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 Section B  
 Subject name opration research

Name  
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 Class  
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 B

Q: 1 There are total of 5 machines and five employment are to be allocated and the related cost network is as per the following. Locate the best possible task.

JOBS	MACHINES				
	A	B	C	D	E
1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	12	23	21
5	9	5	7	6	10

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Solving The problem  
by Hungarian Method

Balanced Problem

Phase-1:-

= Row And Column Reduction

Step-1:- Subtract the minimum  
value of each row from the  
entries of the row -

Step-2:- Subtract the minimum  
value of each column  
from the entries of that column  
operator

						row minim	
	1	6	12	3	11	15	3
Jobs		<del>4</del>	<del>8</del>	<del>7</del>			
	2	4	2	7	1	10	1
	3	8	11	10	7	11	7
	4	16	19	12	23	21	16
	5	9	5	7	6	10	5
	6						

Subtracting values from row  
minimum:-

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1	6-3	12-3	3-3	11-3	15-3
2	4-1	2-1	7-1	11-1	10-1
Jobs	3	8-7	11-7	10-7	7-7 11-7
4	16-16	19-16	122-7	23-16	121-16
5	9-5	5-5	7-5	6-5	10-5

Job	1	3	9	0	8	12
2	3	1	6	0	9	
3	1	4	3	0	2	
4	0	3	11	5	7	5
5	4	0	2	1	5	
	0	0	0	0	0	4

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1	3	9	0	8	8
2	3	1	6	0	5
3	1	4	3	0	0
4	0	3	10	6	7
5	4	0	2	1	11



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Step-1: After Scanning the last column  
Check whether all the zeros are  
Covered with lines.

	Operator					
1	3	9	0	8	8	Row Scanning
2	3	1	6	0	5	
3	1	4	3	0	0	Column Scanning
4	0	3	106	7	5	
5	4	0	2	1	5	

Step-2: - Checked whether the no. of  
sequences marked is equal to No. of  
rows if yes then go to 5-  
otherwise go to Step-3

Step-3: - Identify the minimum value  
of undetected cells value

$$\text{Column} = \text{row}$$

$$5 = 5$$

Solution is optimum.

Step-a add the minimum  
undetected cells values at interaction  
point of the present



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Summary:-

Solution is optimum and  
Feasible job Machine operator

	Job	Time
1	(3) C	12
2	(4) D	1
3	(5) E	11
4	(1) A	18
5	(2) B	5

45 hour

$$\textcircled{1}: 2 \quad \min Z = 2x_1 + 3x_2$$

$$\text{St } \frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1, x_2 \geq 0$$

Solution:-

By Big-M method:-

Convert the problem of minimization to maximization problem by multiplying -ve sign to the right side of the objective function.



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$$\text{Max } Z = -12x_1 - 20x_2$$

$$\text{Max } Z = -2x_1 - 3x_2$$

Step-1<sup>o</sup>:- Convert the system of inequality to equality using Slack variable, Artificial Variable and surplus variable.

$$\frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4$$

becomes:

$$\frac{1}{2}x_1 + \frac{1}{4}x_2 + S_1 = 0$$

Similarly:

$$x_1 + 3x_2 - S_2 + a_1 = 20$$

also

$$x_1 + x_2 + a_1 = 10$$

Step-2<sup>o</sup>:- Set the objective function equal to zero

$$\begin{aligned} \text{Max } Z &= -2x_1 - 3x_2 - M A_1 - M A_2 \\ &= Z + 2x_1 + 3x_2 + M A_1 + M A_2 \\ &= 2x_1 + 3x_2 + M A_1 + M A_2 + Z \end{aligned}$$

Step-3<sup>o</sup>:- Create a Simple table

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	$x_1$	$x_2$	$s_1$	$s_2$	$A_1$	$A_2$	$Z$	
$R_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$R_2$	1	3	0	-1	1	0	0	20
$R_3$	1	1	0	0	0	1	0	10
$Z$	2	3	0	0	$m$	$m$	1	0

In the above table  $A_1$  and  $A_2$  both are basic variable and having coefficient of  $m$  in the objective function. But according to the definition of basic variable it should be zero.

Now we have to manipulate this row in order to get 0 instead of  $m$ .

$$R_4 + (-m)R_3$$

	$x_1$	$x_2$	$s_1$	$s_2$	$A_1$	$A_2$	$Z$	
$R_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$R_2$	1	3	0	-1	1	0	0	20
$R_3$	1	1	0	0	0	1	0	10
$Z$	$2-m$	$3-m$	0	0	$m$	0	1	$-10m$



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also

$$R_2 + (-m)R_1$$

	$x_1$	$x_2$	$S_1$	$S_2$	$A_1$	$A_2$	$b$
$R_1$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	4
$R_2$	1	3	0	-1	1	0	20
	1	1	0	0	0	1	10
	$2-2m$	$3-m$	0	$m$	0	0	$-30m$

Step-4:- Select the pivot column

$x_1$	$x_2$	$S_1$	$S_2$	$A_1$	$A_2$	$b$
$\frac{1}{4}$	1/2	1	0	0	0	4
1	3	0	-1	1	0	20
1	1	0	0	0	1	10
$2-2m$	$3-m$	0	$m$	0	0	$-30m$



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Step-5<sup>o</sup> - Select The Pivot row

$x_1$	$x_2$	$S_1$	$S_2$	$A_1$	$A_2$	$b$
1/2	1/4	1	0	0	0	4/4 = 0
1	3	0	-1	1	0	20/3 = 6.6
1	3	0	-1	1	0	10 = 10
2-2m	3-4m	0	m	0	0	-30

Step-6: Select the pivot which is the entry in the pivot column and pivot row.

$x_1$	$x_2$	<del><math>S_1</math></del>	$S_2$	$A_1$	$A_2$	$b$
1/2	1/4	1	0	0	0	4
1	3	0	-1	1	0	20
1	3	0	-1	1	0	10
2-2m	3-4m	0	m	0	0	-30

Step-7<sup>o</sup>: Perform row operation to make the pivot equal to 1 and the remaining element in the pivot column and pivot row = 0



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$R_3 + (-1)R_2$

	$x_1$	$x_2$	$s_1$	$s_2$	$A_1$	$A_2$	$Z$	
$R_1$ $s_1$	$5/3$	$0$	$4$	$1/3$	$-1/3$	$0$	$0$	$28/3$
$R_2$ $s_2$	$1/3$	$1$	$0$	$-1/3$	$1/3$	$0$	$0$	$20/3$
$R_3$ $s_3$	$2/3$	$0$	$0$	$1/3$	$-1/3$	$1$	$0$	$10/3$
$Z$	$2-2m$	$-3-4m$	$0$	$m$	$0$	$0$	$1$	$-3$

$R_4 + (3+4m)R_2$

	$x_1$	$x_2$	$s_1$	$s_2$	$A_1$	$A_2$	$Z$	
$x_2$	$5/3$	$0$	$4$	$1/3$	$-1/3$	$0$	$0$	$28/3$
$s_1$	$1/3$	$1$	$0$	$-1/3$	$1/3$	$0$	$0$	$20/3$
$a_2$	$2/3$	$0$	$0$	$1/3$	$-1/3$	$1$	$0$	$10/3$
$Z$	$1+2m$	$0$	$0$	$1-3m$	$-1+4m$	$0$	$1$	$3$

$$\int \cdot S \left\{ \begin{array}{l} x_2 = \frac{28}{3} \\ s_1 = \frac{20}{3} \\ a_2 = \frac{10}{3} \\ Z = -10 + 50m \end{array} \right\} \text{Ans}$$



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Q.3: Use Vogel's Approximation Method to obtain the initial feasible solution of

Origin	Destination				Supply
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Supply					
4					
Demand	60	40	30	110	240



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Demand = Supply  
Balanced.

	Supply				Row difference.			
	1	2	3	4				
1	X 20	40 22	X 17	80 4	80 170	13	(13)	-
2	10 24	X 37	30 9	30 7	70	2	2	(17)
3	50 32	X 37	X 20	15 15	50	5	5	17
4								

  

Demand	660	460	360	1160
	4	(15)	8	3
	4	-	8	3
Column difference	8	-	(11)	8
	8	-	-	8

  

The initial feasible Solution:-

$$\begin{aligned}
 & 880 \quad 320 \quad 240 \\
 & (40 \times 22) + (80 \times 4) + (10 \times 24) + \\
 & 270 \quad 210 \quad 1600 \\
 & (30 \times 9) + (30 \times 7) + (50 \times 32)
 \end{aligned}$$

+

$$\Rightarrow 880 + 320 + 240 + 270 + 210 + 1600$$

3520

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