ASSIGNMENT NO:- 01

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Subject:- Advanced mechanics of Materials

Question 01 :-

Application of Mohr's Circle:-

Mohr's circle is a visual or geometrical representation of what is known as the stress state. Now, for 1-D case, there is just one stress state (tension or compression or shear) and doesn't require any geometrical representation.2-D stress state, on the other hand, is quite complex. This complexity is not just because of several stresses in the element but because of how these stresses "transform" if the object or the loads rotate w.r.t. each other. We have those second degree analytical formulae for such stress transformation but they are quite complex themselves. Mohr came with a relatively easy method to "see" what happens to these stresses when the object rotates. Without going into the details, it can be said that Mohr's circle provides an easy way to calculate these rotated stresses even when you don't know the transformation formulae. With computers and software, Mohr's circle may look like somewhat ancient but it is still popular amongst many analysts for quick checks and verification of values being thrown by these software. The Mohr diagram for strain is rarely used in its full form, as a representation of three-dimensional strain. Recent attention has focused on various uses of the Mohr circle to express two-dimensional strain tensors. This contribution re describes the Mohr diagram for three-dimensional strain and illustrates some new applications. The Mohr diagram for any strain ellipsoid provides an immediate method for ellipsoid shape classification. However, its greatest new potential is considered to be in the representation of strain ellipses as sections of ellipsoids. Any plane section of a strain ellipsoid can be plotted on the ellipsoid's Mohr diagram: it is here called a 'Mohr locus' because it is constructed as a locus of points representing the sheaf of lines which can be considered to define the plane. Mohr loci for sectional ellipses have a variety of forms, according to their orientation in the strain ellipsoid. Generally oblique sections are represented by loops bounded by the three principal circles. Their most leftward and rightward points are the plane's principal axes. Any Mohr locus can be transformed into a Mohr circle for the sectional ellipse.

The benefit that Mohr's circle provides is that it creates a visual representation of what is happening, and the relative positioning of the stress states on an element relative to a set of coordinate axes.

Question No. 03 :- Simple bending and Pure Bending

Pure Bending:

Bending will be called as pure bending when it occurs solely because of coupling on its end. In that case there is no chance of shear stress in the beam. But, the stress that will propagate in the beam as a result will be known as normal stress. Normal stress because it not causing any damages to beam.

Simple Bending:

Bending will be called as simple bending when it occurs because of beam self-load and external load. This type of bending is also known as ordinary bending and in this type of bending results both shear stress and normal stress in the beam

Question no. 04:- assumptions made in theory of simple bending

Following are the assumptions made in theory of simply bending

1. Only pure bending can occur - there's no shear force, torsion nor axial load
2. We consider isotropic or orthotropic homogenous material
3. Only linear elasticity (up to proportionality limit) is analyzed
4. Initially there's no deformation and there's no varying cross section
5. Beam is symmetrical in the plane along which bending occurs
6. Appropriate proportions make it impossible for the beam to fail in any other way than because of bending (no buckling and so on)
7. Cross-section of the beam is still plane after (and during) bending - that's the main assumption of Euler-Bernoulli beam theory

The classic formula for determining the bending stress in a beam under simple bending is:
stress $\sigma = My/Ix$

where

σ = is the bending stress
M - the moment about the neutral axis
y - the perpendicular distance to the neutral axis
Ix - the second moment of area about the neutral axis x

Actual beam loading cases and end conditions can be found from tables e.g cantilevered, simply supported point loading uniformly distributed etc.

Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members. Other geometric properties used in design include area for tension and shear, radius of gyration for compression, and moment of inertia and polar moment of inertia for stiffness. Any relationship between these properties is highly dependent on the shape in question. Equations for the section moduli of common shapes are given below. There are two types of section moduli, the elastic section modulus and the plastic section modulus.

Question No. 06:- Elastic section modulus :-

For general design, the elastic section modulus is used, applying up to the yield point for most metals and other common materials.

The elastic section modulus is defined as S = I / y, where I is the second moment of area (or area moment of inertia, not to be confused with moment of inertia) and y is the distance from the neutral axis to any given fiber. It is often reported using y = c, where c is the distance from the neutral axis to the most extreme fiber, as seen in the table below. It is also often used to determine the yield moment (My) such that My = S × σy, where σy is the yield strength of the material.

I: - Plastic section modulus

The plastic section modulus is used for materials where elastic yielding is acceptable and plastic behavior is assumed to be an acceptable limit. Designs generally strive to ultimately remain below the plastic limit to avoid permanent deformations, often comparing the plastic capacity against amplified forces or stresses. The plastic section modulus depends on the location of the plastic neutral axis (PNA). The PNA is defined as the axis that splits the cross section such that the compression force from the area in compression equals the tension force from the area in tension.

Question o7 :- Application of Bending Equation in any object

Bending equation just gives you the variation of bending moment along the length of the beam. And by plotting the variation of bending moment for a beam, you can calculate the value of maximum bending moment and design your beam for that maximum value of bending moment.

When we derive the flexural formula i.e.

M/I=sigma/Y=E/R

if we want to design a beam or any other structural member where bending moment is the dominating one for design. Then we take the value of maximum bending moment (which we get from the bending moment diagram) and for a certain permissible stress, we choose the dimensions of our member.

E.g if the max bending moment coming on a beam with certain kind of loading comes out to be 10kn-m and the permissible stress of the material is 20N/mm^2. Now you need to design your beam such that it is able to resist this moment. Let's say you want to design a beam with rectangular cross-section.

M=allowable stress*I/Y

Question 08:- Moment of resistance

Moment of resistance denotes the resistance offered by the beam to the external moment applied. The moment of resistance of the concrete section is the moment of couple formed by the total tensile force (T) in the steel acting at the center of gravity of reinforcement and the total compressive force (C) in the concrete acting at the center of gravity (C.G.) of the compressive stress diagram. The moment of resistance is denoted by M. The distance between the resultant compressive force (C) and tensile force (T) is called the lever arm, and is denoted by z.

Therefore, center of gravity of the compressive force is at a distance x/3 from the top edge of the section.

Therefore, $z = d - x/3$

Moment of resistance is given by,

$M_r = C \times z$

$= bx\,(\sigma_{cbc}/2)(d - x/3)$

$M_r = T \times z$

$= A_{st}\,\sigma_{st}\,(d - x/3)$

For balanced section, the formula is as follows,

$M_r = b x_c\,\sigma_{cbc}\,(d - x_c/3)$

$= A_{st}\,\sigma_{st}\,(d - x_c/3)$

For under-reinforced section, the formula is as follows,

$M_r = T \times z$

$= A_{st}.\sigma_{st}\,(d - x/3)$

For over-reinforced section, the formula is given as,

$M_r = C \times z$

Question No. 09:- Design of Columns Under Centric Load

Design stages of centric loaded columns: -

Longitudinal / Main Reinforcement

- The main reinforcement in columns is longitudinal, parallel to the direction of the load.
- They are provided to resist bending moment and to take the compression Lateral Ties
- The lateral ties are bars arranged in a square, rectangular, or circular pattern.
- They are provided to resist buckling, to hold the main bars and to resist shear.
- The continuous spiral contain / retain concrete, thus increasing the load taking capacity.

Consider a Rectangular Section subjected to axial.

load Pu.

- To avoid failure; $\Phi Pn \geq Pu$
- Nominal Axial Capacity Pn can be calculated as follows;
- $Cs1 + Cs2 + Cs3 + Cc = Pn$
- $Cs1 = As1 * fs1$
- $Cs2 = As2 * fs2$
- $Cs3 = As3 * fs3$
- $Cc = Ac * fc$
- $As1 * fs1 + As2 * fs2 + As3 * fs3 + Ac * fc = Pn$

The section will reach its axial capacity when strain in concrete reaches

a value of 0.003.

The yield strain values of steel for grade 40 and 60 are 0.00138 and

0.207 ectively.

Therefore steel would have already yielded at 0.003 strain.

Hence $fs1 = fs2 = fs3 = fs4 = fy$ and $fc = 0.85\, fc'$

- Let $As1 + As2 + As3 = Ast$ and $Ac = Ag - Ast$,
  where Ag = gross area ofcolumn section, Ast = total steel area
- $Ast\, fy + 0.85\, fc'(Ag - Ast) = Pn$ ------------(A)
- $Pn = Ast\, fy + 0.85\, fc'(Ag - Ast)$
- $\Phi\, Pn = Pu$
- As per ACI code (22.4.2.1), $\Phi = 0.65$ for tied column and $\Phi =$
- 0.75 for spiral column

Additional reduction factor 'α' are used to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than fc′ under sustained high loads.

Axial capacity for Tied Columns

- $\alpha \Phi Pn = Pu$; $\alpha = 0.80$, $\Phi = 0.65$
- 0.80 x 0.65 $[0.85fc'(Ag - Ast) + fy\ Ast] = Pu$
- Axial capacity for Spiral Columns
- $\alpha \Phi Pn = Pu$; $\alpha = 0.85$, $\Phi = 0.75$
- 0.85 x 0.75 $[0.85fc'(Ag - Ast) + fy\ Ast] = Pu$

Ratio of Longitudinal Reinforcement

- According to ACI Code 10.6.1.1, For columns, area of longitudinal reinforcement shall be at least 0.01Ag but shall not exceed 0.08Ag.
- Most columns are designed with ratios below 0.04
- Lower limit To prevent failure mode of plain concrete
- Upper limit To maintain proper clearance between bars

Minimum number of Bars

- According to ACI Code 10.7.3.1, the minimum number of longitudinal bars shall be;
- A minimum of four longitudinal bars is required when the bars are enclosed by spaced rectangular or circular ties
- A minimum of six bars must be used when the longitudinal bars are enclosed by a continuous spiral.

spacing between Bars

According to ACI Code 25.2.3, For longitudinal reinforcement in columns, clear spacing between bars shall be at least the greatest of;

i. 1.5 in.

ii. 1.5db

Clear cover to Bars

Cover shall be 1.5 in. minimum over primary reinforcement, ties or Spirals

Maximum Spacing (minimum reinforcement) of Lateral Ties According to ACI 25.7.2.1;

Center-to-center spacing shall not exceed the least of;

i. 16db of longitudinal bar
ii. 48db of tie bar
iii. Smallest dimension of member

Minimum diameter of Lateral Ties

According to ACI 25.7.2.2; Diameter of tie bar shall be at least (a) or (b):

a. No. 3 enclosing No. 10 or smaller longitudinal bars

b. No. 4 enclosing No. 11 or larger longitudinal bars or bundled longitudinal bars

Spacing and diameter of Spiral reinforcement

According to ACI 25.7.3.1, Spacing/pitch of spiral must not be less than 1 in. and greater than 3 in.

According to ACI 25.7.3.2 the minimum spiral size is 3/8 in.

Question no. 10:- deflections of beam by castigliano's theorem

Italian engineer Alberto Castigliano (1847 – 1884) developed a method of determining deflection of structures by strain energy method. His Theorem of the Derivatives of Internal Work of Deformation extended its application to the calculation of relative rotations and displacements between points in the structure and to the study of beams in flexure.

Energy of structure is its capacity of doing work and strain energy is the internal energy in the structure because of its deformation. By the principle of conservation of energy,

U=Wi

where U denotes the strain energy and Wi represents the work done by internal forces. The expression of strain energy depends therefore on the internal forces that can develop in the member due to applied external forces.

Castigliano's Theorem for Beam Deflection

For linearly elastic structures, the partial derivative of the strain energy with respect to an applied force (or couple) is equal to the displacement (or rotation) of the force (or couple) along its line of action.

$\delta=\partial U\partial P$ or $\theta=\partial U\partial \bar{M}$

Where δ is the deflection at the point of application of force P in the direction of P, θ is the rotation at the point of application of the couple $\bar{M}$ in the direction of $\bar{M}$, and U is the strain energy.

The strain energy of a beam was known to be $U=\int_0^L \frac{M^2}{2EI}dx$.

Finding the partial derivative of this expression will give us the equations of Castigliano's deflection and rotation of beams. The equations are written below for convenience.

$$\delta=\int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{M}{EI}dx$$

and

$$\theta=\int_0^L \left(\frac{\partial M}{\partial \bar{M}}\right)\frac{M}{EI}dx$$