w.r.t "x"

$$F(y)' = \frac{d}{dx}(x \cdot \sin x)$$

Now; By product Rule.

$$\Rightarrow F(y)' = x \cdot \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x)$$

$$= x \cdot (\cos x) + \sin x\,(1)$$

$$\boxed{F(y)' = x \cdot \cos x + \sin x}$$ **Answer**

(b) $F(y) = x^2 \cdot \cos x$

Sol:- $F(y) = x^2 \cdot \cos x$

Differentiate w.r.t "x" B. Side.

Applying product Rule.

$$\Rightarrow F(y)' = \frac{d}{d}(x^2 \cdot \cos x)$$

$$\Rightarrow F(y)' = \left[x^2 \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(x^2)\right]$$

$$\Rightarrow F(y)' = \left[x^2(-\sin x) + \cos x . (2x)\right]$$

$$\Rightarrow \boxed{F(y)' = -x^2 . \sin x + \cos x . 2x}$$ **Ans:-**

(C) $F(t) = Z.(2Z-2)^2$

<u>Solution:</u> $F(t) = Z.(2Z-2)^2$

Now differentiate Both side w.r.t "Z"

$$\Rightarrow F(t)' = \frac{d}{dZ}\left(Z.(2Z-2)^2\right)$$

Now Applying product Rule.

$$\Rightarrow F(t)' = Z . \frac{d}{dZ}(2Z-2)^2 + (2Z-2)^2 . \frac{dz}{dz}$$

$$= Z . 2(2Z-2)\frac{d}{dx}(2Z-2) + (2Z-2)^2(1)$$

$$= Z . 2(2Z-2)(2) + (2Z-2)^2$$

$$\Rightarrow \boxed{F(t)' = 4Z(2Z-2) + (2Z-2)^2}$$ **Answer**

End