D.E FINAL ASSIGNMENT PAPER.

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Submitted To:
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Semester:
3rd.

Q1) Define 2nd order linear homogenous/ non-homogenous d.e along with example.

A) Homogenous differential Equation:

The one which is all the terms involing the unknown function are collected together on side of equation, the other side is 0.

Example:

$y'' - 2y' + y = 0$, is homogenous -(1)

but $y'' - 2y' + y = x$ -(2) is not homogenous

equation (2) can be converted in homogenous equation by replacing hand side by 0.

$$a(x)y'' + b(x)y' + c(x)y = 0 \quad (**).$$

Non-homogenous differential Equation.

The non-homogenous differential equation has type form of:

$$y'' + Py' + qy = f(x).$$

where P & q are constant, for each equation, we can related homogenous & complementary equation

The general solution of non-homogenous equation is sum of general solution related to particular solution

$$y(x) = y_0(x) + y_1(x).$$

Example:

(1) $y'' + y = \sin 2x$

(2) $y'' + y' - 6y = 36x.$

(b)

(1) $4y'' - 6y' + 7y = 0.$

Sol:

$4y'' - 6y' + 7y = 0.$

A second order linear, homogenous ODE has form of $ay'' + by' + cy = 0$'

for equation $ay'' + by' + cy = 0,$

assume e^{rt}

$y = e^{rt}$

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$$4((e^{rt}))'' - 8((e^{rt}))' + 7e^{rt} = 0.$$

$$4\,(((e^{rt})'' - 6((e^{rt}))' + 7e^{rt} = 0.$$

$$e^{rt}(4y^2 - 6y + 7) = 0.$$

$$e^{rt}(4y^2 - 6y + 7) = 0; \quad r = \frac{3}{4} + i\frac{\sqrt{19}}{4}, \; r = \frac{3}{4} - i\frac{\sqrt{19}}{4}$$

$$r = \frac{3}{4} + i\frac{\sqrt{19}}{4}, \quad r = \frac{3}{4} - i\frac{\sqrt{19}}{4}.$$

for two complex root

$$r_1 \neq r_2$$

$$e^{\frac{3}{4}t}\left(c_1 \cos \frac{\sqrt{19}}{4} t\right) + c_2 \sin\left(\frac{\sqrt{19}}{4} t\right)$$

Solution is

$$y = e^{\frac{3t}{4}}\left(c_1 \cos \frac{\sqrt{19}\,t}{4}\right) + c_2 \sin\left(\frac{\sqrt{19}\,t}{4}.\right)$$

(ii) $y'' - 4y' - 12y = 3e^{5t}$

Sol:

$$y'' - 4y' - 12y = 0$$

characteristic roots are

$$r^2 - 4r - 12 = (r-6)(r+2) = 0$$

$$\Rightarrow r_1 = -2, \; r_2 = 6$$

$$y_c(t) = c_1 e^{-2t} + c_2 e^{6t}$$

$$y_p(t) = Ae^{5t}$$

$$25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t})$$

$$= 3e^{5t} - 7Ae^{5t} = 3e^{5t}$$

$$-7A = 3, \quad A = -\frac{3}{7}$$

solution

$$y_p(t) = -\frac{3}{7} e^{5t}$$

Q2

(i) $16y'' - 40y' + 25y = 0, \; y(0) = 3, \; y'(0) = -\frac{9}{4}$

Sol:

The characteristic solution is

$$16r^2 - 40r + 25 = (4r-5)^2 = 0, \quad r_{1,2} = \frac{5}{4}$$

The general solution is

$$16r^2 - 40r + 25 = (4r-5)^2 = 0, \quad r_{1,2} = \frac{5}{4}.$$

The general derivative is

$$y(t) = c_1 e^{\frac{5t}{4}} + c_2 t e^{\frac{5t}{4}}$$

$$y'(t) = \frac{5}{4} c_1 e^{\frac{5t}{4}} + c_2 e^{\frac{5t}{4}} + \frac{5}{4} c_2 t e^{\frac{5t}{4}}$$

$$3 = y(0) = c_1$$

$$-\frac{9}{4} = y'(0) = \frac{5}{4} c_1 + c_2$$

$c_1 = 3$, $c_2 = -6$.

$$y'(t) = 3e^{\frac{5t}{4}} - 6te^{\frac{5t}{4}}.$$

(6)

(ii) $y'' + 14y' + 49y = 0,\ y(-4) = -1,\ y'(-4) = 5$

Sol:

$$r^2 + 14r + 49 = (r+7)^2 = 0,\ r_{1,2} = -7$$

$$y(t) = c_1 e^{-7t} + c_2 t e^{-7t}$$

$$y'(t) = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 t e^{-7t}$$

$$-1 = y(-4)\ c_1 e^{28} - 4c_2 e^{28}$$

$$5 = y'(-4) = -7c_1 e^{28} + c_2 e^{28} + 28c_2 e^{28} = -7c_1 e^{28} + 29c_2 e^{28}$$

$$c_1 = -9e^{-28},\quad c_2 = -2e^{-28}$$

$$y(t) = -9e^{-7(t+4)} - 2te^{-7(t+4)}.$$

(iii) $y'' - 4y' + 9y = 0,\ y(0) = 0\ y'(0) = -8$

Sol:

$$r^2 - 4r + 9 = (r-2)^2 = 0,\ r_1, r_2 = 2$$

$$y(t) = c_1 e^{2t} + c_2 t e^{2t}$$

$$y'(t) = 2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t}$$

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$12 = y(0) = c_1$

$-3 = y'(0) = 2c_1 + c_2$

$c_1 = 12, \quad c_2 = -27,$

$$\underline{y(t) = 12e^{t} - 27te^{2t}}$$

(iv) $y'' - 8y' + 17y = 0, \quad y(0) = -4, \; y'(0) = -1.$

$r_1 = r_2 = 4.$

$y(x) = c_1 e^{4x} + c_2 e^{4x} x$

$y(t) = c_1 e^{4t} + c_2 t e^{4t}, \quad \frac{d}{dt}\left[te^{4t}\right]$
$= 4te^{4t} + e^{4t}$

$y'(t) = 4c_1 e^{4t} + 4c_2 te^{4t} + c_2 e^{4t}.$

$-2 = c_1 e^{4(0)} + c_2(0)e^{4(0)} \qquad -2 = c_1, \; c_1 = -2$

$-\frac{22}{3} = 4c_1 e^{4(0)} + 4c_2(0)e^{4(0)} + c_2 e^{4(0)}.$

$-\frac{22}{3} = 4(-2) + c_2 \qquad c_2 = \frac{1}{3}$

Sol:

$$\underline{y(x) = \frac{1}{3} e^{4x}(x - 6),}$$

(P) 8

Q3) Define Laplace form? with example

(A) Laplace form:

It's a technique for solving d.e. Here d.e of first domain is transformed in algebric equation of frequency domain form.

$$\mathcal{L} [g(t)] = 1$$

$$\mathcal{L} [u(t)] = \frac{1}{s}$$

and so on.

$$F(s) = \int_0^\infty F(t) \cdot e^{-st} dt$$

Example:

(1) $\int_0^\infty e^{ct} dt$

(2) $\mathcal{L} \{[\sin(at)]\}$

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(i) $f(t) = 6(e^{-5t}) + e^{3t} + 5(t^3) - 9.$

Sol:

$$F(s) = 6\,\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\,\frac{3!}{s^{3+1}} - 9\,\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}.$$

(ii) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t).$

Sol:

$$G(s) = 4\,\frac{s}{s^2+(4)^2} - 9\,\frac{4}{s^2+(4)^2} + 2\,\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}.$$

(iii) $h(t) = e^{3t} + \cos(6t) - e^{n3t}\cos 6t.$

$$H(s) = 3\,\frac{2}{s^2-(2)^2} + 3\,\frac{2}{s^2+(2)^2}$$

$$= \frac{6}{s^2-4} + \frac{6}{s^2+4}.$$

(9) (10)

(Q4)

(1) $y'' - 10y' + 9y = 5t \qquad y(0) = -1,\ y'(0) = 2.$

Sol:-

first convert into d.e

$$\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{5t\}$$

$$s^2Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = \frac{5}{s^2}$$

$$(s^2 - 10s + 9)Y(s) + s - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-s}{(s-9)(s-1)}$$

Combining terms.

$$Y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}.$$

$5 + 12s^2 - s^3 = As(s-9)(s-1) + B(s-9)(s-1) + C^2 s(s-1) + Ds^2(s-9)$.

$s=0 \qquad 5 = 9B$

$s=1 \qquad 16 = -8D$

$s=9 \qquad 248 = 648C$

$s=2 \qquad 45 = -14A + \frac{4345}{81}$

$B = \frac{5}{9}$, $D = -2$, $C = \frac{31}{81}$, $A = \frac{50}{81}$.

plugging the

$$Y(s) = \frac{\frac{50}{81}}{s} + \frac{5/9}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}.$$

Solution is

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{9t} - 2e^{t}.$$

(11) (12)

(ii) ~~$y'' - 10y' + 9y = 5t$~~, $y(0) = -1$, $y'(0) = 2$

(ii) $y'' - 6y' + 15y = 2\sin(3t)$, $y(0) = -1$, $y'(0) = -4$.

Sol.

$s^2 Y(s) - sy(0) - y'(0) - 6(sY(s) - y(0))$
$+ 15Y(s) = 2 \frac{3}{s^2+9}$.

$(s^2 - 6s + 15) Y(s) + s - 2 = \frac{6}{s^2+9}$.

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}.$$

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}.$$

$-s^3 + 2s^2 - 9s + 24 = (As+B)(s^2-6s+15) + (Cs+D)(s^2+9)$

$= (A+C)s^3 + (-6A + B + D)s^2 + (15A - 6B + 9C)s + 15B + 9D$.

s^3 : $A + C = -1$
s^2 : $-6A + B + D = 2$
s^1 : $15A - 6B + 9C = -9$
s^0 : $15B + 9D = 24$.

$A = \frac{1}{10}$
$B = \frac{-11}{10}$
$C = \frac{1}{10}$
$D = \frac{5}{2}$.

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11\,(s-3+3)+25}{(s-3)^2+6} \right)$$

$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{\frac{1}{3}\cdot 3}{s^2+9} - 11\,\frac{(s-3)}{(s-3)^2+6} - \frac{\frac{8\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6} \right)$$

Solution is

$$y(t) = \frac{1}{10}\left(\cos 3t + \frac{1}{3}\sin 3t - 11 e^{3t} \cos(\sqrt{6}t) - \frac{8}{\sqrt{6}} e^{3t} \sin(\sqrt{6}t) \right).$$