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Subject: Linear Algebra

(Question No: 1) 1)

let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

identify the (3,2) entry of AB

Solution

$Row_3(A)$ and $Col_2(B)$

$= [0, 1, -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} = -5$

QUESTION (3):-

$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$ Find A^{-1}

Solution

$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$

$|A| = 3 \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$

$|A| = 3(-4-6) + 2(-15-2) + (0-6)$

$|A| = -94$

2) Now

$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

(3) $$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$

QUESTION (2)

Part b Estimate the linear system of equation.

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

SOLUTION

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

$$Ax = b$$

$$A.b = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -4 \\ 3 & 1 & 1 & 3 \end{array}\right] \quad R_2 - R_1$$

(4)

$$R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & 5 & 6 \end{array}\right] \quad R_2 - 3R_1$$

$$R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & -13 & 26 \end{array}\right] \quad \begin{array}{l} R_3 - R_1 \\ R_3 + 2R_2 \end{array}$$

$$R = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \frac{-1}{13} R_3$$

$$x + y + 2z = 1 \quad \text{— (1)}$$
$$0x + 1y - 4z = 10 \quad \text{— (2)}$$
$$0x + 0y + 1z = -2 \quad \text{— (3)}$$

$$1z = -2$$
$$z = -2$$

Put $(z=-2)$ in eq (2)

$$+y-4z=10$$
$$+y-4(-2)=10$$
$$+y+8=10$$
$$y=10-8$$
$$y=2$$

Put these values in eq (1)

$$x+y+2z=1$$
$$x-2+2(-2)=1$$
$$x-2-4=1$$
$$x-6=1$$
$$x=1+6$$
$$x=7$$

(QUESTION 2)

Part a:-

If A and B are non matrices $|A| = 2$ and $|B| = -3$ calculate $|A^{-1} B^{-1}|$

SOLUTION

Since $|A - B^t| = |A^{-1}|\ |B^t|$

$= \frac{1}{|A|} |B|$

So

$|A^{-1} B^t| = \frac{1}{|A|}\ |B|$

$= \frac{1}{2} \cdot 3$

$= 3/2$ Ans

(QUESTION :-1)

(Part b:)

Label the quadratic polynomial the interpolate the point (1, 3), (2, 4), (3, 4)

Solution

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_2 x_2^2 + a_1 x_3 + a_0 = y_3$$

Now

$$(x_1, y_1) = (1, 3)$$
$$(x_2, y_1) = (2, 4)$$
$$(x_3, y_3) = (3, 4)$$

$$a_2 + a_1 + a_0 = 3$$
$$4a_2 + 2a_1 + a_0 = 4$$
$$4a_2 + 3a_1 + a_0 = 4$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} \quad R_2 - 4R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -3 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -8 \\ 4 \end{bmatrix} \quad R_3 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -1 & -3 & 8 \end{array}\right] \quad R_3 + 1R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & -2 & -5 \end{array}\right]$$

$a_2 + a_1 + a_0 = 3 \longrightarrow ①$

$-2a_1 - 3a_0 = -8 \longrightarrow ②$

$$\frac{-\cancel{2}a_0}{\cancel{2}} = \frac{-5}{-2}$$

$$a_0 = 2.5$$

$2a_1 - 3(2.5) = -8$

$-2a_1 - 7.5 = -8$

$-2a_1 = -8 + 7.5$

$$\frac{2a_1}{-2} = \frac{-0.5}{-2}$$

$$\boxed{a_1 = 0.25}$$

$a_2 + a_1 + a_0 = 3$

$a_2 + 0.25 + 2.5 = 3$

$a_2 + 2.75 = 3$

$a_2 = 3 - 2.75$

$$\boxed{a_2 = 0.25}$$