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Sec - B

Subject- Mechanics of Solid 2

To - Engr Moh Saqib.

Prob: Construct the Mohr's circle diagram and find the pripiple stress and maximum in plane shear stress for the stress state of a point C located at the center of uniformly distributed load and 1 inches below the top fiber of beam cross section shown in figure. However to construct the Mohr's circle it is necessory to draw the shear stress and fluxural stress variation diagram for maximum force and bending movement respectively. Compare the result obtained from the Mohr's circle with the stress transformation equations.

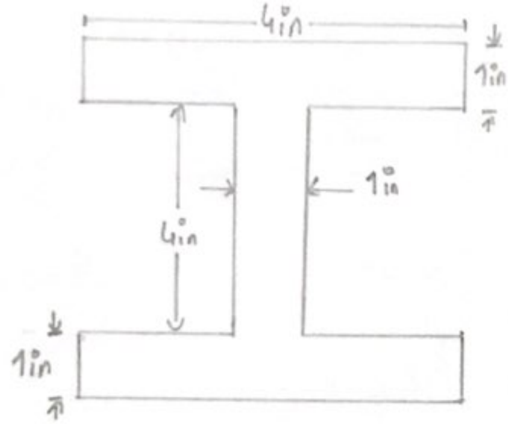
Hint:

To calculate the stress in the beam crossection the movement of intertia must be known.

Where,

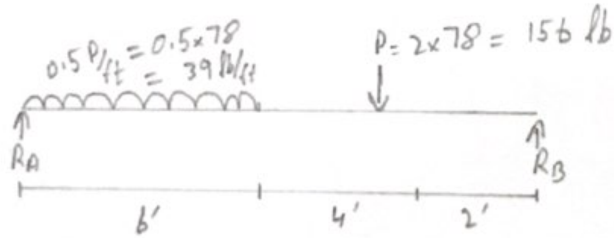
$P = 78lb$ (ID-7978).

(2)



<u>Solution:</u>

First we have to find the reactions,



$$\Sigma F_y = 0 \ (\uparrow +)$$

$$R_A + R_B - (39 \times 6) - (156) = 0$$

$$R_A + R_B = 234 + 156$$

$$\boxed{R_A + R_B = 390 \text{ lb}}$$

$$\Sigma M_A = 0 \quad (+)$$

$$(R_B \times 12) - (156 \times 10) - (39 \times 6 \times \frac{6}{2}) = 0$$

$$12 R_B = 1560 + 702$$

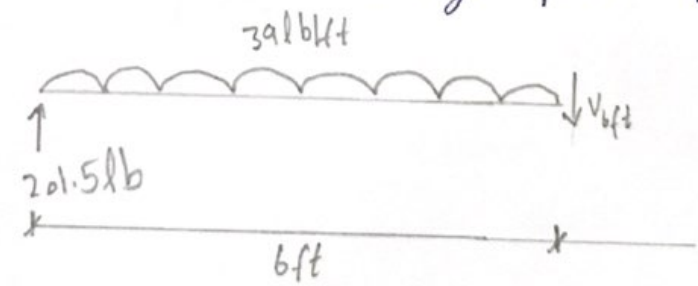
$$\boxed{R_B = 188.5\ lb}$$

$$R_A = 390 - 188.5$$

$$\boxed{R_A = 201.5\ lb}$$

Now we will draw shear force & bending movement diagram.

-> Shear force at change point of beam.



Shear force at 6ft from left support

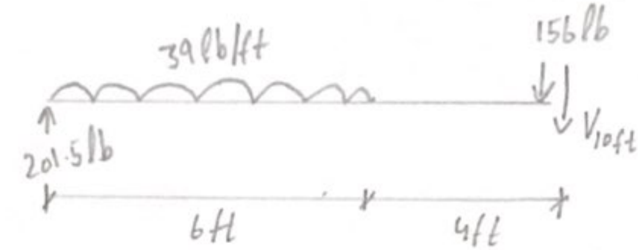
$$\Sigma F_y = 0 \ (\uparrow +)$$

$$-V_{6ft} + 201.5 - 39 \times 6 = 0$$

$$-V_{6ft} - 32.5 = 0$$

$$\boxed{V_{6ft} = -32.5\ lb}$$

Now shear force at 10ft from left end, $V_{10ft} = ?$

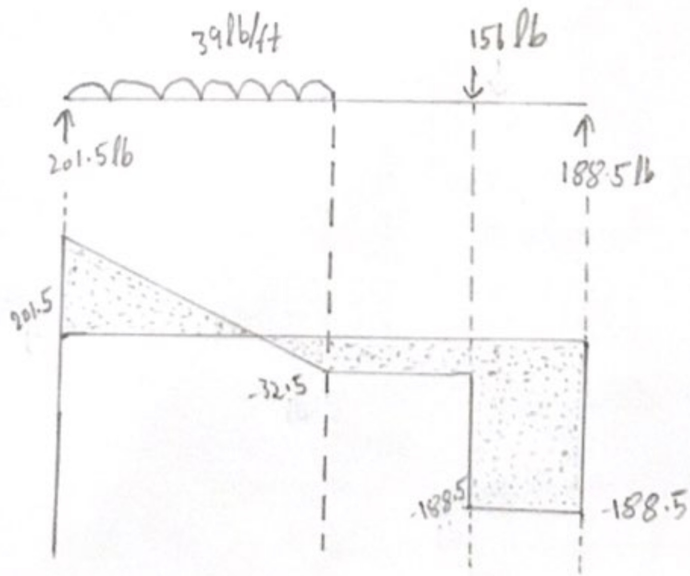


$$\Sigma F_y = 0\ (\uparrow +)$$

$$201.5 - (39 \times 6) - 156 - V_{10ft} = 0$$

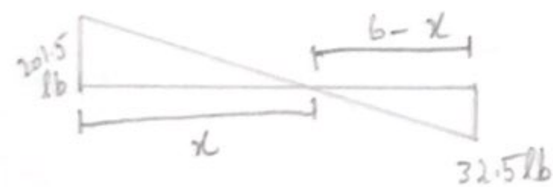
$$-188.5 - V_{10ft} = 0$$

$$\boxed{V_{10ft} = -188.5\ lb}$$



Now for movement diagram we find movement at change points

First finding movement at zero shear point



$$\frac{201.5}{x} = \frac{32.5}{6-x}$$

$$(201.5)(6-x) = 32.5x$$

$$1209 - 201.5x = 32.5x$$

$$1209 = 201.5x + 32.5x$$

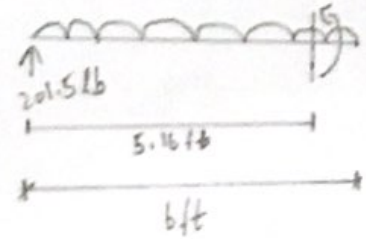
$$1209 = 234x$$

$$\boxed{x = 5.166\ ft}$$

As we know that movement is max where shear force is zero

Taking section at 5.16ft from left support and find movement

$$\Sigma M_{5.16} = 0 \ (+)$$



(b)

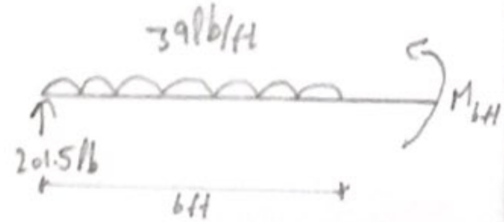
$$M_{5.16} - (201.5 \times 5.16) + \left(39 \times 5.16 \times \frac{5.16}{2}\right) = 0$$

$$M_{5.16} - 1039.74 + 519.19$$

$$\boxed{M_{5.16ft} = 1558.93 \text{ lb-ft}}$$

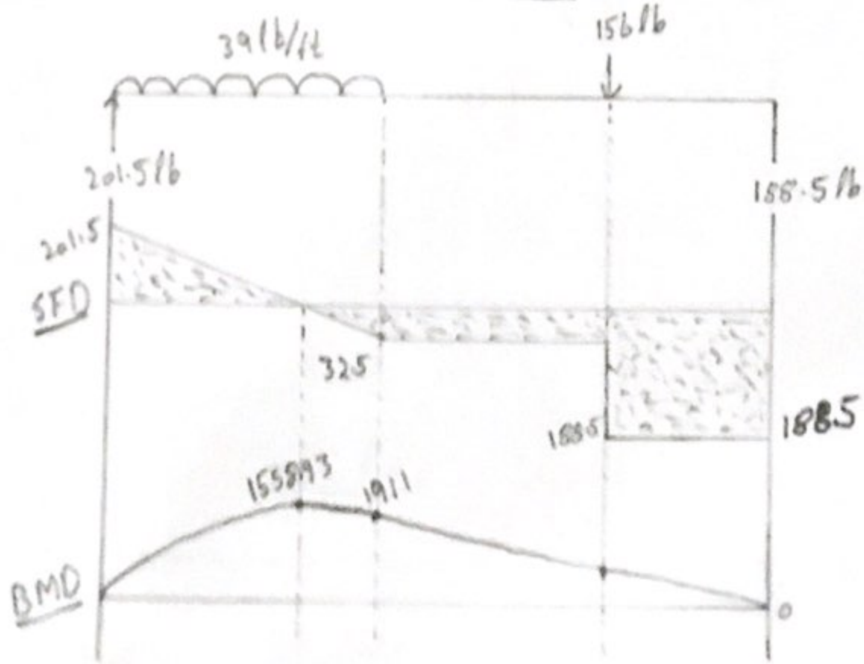
Movement at 6ft from left side

$\Sigma M_{6ft} = 0$ (+)

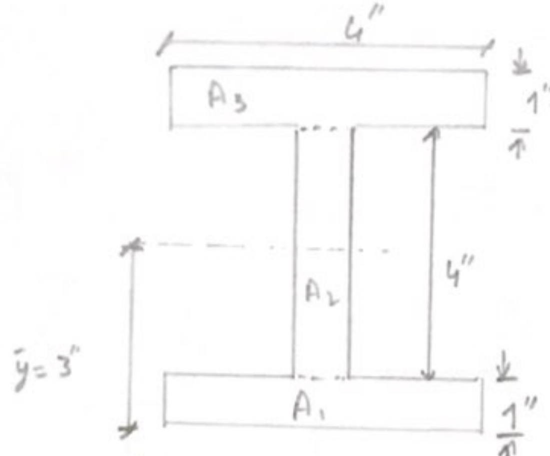


$$M_{6ft} - (201.5 \times 6) + \left(39 \times 6 \times \frac{6}{2}\right) = 0$$

$$\boxed{M_{6ft} = 1911 \text{ lb. ft}}$$



Now we find moment of inertia of the section.



As the section is symetric

so $\bar{y} = 1 + \frac{4}{2}$

$$\boxed{\bar{y} = 3 in}$$

$$I_x = \left[\bar{I}_1 + A_1 d_1^2\right] + \left[\bar{I}_2 + A_2 d_2^2\right] + \left[\bar{I}_3 + A_3 d_3^2\right]$$

$$I_x = \left[\frac{4 \times 1^3}{12} + (4 \times 1)(2.5)^2\right] + \left[\frac{1 \times 4^3}{12} + (1 \times 4)(0)^2\right]$$

$$+ \left[\frac{4 \times 1^3}{12} + (4 \times 1)(2.5)^2\right]$$

$$I_x = (0.33 + 25) + (5.33) + (0.33 + 25)$$

$$\boxed{\bar{I}_x = 56 in^4}$$

Now we calculate shear stresses & flexural stresses at different points in the beam.

For shear stress

$$\tau = \frac{VQ}{Ib}$$

$V_{max} = 188.5\,lb$

$I = 56\,in^4$

Case 1:

τ at top fiber

$$\tau_{top} = \frac{VQ}{Ib} = \frac{188.5 \times 0}{56 \times 4}$$

$$\boxed{\tau_{top} = 0}$$

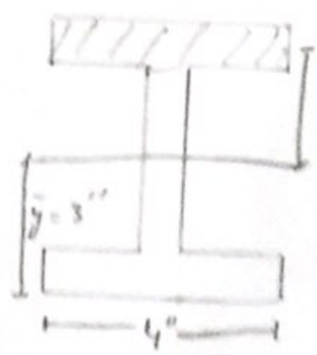
Case 2:

τ in below the top fiber

$$\tau_A = \frac{VQ}{Ib}$$

$$\tau_A = \frac{188.5 \times (4 \times 1)(2.5)}{56 \times 4}$$

$$\boxed{\tau_A = 8.41\,psi}$$



$$\tau_B = \frac{188.5 \times (4 \times 1)(2.5)}{56 \times 1}$$

$$\boxed{\tau_B = 33.66\,psi}$$

Case 3:

"T" at centroidal axis at the section which will be the max. shear stress,

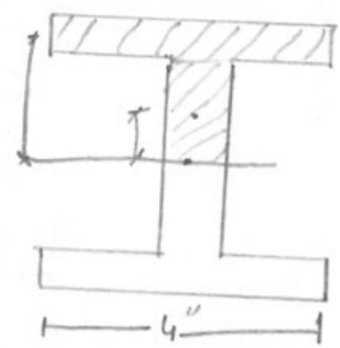
$$Q = Q_1 + Q_2$$

$$Q = A_1 y_1 + A_2 y_2$$

$$Q = (4\times1)(2.5) + (2\times1)(1)$$

$$Q = 10 + 2$$

$$\boxed{Q = 12}$$

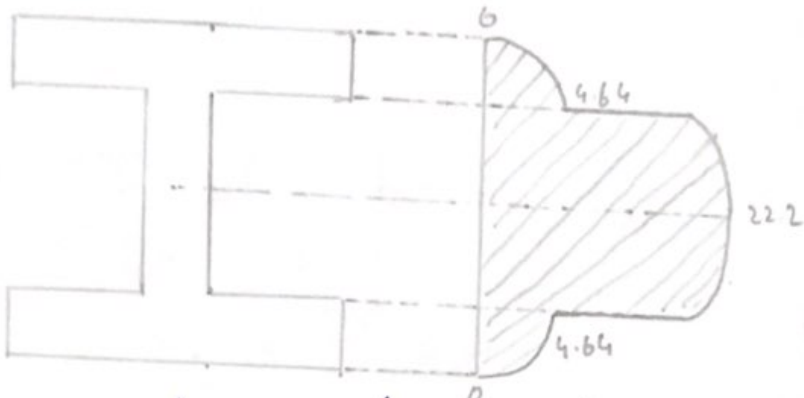


$$\tau_{max} = \frac{188.5\times12}{56\times1}$$

$$\boxed{\tau_{max} = 40.39\,Psi}$$

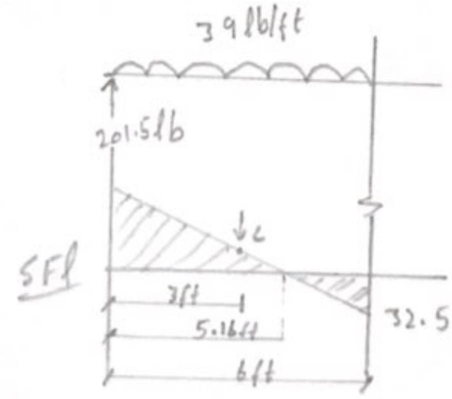
Rest of the section is symetric as above

Shear stress variation diagram along the depth of beam



Now shear stress at point "c" located at the centre of uniformly distributed load and in below the top fiber of beam crossection

Now "V" at point "c" is



$$\frac{V_c}{3} = \frac{201.5}{5.16} \Rightarrow V_c = \frac{201.5 \times 3}{5.16}$$

$$\boxed{V_c = 117.15}$$

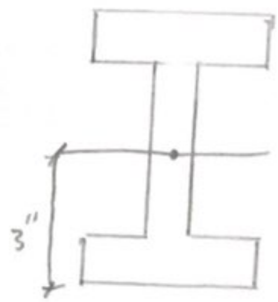
"τ" at point "C" and in below the top fiber is

$$\tau_c = \frac{V_c Q}{I b}$$

To get higher value of τ_c, we take $b = 1''$

$$\tau_c = \frac{117.15 \times 4 \times 1 \times 2.5}{56 \times 1}$$

$$\boxed{\tau_c = 20.98 \text{ psi}}$$



<u>Now</u> <u>flexural</u> <u>stress</u> <u>Analysis</u>:

We consider maximum movement From BMD

Max. movement = 1558.93 lb-ft

Flexural stress, $\sigma = \frac{My}{I}$

<u>Case 1</u>:

Stress at top fiber;

$$\sigma_{Top} = \frac{1558.93 \times 12 \times 3}{56}$$

$$\boxed{\sigma_{Top} = 1002.16\ psi}$$

Case 2:

Stress at 1" below top fiber;

$$\sigma_1 = \frac{155.93 \times 12 \times 2}{56}$$

$$\boxed{\sigma_1 = 668.11\ psi}$$

Case 3:

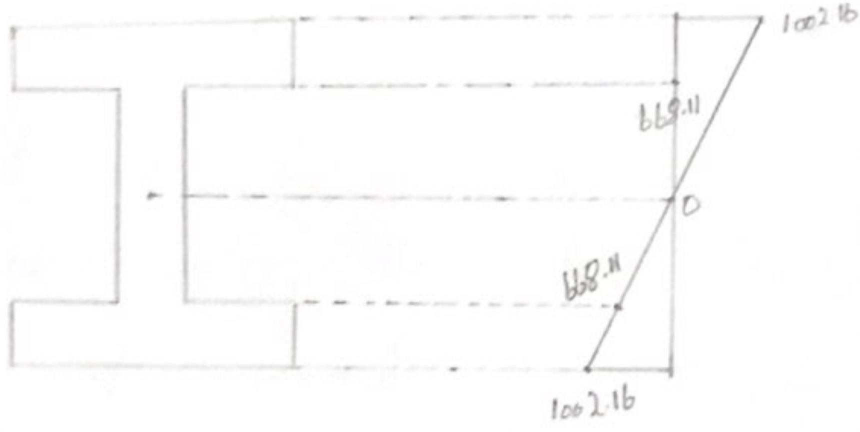
Stress at centroidal axis;

$$\sigma_{center} = \frac{1558 \times 12 \times 0}{56}$$

$$\boxed{\sigma_{center} = 0\ psi}$$

Rest of the section is symetory of above.

Flexural stress variation diagram;



Stress state of a point element:

Now the stress state of a point element located at the center of unformly distributed load and 1". below the top fiber of beam cross section.

All applied stresses are required at point C.

We have found the shear stress at the required point which is;

$$\boxed{\tau_c = 20.98 \text{ psi}}$$

Flexural stress at required point is

$$\sigma_c = \frac{M_c y}{I}$$

moment at c is approximatly the area under the shear force diagram of concerned

$$M_c = (3 \times 117.15) + (3 \times \frac{46.51}{2})$$

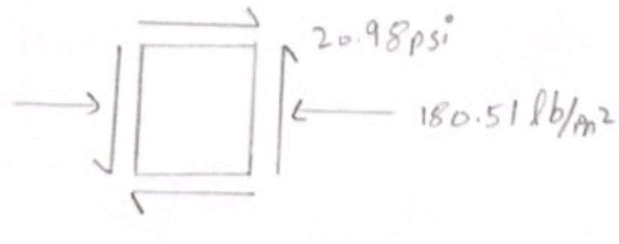
$$M_c = 351.45 + 69.765$$

$$M_c = 421.215$$

$$\sigma_c = \frac{421.21 \times 2 \times 12}{56}$$

$$\boxed{\sigma_c = 180.51\ lb/in^2}$$

Combine stress on 2D element



Principle stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-180.51 + 0}{2} \pm \sqrt{\left(\frac{180.51 - 0}{2}\right)^2 + (20.98)^2}$$

$$= -90.25 \pm \sqrt{(90.25)^2 + (20.98)^2}$$

$$= -90.25 \pm 92.65$$

$$\sigma_y = \sigma_1 = 2.4 \text{ psi}$$

$$\sigma_x = \sigma_2 = 182.9 \text{ psi}$$

Max. In plane shear stress

$$\tan 2\theta_s = \frac{(-\sigma x - \sigma y)/2}{\tau xy}$$

$$\tan 2\theta_s = \frac{-(-180.51 - 0)/2}{20.98}$$

$$\theta_s = 76.908$$

As General equation of $\tau_{x'y'}$ is

$$\tau_{x'y'} = \left(-\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = \left(-\frac{180.51-0}{2}\right)\sin(2 \times 76.908) + 20.98 \cos 2(76.908)$$

$$\tau_{x'y'} = (-90.25)(0.441) + 20.98(-0.897)$$

$$= -39.818 - 18.819$$

$$\boxed{\tau_{x'y'} = -58.63 \text{ psi}}$$

max. in plane shear stress