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INTEGRATION :-

Integration is the calculation of an integral, Integral in maths are used to find many useful quantities such as area volumes, displacement etc, When we speak about integrals, it is related to usually definate Integrals.

Integration is a way of adding slices to find the whole.

Integration is the reverse of differentiation. The fundamental use of integration is as a continuous version of summing, But paradoxically, often integrals are computed by viewing integration as essentially an inverse operation to differentiation.

The notation which were stuck which for historical reasons, is as the notation for derivatives the integral of a function $f(x)$ with respect to x is written as

$$\int f(x)\, dx$$

the remark that integration is (almost) an inverse to the operation of differentiation means that if

$$\frac{d}{dx} f(x) = g(x)$$

then

$$\int g(x)\, dx = f(x) + C$$

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Trapezoidal Rule:-

We know from a previous lesson that we can use Riemann Sums to evaluate a definite integral

$$\int_a^b f(x)\,dx.$$

Riemann Sums use rectangles to approximate the under a curve.

Another useful integration rule is the trapezoidel Rule. Under this rule is the area under a curve is evaluated by dividing the total area into little trapezoids rather than rectangles

Let $f(x)$ be continuous on $[a, b]$, we partition the interval $[a, b]$ into n equal subintervals, each of width

$$\Delta x = \frac{b-a}{n}$$

Such that.

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b$$

The trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

As $n \to \infty$, the right-hand side of the expression approaches the definite integral $\int_a^b f(x)\,dx$

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Simpson's Rule:

Simpson's Rule is a numerical method that approximates the value of a definite integral by using quadratic functions.

This method is named after the English mathematicatician

Simpson's Rule is based on the fact that given three points, we can find the equation of a quadratic through those points

to obtain an approximation of the definite integral $\int_a^b f(x)\,dx$ using Simpson's Rule, we partition the interval $[a, b]$ into an even number n of subinterval, each of width

$$\Delta x = \frac{b-a}{n}$$

On each pair of consecutive subintervals. $[x_{i-1}, x_i]$, $[x_i, x_{i+1}]$ we consider a quadratic function $y = ax^2 + bx + c$ such that it passes through the point

If the function $f(x)$ is continous on $[a, b]$ then.

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

the co-efficients in Simpson's Rule have the following pattern

$$\underbrace{1, 4, 2, 4, 2 \cdots 4, 2, 4, 1}_{n+1 \text{ point.}}$$