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FINAL PAPER SUBMISSION :-

INTRO TO STRUCTURAL Dynamics & Earthquake Engineering.

Question 2 ANSWER:-

* DATA:-

Pulled for 1/2 inch.

$E = 29,000$ ksi

$I = 150 \ in^4$

Deflection = 7744.

Sol:-

The general E.O.M for SDOF system is,

$Ku + c\dot{u} + m\ddot{u} = P(t)$

In this case system is undamped $c = (0)$ Undergoing free vibration $P(t) = 0$

Hence general EOM become $Ku + m\ddot{u} = 0$

$K = 3EI/L^3$

$$= \frac{3 \times 2900 \times (150)^4 \text{ in}}{(10 \times 12)^3} = 7.55 \text{ k/in}.$$

In order to determine the chances of mistake during calculation if it is more appropriate then use fundamental units like lb, ft, sec, kg. m.

$K = 7.55 \text{ k/in} = 90600 \text{ lb/ft}.$

$$\Rightarrow m = \frac{7744 \text{ lb/sc}^2}{32.2 \text{ ft}} = 240.49 \text{ slug}.$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90600}{240.49}} = 6.31 \text{ rad/sec}.$$

$$T_n = 2\pi/\omega_n = 2(3.14)/6.31 = 0.995 \text{ sec}.$$

Substituting the corresponding value in eq(1)

$90600 + 240.49 = 90840.49$

General solution to EOM for Undamped

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = 1/2'' = 1/24$ ft & $u'(0) = 0$.

$$u(t) = (1/24) \times \cos(19.38t) + 0$$

Equivalent static force at and time "t" is;

$$f_s(t) = K \cdot u(t) = \frac{90600 \times \cos(19.38t)}{24}$$

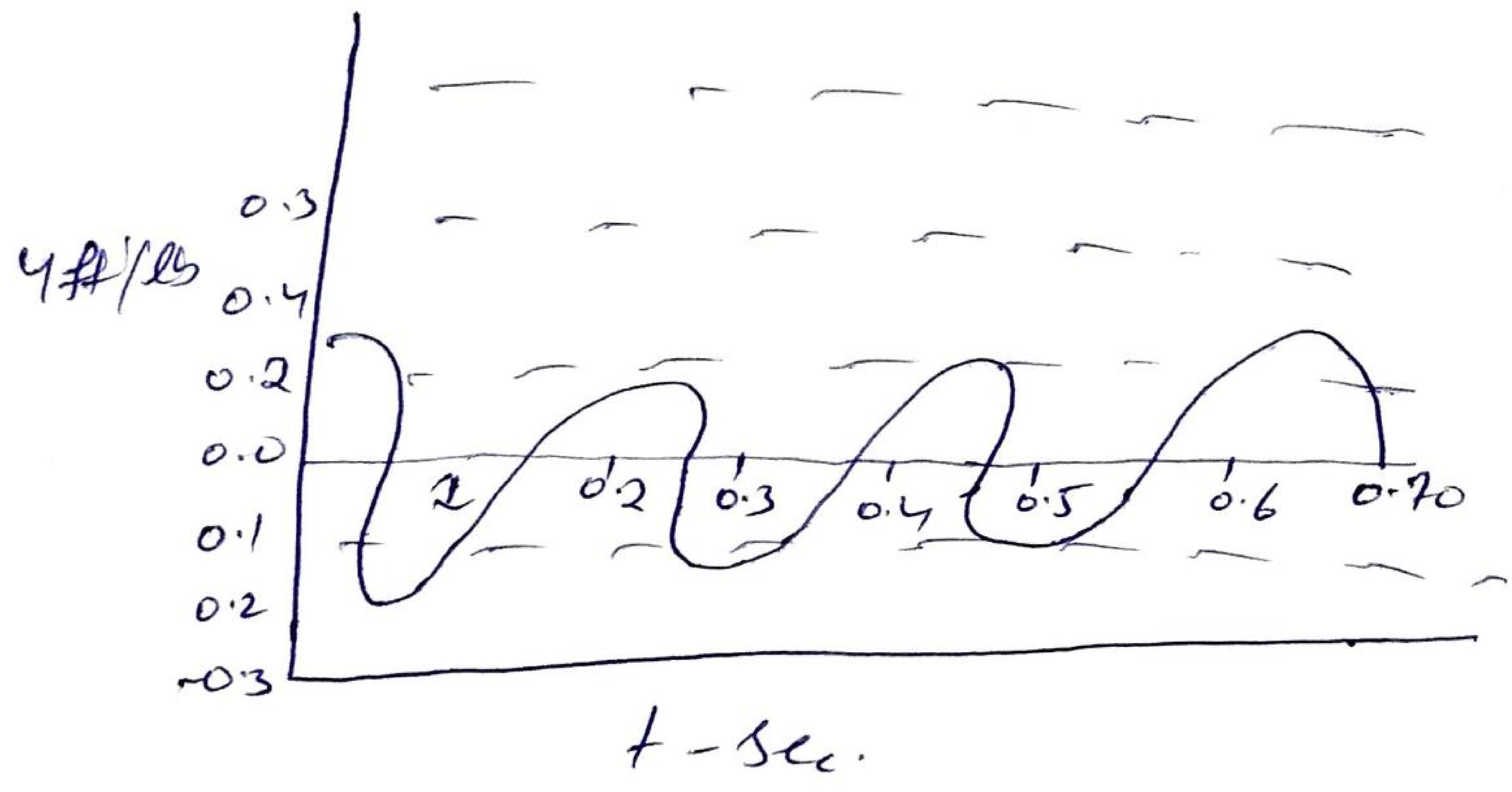
$$f_s(t) = 3776.04 \cos(19.38t)$$

Amplitude of dynamic displacement

$$u_o = \sqrt{[(u(0)^2 + (\dot{u}(0)/\omega_n)^2} = \sqrt{(1/48)^2 + 0} = 1/24 \text{ ft}$$

$$K u_0 = 90600 \times 1/24 = \underline{\underline{3775 \text{ lb.}}}$$

* UNDAMPED FREE VIBRATION :-



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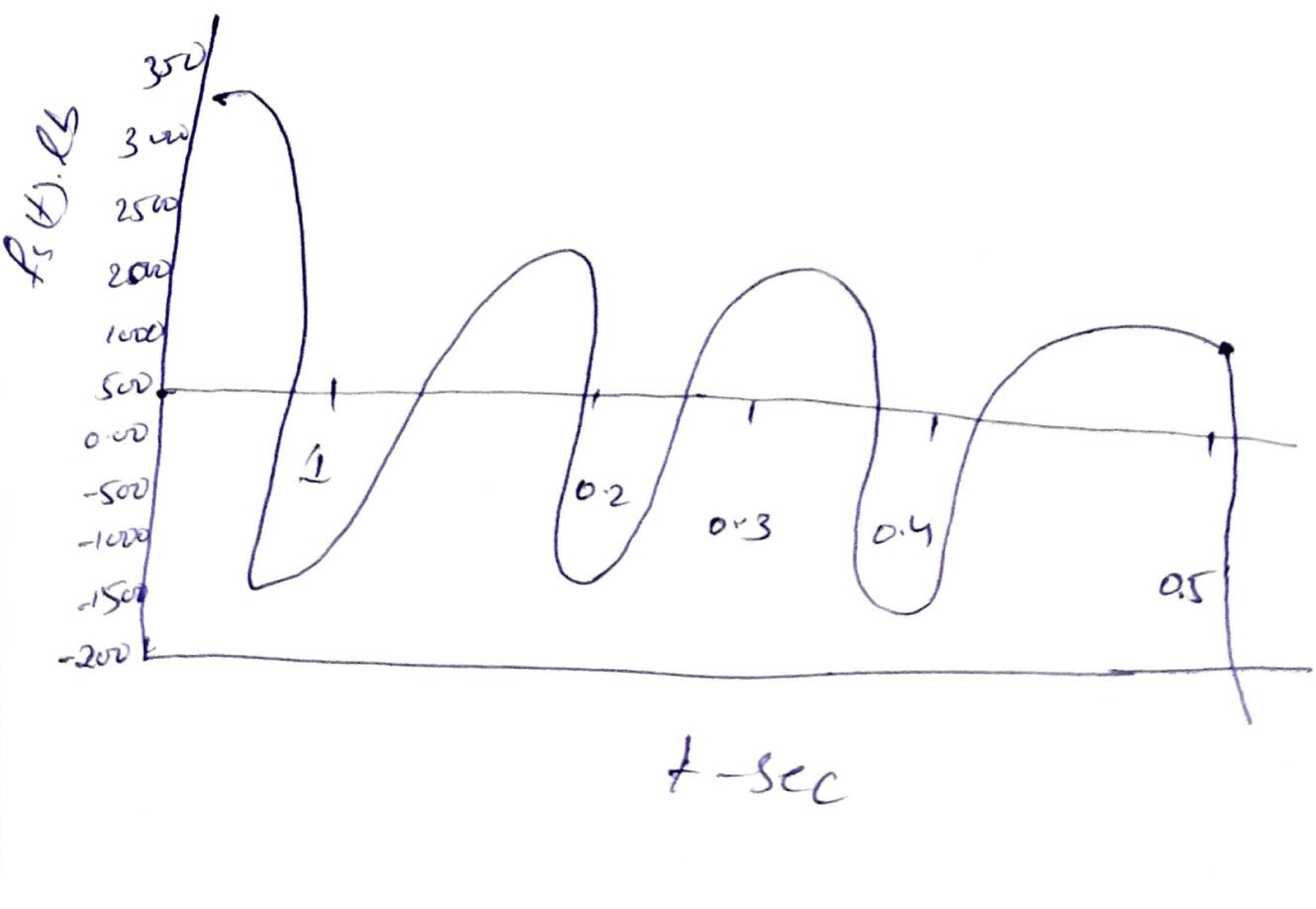


Question : 2 ANSWER :-

* Solution :-

E.O.M for Demand Vibration is, $ku + c\dot{u} + m\ddot{u} = 0$ — (i).

It is known from question 2 that $k = 90625$ lb/ft & $m = 240.49$ lb.sec²/ft.

$C = \xi \times 2m\omega_D = 2 \times 240.49 \times 6.31 \times 0.025$

$C = 75.874$ lb/ft

By substituting value of k, c & m in (i) we get,

$\Rightarrow 90625u + 75.874 + 240.49\ddot{u} = 0.$

Solution to the E.O.M for damped free vibration ;-

$\Rightarrow u(t) = e^{-\xi \omega_0 t} \left[u(0) \cos[\omega_D t] + \frac{1}{\omega_D}[\dot{u}(0) + u(0)\xi\omega_0] \sin[\omega_D t] \right]$.

$\omega_D = 6.31$ rad/sec.

$\Rightarrow u(t) = e^{-0.025 \times 6.31} \left[1/24 \times \cos(6.31 \times t) + 1/6.31 \times [0 + 1/24 \times 0.25 \times 6.31 \times \sin(6.31)] \right]$

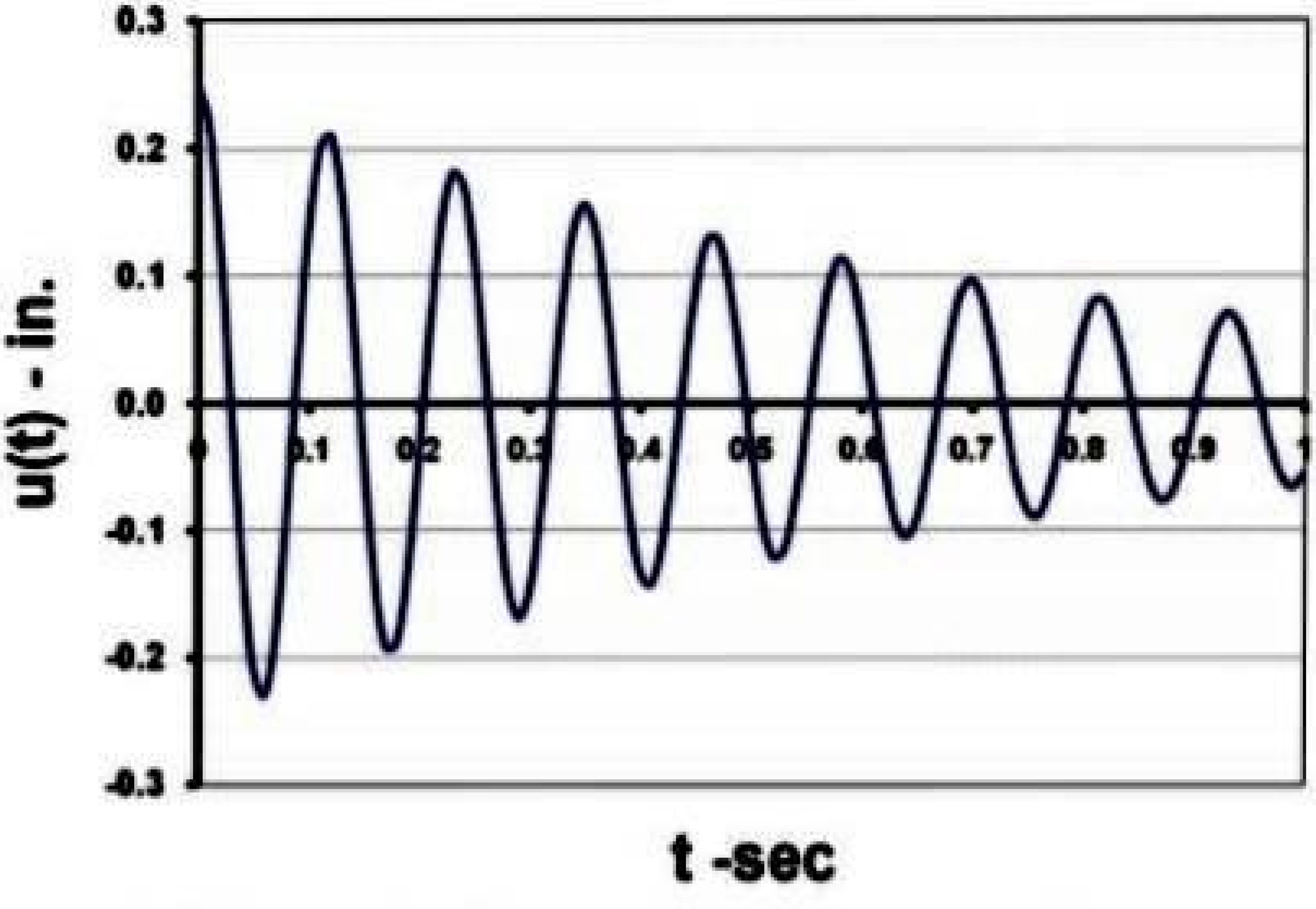
$\Rightarrow u(t) = e^{-0.157t} \left[0.0416 \times \cos(6.31t) + 0.0515 \times \sin(6.31 \times t) \right]$.

$f_s(t) = k \cdot u(t) = 90625 \times u(t)$

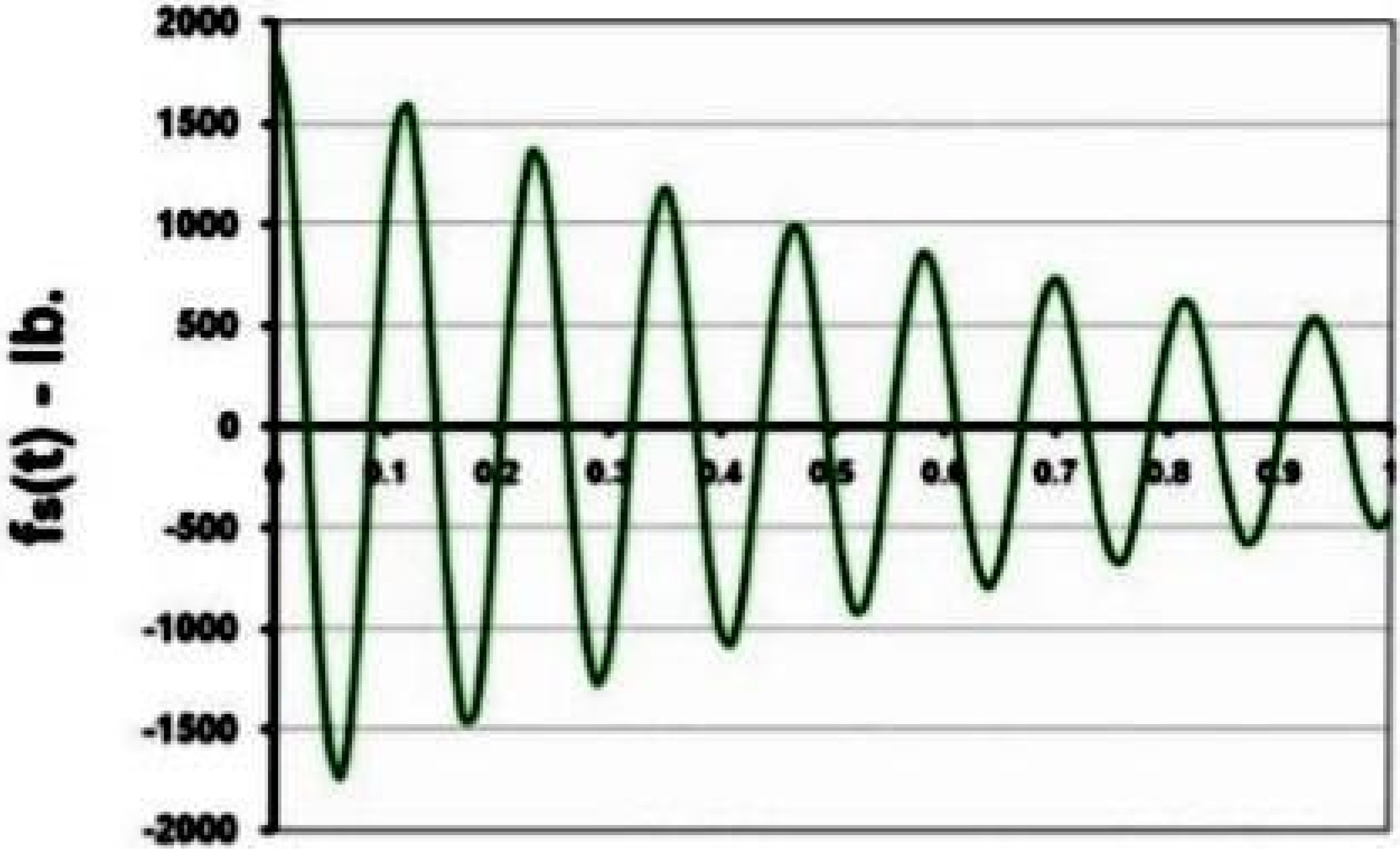
$f_s(t) = e^{-0.157t} \left[3776 \cos(6.31t) + 47.13 \sin(6.31t) \right]$.

Ans.

Damped Free Vibration



Damped Free Vibration



Question #3 ANSWER.

* GIVEN DATA:-

=> Force = 60 kips.

=> u_k = ~~6872~~ 7744/1000 = ~~6.872~~ 7.744 in.

=> After, j = 7 (cycles)

=> Completed = 3.57 sec.

=> u_{j+1} = 2.286 cm = 0.9 in

=> ignore the vertical vibration.

* REQUIRED:-

a) Damping ratios.

b) Natural period of undamped vibration

c) Stiffness of structures.

d) weight of tank.

e) damping coefficient.

f) No of cycles to reduce the displacement amplitude to 0.5".

Sol:-

q) ζ = Damping ratio = ?

As; $\delta = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$

By putting values

$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.744}{0.9}\right]$

$\zeta(7 \times 2 \times 3.14) = 2.152.$

$\zeta(43.96) = 2.152.$

$\zeta = 2.152/43.96$

$= 0.0489$

$\zeta = 0.0489\%$

(b) $T_n = ?$

As seven cycles are completed in "3.57" sec.

Thus time required to complete one cycle $= 7/3.57 = 1.96$ sec.

$T_D = 1.96$ sec.

Now;

$$\omega_D = \omega_n\sqrt{(1-\zeta^2)}$$

$$\Rightarrow 2\pi/\omega_D = 2\pi/\omega_n\left(\sqrt{1-\zeta^2}\right)$$

As;

$$T_D = T_n/\sqrt{1-\zeta^2}$$

$$= T_n = T_D\left(\sqrt{1-\zeta^2}\right)$$

$$= 1.96\left(\sqrt{1-(0.0489)^2}\right)$$

$$\Rightarrow T_D = 1.957 \text{ sec}$$ "Natural period of Undamped vibration."

(C) Stiffness of structure $K = ?$

As, $K = \frac{f \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60)}{2}$ $\left(\begin{matrix} f = 60 \text{ kips} \\ \theta = 60^\circ \end{matrix} \right)$

$= 15 \text{ K/in}$

$K = 18000 \text{ lb/ft.}$

(D) WEIGHT of TANK:-

$W = ?$

As, $\omega_D = \sqrt{K/m} = \sqrt{K/W/g} = \sqrt{K \cdot g/W}$

$\Rightarrow \omega_D^2 = K \cdot g/W \Rightarrow (W = K \cdot g/\omega_D^2)$

By putting values of $\omega_D = 2\pi/T_D$

$W = K \cdot g/(4\pi^2/T_D^2) = K \cdot g\left(\frac{T_D^2}{4\pi^2}\right)$

$W = \frac{18000 \text{ lb}}{\text{ft}} \cdot \frac{32.2 \text{ ft}}{\sec^2} \left(\frac{(1.957)^2}{4(3.14)^2}\right)$

$W = 56284.75$ lb. or $\boxed{56.284 \text{ klb}}$

(e) Damping Co-efficient "$c = ?$"

we know that; $\zeta = \dfrac{c}{2m\omega_D}$

$\Rightarrow c = \zeta(2m\omega_D) = \zeta(2m(2\pi/T_D))$

By putting values;

$$c = \frac{0.0462\left(2\left(\frac{56284}{32.2}\right)\left(2(3.14)\right)\right)}{1.957}$$

$\boxed{c = 518.286 \text{ lb/sec/ft}}$

(f) No of cycles to reduce displacement altitude from "6.872 in to 0.5 in".

$J = ?$

$$J = \frac{1}{2\pi \times \zeta} \ln\left(\frac{u_1}{u_{J+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0484)} \ln\left[\frac{7.744}{0.4}\right]$$

$\boxed{J = 7 \text{ cycles}}$ Ans