Name------------------- Hidayatullah
ID 16495

Section (B)

Department--------- BS Software Engineering
Semester------------- 2nd
Subject---------------- Discrete structure
Examination--------- Mid-Term
Date
April 14th, 2020
Teacher Muhammad Ibrar Khan

Note: Attempt all questions. Use examples and diagrams where necessary.
Q. 1

Which of the following are propositions?
a) Buy Premium Bonds!
b) The Apple Macintosh is a 16 bit computer.
c) There is a largest even number.
d) Why are we here?
e) $8+7=13$
f) $\mathrm{a}+\mathrm{b}=13$

## Answer : b) , c) and e) are propositions

Q. 2
p is " $\mathrm{x}<50$ "; q is " $\mathrm{x}>40^{\text {". }}$
Write as simply as you can:
(a) $\neg p \quad \rightarrow$ " $X$ is not less than 50 " or " $X \geq 50$ "
(b) $\neg \mathrm{q} \quad \rightarrow$ " $X$ is not greater than 40 " or " $X \leq 40$ "
(c) $\mathrm{p} \wedge \mathrm{q} \quad \rightarrow$ " X is less than $\mathbf{5 0}$ and greater than $\mathbf{4 0}$ "
(d) $\mathrm{p} \vee \mathrm{q} \quad \rightarrow$ " $\mathbf{X}$ is less than $\mathbf{5 0}$ or greater than $\mathbf{4 0}$ "
(e) $\neg \mathrm{p} \wedge \mathrm{q} \rightarrow$ " X is not less than $\mathbf{5 0}$ but less than $\mathbf{4 0}$ "
(f) $\neg p \wedge \neg q \rightarrow$ " $X$ is not less than $\mathbf{5 0}$ and $\mathbf{4 0}$ "
Q. 3

In each part of this question a proposition $p$ is defined. Which of the statements that follow the definition correspond to the proposition $\neg$ p? (There may be more than one correct answer.)
(a)
p is "Some people like Maths".
(a) "Some people dislike Maths"
(b) "Everybody dislikes Maths"
(c) "Everybody likes Maths"

Answer : (a) "Some people dislike Maths"
b)
p is "The answer is either 2 or 3 ".
(a) "Neither 2 nor 3 is the answer"
(b) "The answer is not 2 or it is not 3 "
(c) "The answer is not 2 and it is not 3 "

Answer : (a) "Neither 2 nor 3 is the answer" (c) 'The answer is not $\mathbf{2}$ and it is not $\mathbf{3}$ "
c)
p is "All people in my class are tall and thin".
(a) "Someone in my class is short and fat"
(b) "No-one in my class is tall and thin"
(c) "Someone in my class is short or fat"

Answer : (b) "No-one in my class is tall and thin"
Q. 4

Construct truth tables for:
a) $\neg p \vee \neg q$

| $\mathbf{p}$ | $\mathbf{q}$ | $\neg \mathbf{p}$ | $\neg \mathbf{q}$ | $\neg \mathbf{p} \vee \neg \mathbf{q}$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

b) $\mathbf{q} \wedge(\neg \mathbf{p} \vee \mathbf{q})$

| $\mathbf{p}$ | $\mathbf{q}$ | $\neg \mathbf{p}$ | $(\neg \mathbf{p} \vee \mathbf{q})$ | $\mathbf{q} \wedge(\neg \mathbf{p} \vee \mathbf{q})$ |
| :--- | :--- | :--- | :--- | :--- |
| T | T | F | T | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | F |

c) $\mathbf{p} \wedge(\mathbf{q} \vee \mathbf{r})$

| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{r}$ | $(\mathbf{q} \vee \mathbf{r})$ | $\mathbf{p} \wedge(\mathbf{q} \vee \mathbf{r})$ |
| :--- | :--- | :--- | :--- | :--- |
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | T | F |
| F | F | T | T | F |
| F | F | F | F | F |

d) $(\mathbf{p} \wedge \mathbf{q}) \vee \mathbf{r}$

| $\mathbf{p}$ | q | r | $(\mathbf{p} \wedge \mathbf{q})$ | $(\mathbf{p} \wedge \mathbf{q}) \vee \mathbf{r}$ |
| :--- | :--- | :--- | :--- | :--- |
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | F | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

## Q. 5

Use truth tables to show that:
$\neg((\mathrm{p} \vee \neg \mathrm{q}) \vee(\mathrm{r} \wedge(\mathrm{p} \vee \neg \mathrm{q}))) \equiv \neg \mathrm{p} \wedge \mathrm{q}$
L.H.S $\quad \neg((p \vee \neg q) \vee(r \wedge(p \vee \neg q)))$

| p | q | r | $\neg \mathrm{p}$ | $\neg \mathrm{q}$ | $\neg \mathrm{r}$ | $(\mathrm{p} \vee \neg \mathrm{q})$ | $\neg(\mathrm{p} \vee \neg \mathrm{q})$ | $(\mathrm{r} \wedge(\mathrm{p} \vee \neg \mathrm{q})$ | $\neg(\mathrm{p} \vee \neg \mathrm{q}) \vee(\mathrm{r} \wedge(\mathrm{p} \vee \neg \mathrm{q})))$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T | T | T | F | F | F | T | F | T | T |
| T | T | F | F | F | T | T | F | F | F |
| T | F | T | F | T | F | T | F | T | T |
| T | F | F | F | T | T | T | F | F | F |
| F | T | T | T | F | F | F | T | F | T |
| F | T | F | T | F | T | F | T | F | T |
| F | F | T | T | T | F | T | F | T | T |
| F | F | F | T | T | T | T | F | F | F |

R.H.S $\quad \neg \mathrm{p} \wedge q$

| p | q | $\neg \mathrm{p}$ | $\neg \mathrm{p} \wedge \mathrm{q}$ |
| :--- | :--- | :--- | :--- |
| T | T | F | F |
| T | F | F | F |
| F | T | T | T |

## Q. 6

Use the laws of logical propositions to prove that:
$(\mathrm{z} \wedge \mathrm{w}) \vee(\neg \mathrm{zw}) \vee(\mathrm{z} \wedge \neg \mathrm{w}) \equiv \mathrm{z} \vee \mathrm{w}$

State carefully which law you are using at each stage.

Answer : $(z \wedge \mathbf{w}) \vee(\neg z w) \vee(z \wedge \neg w)=(z \wedge \mathbf{w}) \vee(z \wedge \neg w) V(\neg z \wedge w) \quad$ Communicative Law

| $=(\mathbf{z} \wedge(\mathbf{w} \vee \neg \mathbf{w})) \mathrm{V}(\neg \mathbf{z} \wedge \mathbf{w})$ | Distributive Law |
| :---: | :---: |
| $=(z \wedge T) \vee(\neg \mathrm{Z} \wedge \mathrm{w})$ | Complement Law |
| $=\mathrm{zV}(\neg \mathrm{z} \wedge \mathbf{w})$ | Identity Law |
| $=(\mathrm{zV} \neg \mathrm{z}) \wedge(\mathrm{z} \vee \mathbf{w})$ | Distributive Law |
| $=\mathbf{T} \wedge(\mathbf{z} \vee \mathbf{w})$ | Complement Law |
| $=(\mathrm{z} \vee \mathbf{w}) \wedge$ T | Commutative Law |
| $=\mathrm{z} V \mathrm{w}$ | Identity Law |

