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BS(SE)

Subject Calculus

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Q1 Part (A)

Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$

with respect to x.

Sol

$$\frac{3x^4-2x^3+5}{x^3+1}$$

Now apply Quotient Rule.

$$\frac{x^3+1\ \frac{d}{dx}(3x^4-2x^3+5)-3x^4-2x^3+5\ \frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\frac{(x^2+1)(12x^4-6x^2+0)-3x^4-2x^3+5\ (3x^2+0)}{(x^3+1)^2}$$

$$\frac{(x^3+1)(12x^4-6x^2)-(3x^4-2x^3+5)3x^2}{(x^3+1)^2}$$

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Q 2) a) Find integration of $\int \frac{1}{\sqrt{x^5}} dx$.

SOLUTION:

Let $x^5 = u \rightarrow$ putting in given equation

$\int \frac{1}{\sqrt{u}} dx \quad \rightarrow$ By substitution method.

$$= \int \frac{1}{u^{1/2}} dx$$

$$= \int u^{-1/2} dx$$

$$= \frac{u^{-1/2+1}}{-1/2+1}$$

$$= \frac{u^{1/2}}{+1/2} = \underline{2}u^{1/2}$$

putting $u = x^5$

$$= 2(x^5)^{1/2}$$

$$= \sqrt{2x^5} \text{ Ans.}$$

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2) b) Find integration of $\int \frac{1}{(8x+7)^8} dx$

SOLUTION:

$$\int \frac{1}{(8x+7)^8} dx \longrightarrow ①$$ By substitution method.

let $(8x+7) = u$ putting in ① we get

$$= \int \frac{1}{u^8} dx$$

$$= \int u^{-8} dx$$

$$= \frac{u^{-7}}{-7}$$ putting value of u.

$$\frac{(8x+7)^{-7}}{-7}$$

$$= \frac{7}{(8x+7)^7} = \frac{7}{(8x+7)^7}$$ Ans

$$x^3-1 \quad \frac{2(x^3+1)2x^2-(x^3+1)^2 3x^2}{(x^3-1)^2}$$

$$\frac{(2x^3-2)(2x^5+2x^2)-((x^3)^2+(1)^2+2x^3)\cdot 3x^2}{(x^3-1)^2}$$

$$\frac{4x^8+\cancel{4}x^5-\cancel{4}x^5-4x^2-3x^8-3x^2-6x^5}{(x^3-1)^2}$$

$$\frac{x^8-6x^5-7x^2}{(x^3-1)^2}$$

Ans

$$\frac{12x^7 - 6x^5 + 12x^4 - 6x^2 - 9x^6 + 6x^5 - 15x}{(x^3+1)^2}$$

$$\frac{12x^7 - 6x^5 + 12x^4 - 6x^2 - 9x^6 + 6x^5 - 15x}{(x^3+1)^2}$$

$$\frac{12x^7 - 9x^6 + 12x^4 - 21x^2}{(x^3+1)^2}$$ Ans

Q7 Part (B)

$\frac{(x^3+1)^2}{x^3-1}$ with Respect to x

Sol

By Quotient Rule -

$$\frac{x^3-1 \ \frac{d}{dx}(x^3+1)^2 - (x^3+1)^2 \ \frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$\frac{x^3-1 \cdot 2(x^3+1) \ \frac{d}{dx}(x^3+1) - (x^3+1)^2 \, 3x^2}{(x^3-1)^2}$$

Q5 a) If $A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} -3 & 2 \\ 4 & 0 \end{pmatrix}, C \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

Find $A^2 + BC$.

Sol. $A^2 + BC =$

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot1 + 4\cdot2 & 1\cdot4 + 4\cdot1 \\ 2\cdot1 + 1\cdot2 & 2\cdot4 + 1\cdot1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 9 & 8 \\ 4 & 9 \end{pmatrix} \rightarrow A^2$$

$$A^2 + BC = \begin{pmatrix} 9 & 8 \\ 4 & 9 \end{pmatrix} + \begin{pmatrix} -3 & 2 \\ 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$$

$$A^2 + BC = \begin{pmatrix} 9 & 8 \\ 4 & 9 \end{pmatrix} + \begin{pmatrix} -3\times1 + 2\times0 & -3\times0 + 2 \\ 4\times1 + 0\times0 & 4\times0 + 2 \end{pmatrix}$$

$$A^2 + BC = \begin{pmatrix} 9 & 8 \\ 4 & 9 \end{pmatrix} + \begin{pmatrix} -3 & 4 \\ 4 & 0 \end{pmatrix}$$

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$$A^2 + BC = \begin{pmatrix} 6 & 12 \\ 8 & 9 \end{pmatrix}$$

Ans

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Part (b)

$$x + \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

Sol-

if $x \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

then L.H.S = R.H.S

put x value

$\Rightarrow$

$$\begin{bmatrix} -1 & -2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ & 5 \end{bmatrix} + \begin{bmatrix} -4 & \\ -2 & \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

So L.H.S = R.H.S

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Qu

(a) $x + \begin{bmatrix}3 & -1\\ 2 & 2\end{bmatrix} = \begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix}$

if $x = \begin{bmatrix}2 & 2\\ -5 & -1\end{bmatrix}$

then $x + \begin{bmatrix}3 & -1\\ 2 & 2\end{bmatrix} = \begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix}$

so $\begin{bmatrix}2 & 2\\ -5 & -1\end{bmatrix} + \begin{bmatrix}3 & -1\\ 2 & 2\end{bmatrix} = \begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2+3 & 2-1\\ -5+2 & -1+2\end{bmatrix} = \begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix}$

$\begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix} = \begin{bmatrix}5 & 1\\ -3 & 1\end{bmatrix}$

L.H.S = R.H.S

Part (c)

$$x + yI = \begin{bmatrix} 3 & -1 \\ 1 & y \end{bmatrix}$$

Sol.

if $x = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

then L.H.S = R.H.S

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

put values to make L.H.S = R.H.S

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -1 \\ 1 & y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & y \end{bmatrix}$$

thus L.H.S = R.H.S