Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Engr Phir Meher Ali Shah**	**Total Marks:**	30

Student Details

Name: **Irshad khan** **Student ID: 12403**

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n \ , & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{ 2, \underset{\uparrow}{1}, -2, 3, -4 \right\} \quad , h[n] = \left\{ \underset{\uparrow}{3}, 1, 2, 1, 4 \right\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Q1:

(a) consider the following analog signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine minimum sampling rate required to avoid alaising.

$$f_s \geq 2 f_{max}$$

$$f = \frac{\omega}{2\pi}$$

$$f_1 = \frac{100\pi}{2\pi}$$

$$f_2 = \frac{200\pi}{2\pi}$$

According to sampling Theorm

$f_1 = 100HZ$. $f_2 = 200HZ$

$Fs \geq f_{max}$

f_2 is (greater than f_1)

$fs \geq 2 \times 100$

$fs = 200HZ$

$f_1 = 50\ HZ$, $f_2 = 100\ HZ$

so f_2 is max and greater them f_1)

$fs \geq 2 \times 100HZ$ sample frequency to avoid alising

(ii) $Fs = 100\ HZ \Rightarrow f = \frac{100}{2} \Rightarrow \boxed{50HZ}$

so. f_1 become

$$f_1' = \frac{f_1}{f_s} = \frac{50}{100} = 0.5\ HZ$$

Now f_2 becomes.

$$f_2 = \frac{f_2}{100} = \frac{100}{100} = 1\ HZ$$

This is the max frequency that can be represented uniquely by the sampled signal.

$$x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

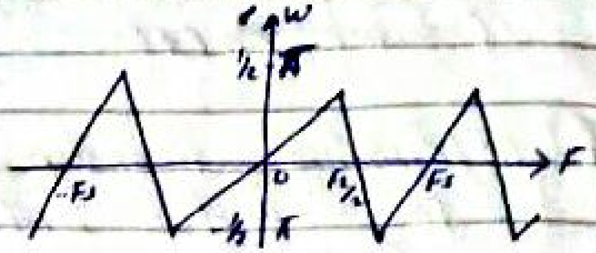
$$3\cos \pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n.$$

So.

$$w_1' = 2\pi f_1$$

$$w_1' = 2\pi \times 0.5$$

$$\boxed{w_1' = \pi}$$



Now,

$$\omega_2' = 2\pi f_2'$$

$$\omega_2' = 2\pi \times 1$$

$$\boxed{\omega_2' = 2\pi}$$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The signal are.

$x[n] = 3\cos \pi n + 4\sin 2\pi n.$

The effect of sampling rate on the newly generated discrete time signal is that there will be no alising phenomenon. Means there will not present unwanted Component in the re-construction of the signals. The re construct original signal.

[illegible] = [illegible] , [illegible] = [illegible]

[illegible] = $\frac{[illegible]}{[illegible]}$, [illegible] = [illegible]

[illegible]

(iii) what is the analog signal $y_a(t)$ we can re constructed from the sampling if we use ideal interpolation?

Sol: Folding frequency of the sampled signal is.

Folding frequency $= F_s/2 \Rightarrow \frac{100}{2}$

$= \boxed{50HZ}$

we have frequency of the original signal

$f_1 = 50HZ$, $f_2 = 100HZ$

Both the frequency are either equal are greater than folding frequency.

Hence for ideal interpolation we can construct the original signal.

$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t$

The original signal is constructed b/c we use sampling frequency at Nyquist rate

we can also reconstruct the signal for sampling frequency above the nyquist rate

The analog signal we can remove or re construct is:

$y_a(t) = 3\cos 100\pi t$: Ans

Q1:

(b) Consider a discrete time signal which is given by.

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

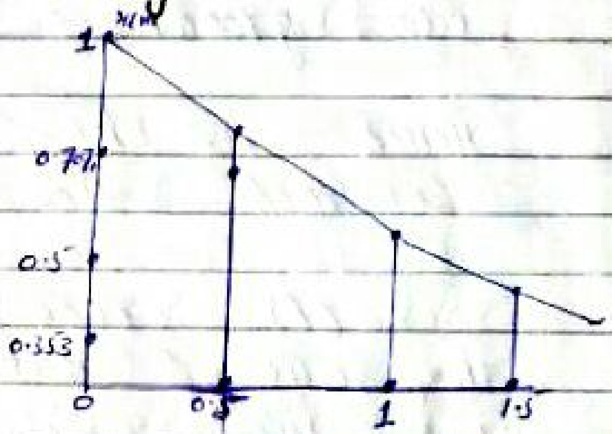
$F_s = 2\,Hz$

$F_s = 1/T$

$T = 1/F_s \Rightarrow 1/2 = 0.5$ sec.

(i) Draw the sampled signal.

x_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 sec.

(ii) Soln:

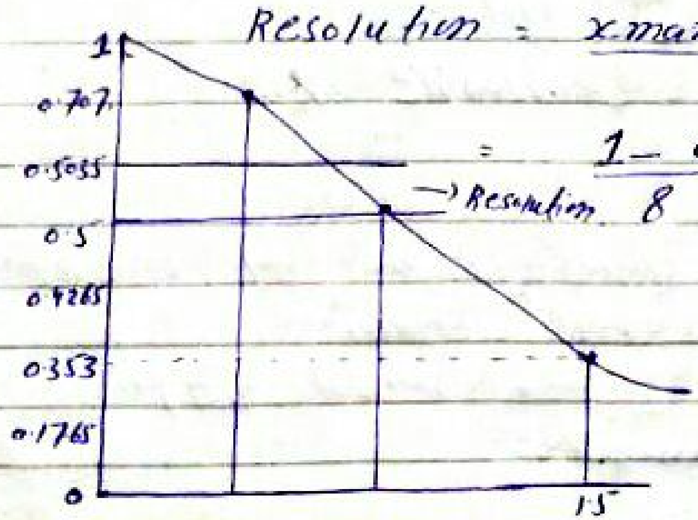
$L = 2^n$

$n = $ bits $= 3$

$L = 2^3 = 8$ levels.

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1 - 0}{8} \Rightarrow 0.125$$



(iii)

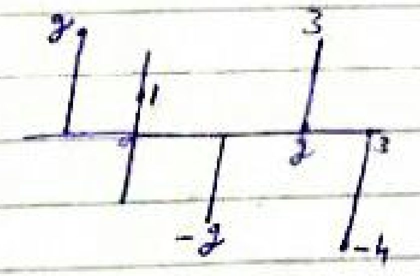
	Discrete time signal	Truncation	Rounding	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

Q. 2:

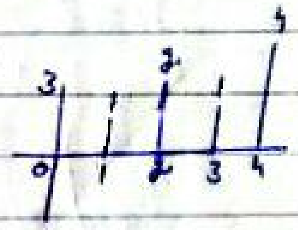
(a) Determine the response of the system to the following input signal with given impulse response.

$$x[n] = 2, \frac{1}{\uparrow}, -2, 3, -4, \quad h[n] = \{\frac{3}{\uparrow}, 1, 2, 1, 4\}$$

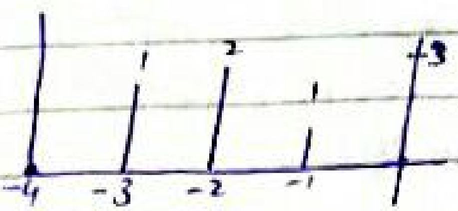
Soln: $y[n] = \sum x[k]\, h(n-k)$



h (K)



h(-K) Folded signal.

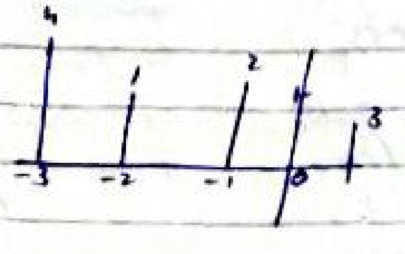


$$y[0] = \sum_{k=1}^{0} x(-1)\, h(-1) + x(0)\, h(0)$$

putting the values

$y(0) = 2(1) + (1)(3)$
$= 2 + 3 = 5$

for $n = 1$
$h(1-k)$



$y(1) = \sum_{k=1}^{1} x(n)\, h(1-k)$

$= x(-1)\, h(-1) + x(0)\, h(0) + x(1)\, h(1)$
$= +(x)\, h(1) + x(2)\, x(2) + 0(x)(3)\, h(3)$

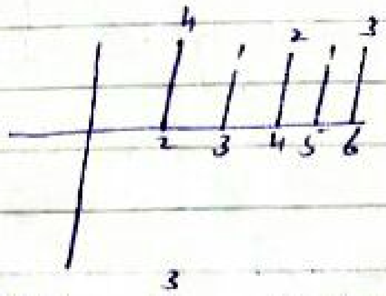
putting the values

$2(2) + (1)(1) + (3)(-2) \Rightarrow 4 + 1 - 6 = \boxed{-1}$

$n = 2$
$h[2-k] \Rightarrow y(2) = 2(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$8 + 1 - 4 + 3 - 12 \boxed{= -4}$

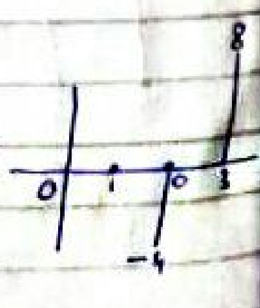
$n = 3$
$h(3-k)$



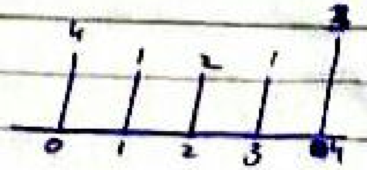
$y(3) = \sum_{k=2}^{3} x(n)\, h(n-k)$

$x(2)\, h(2) + x(3)\, h(3)$

$(3)(4) + (-4)(1) = 12 - 4 \boxed{= 8}$



For $n = 4$



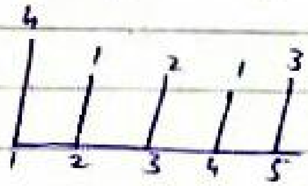
$h[4-k]$

$f(4) = \sum_{k=0}^{3} x[n]\, h(4-k)$

$x(0)\, h(0) + x(1)\, h(1) + x(2)\, h(2)$
$+ x(3)\, h(3)$
$(1)(4) + (-2)(1) + (3)(2) + (-4)(1)$

$y[4] = 4 - 2 + 6 - 4 \;\boxed{= 4}$

For $n = 5$
$h[5-k]$



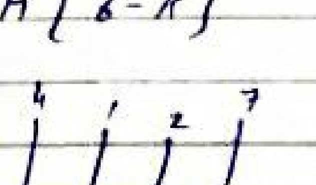
$y[5] = \sum_{k=1}^{3} x(n)\, h[5-k]$

$x(1)\, h(1) + x(2)\, h(2) + x(3)\, h(3)$

$-2(4) + (3)(1) + (-4)(2)$

$-8 + 3 - 8$
$y[5] = \boxed{-13}$

For $n = 6$
$h[6-k]$



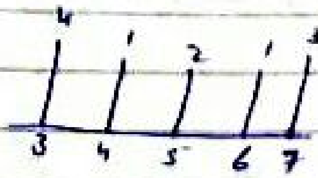
$y[6] = 3(4) + (1)(-4)$

$12 - 4$

$y[6] \boxed{= 8}$

For $n = 7$

$h[7-k]$

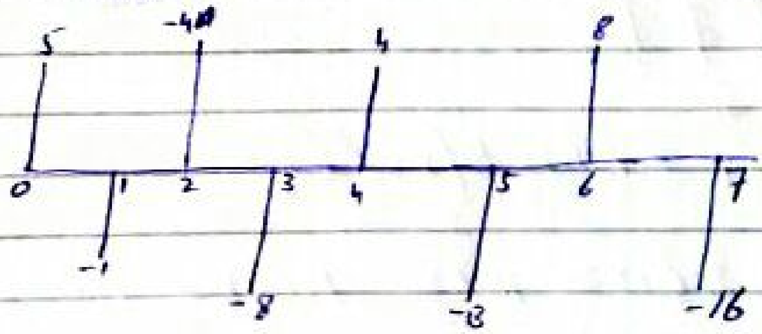


$y[7] = x(3)\ h(3)$

$4 \times (-4)$

$= -16$

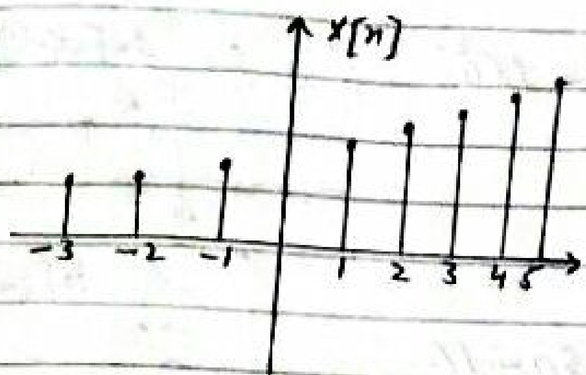
$y[n] =$

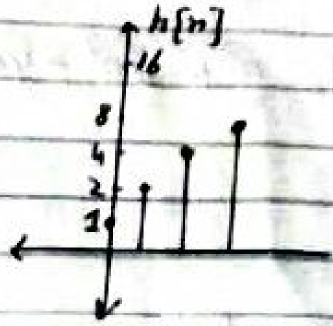


Q9: (b) Compute the convolution of

$$x[n] = \begin{cases} a^{n+1} & , -3 \le n \le 5 \\ 0 & , \text{elsewhere} \end{cases}$$

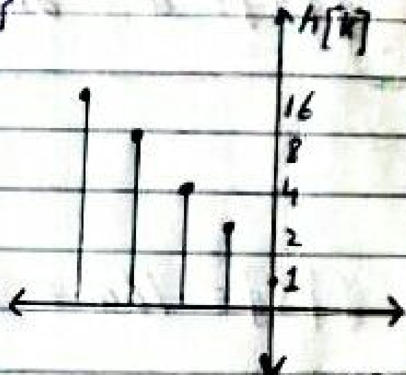
$$h[n] = \begin{cases} 2^{n} & , 0 \le n \le 4 \\ 0 & , \text{else where} \end{cases}$$



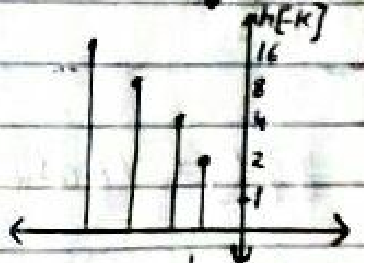


$$x[n] = \{ a^{-2}, a^{-1}, 1, \underset{\uparrow}{a}, a^{2}, a^{3}, a^{4}, a^{5}, a^{6} \}$$

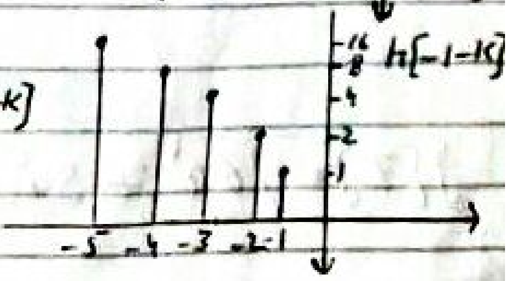
$$h[n] = \{ \underset{\uparrow}{1} \; 2 \; 4 \; 8, 16 \}$$



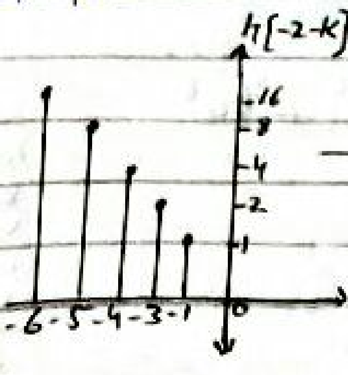
$$y[n_0] = \sum_{k=-\infty}^{\infty} x[k]\, h[n_0 - k]$$



$$y[0] = a - 2 + 4a^{-1} + 8a^{-2}$$

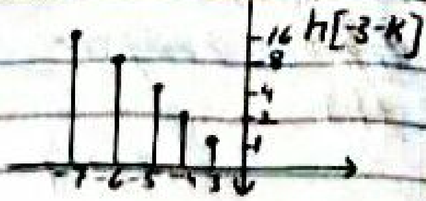


$$y[-1] = 1 + 2a^{-1} + 4a^{-2}$$

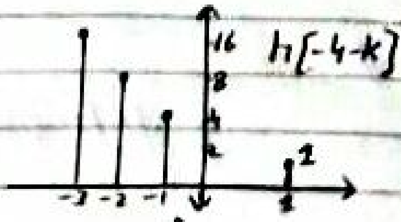


$$y[-2] = 2a^{-2} + a^{-1}$$

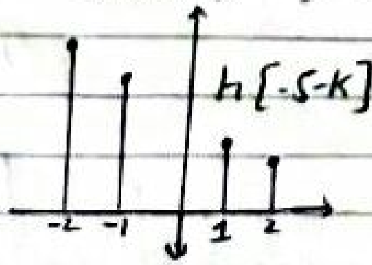
$y[-3] = a^{-2}$



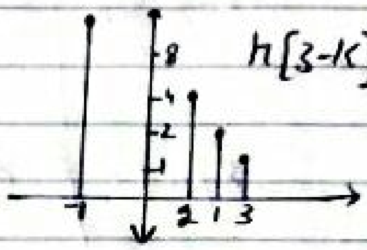
$y[1] = a^2 + 2a + 4 + 8a^{-1} + 16a^{-2}$



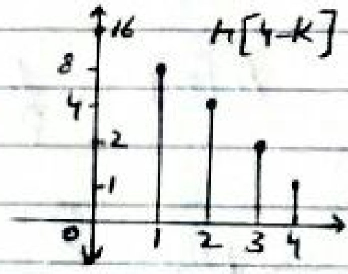
$y[2] = a^3 + 2a^2 + 4a + 8 + 16a^{-1}$



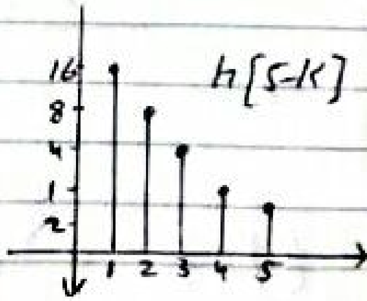
$y[3] = a^4 + 2a^3 + 4a^2 + 8a + 16$



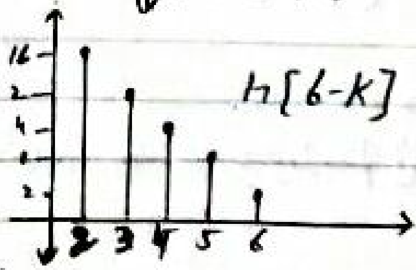
$y[4] = a^5 + 2a^4 + 4a^3 + 8a^2 + 16a$



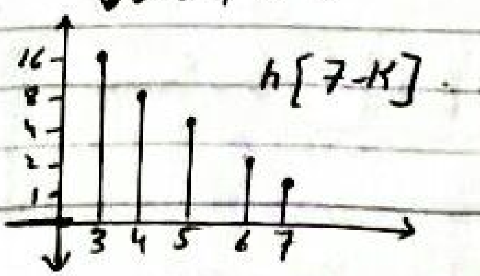
$y[5] = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$



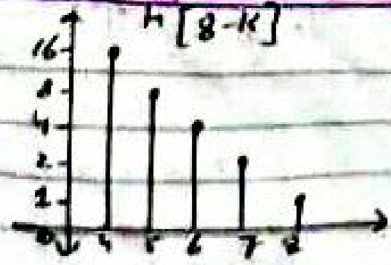
$y[6] = 16a^3 + 8a^4 + 4a^5 + 2a^6$



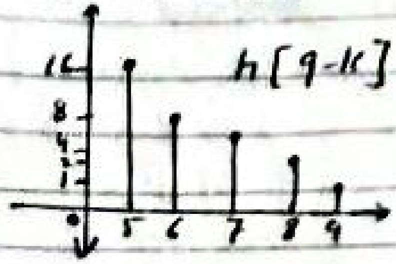
$y[7] = 16a^4 + 8a^5 + 4a^6$



$y[8] = 16a^5 + 8a^6$



$y[9] = 16\,a^6$.



Q3:-

(ii) $x(n) = \left\{ \begin{array}{ll} (\frac{1}{4})^n & n \geq 0 \\ (\frac{1}{3})^n & n < 0 \end{array} \right\}$

Soln:- $x(n) = \left\{ \begin{array}{ll} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{array} \right\}$

As we know that
Z - Transform pair is.

$x(n) = a^n u(n) \longleftrightarrow X(Z) = \frac{1}{1-\alpha Z^{-1}}$ ROC $|Z| >$ eq (A).

By using eq (A) we can put values.

$$X(Z) = \sum_{n=0}^{\infty} (\tfrac{1}{4})^n Z^{-n} + \sum_{n=-\infty}^{0} (\tfrac{1}{3})^n Z^{-n} - 1$$

$$\frac{1}{1-\frac{1}{4}Z^{-1}} + \sum_{n=0}^{\infty} (\tfrac{1}{3})^n Z^n - 1$$

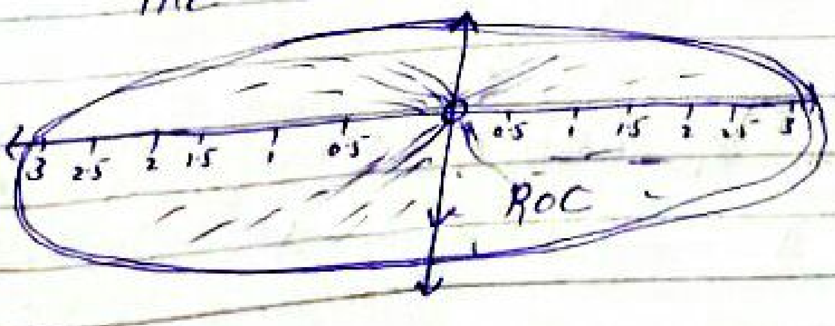
in simplest form.

$$\frac{1}{1-\frac{1}{4}Z^{-1}} + \frac{1}{1-\frac{1}{3}Z} - 1$$

$$= \frac{7/12}{(1-\frac{1}{4}Z^{-1})(1-\frac{1}{3}Z)}$$

R.W
$\frac{1}{4} + \frac{1}{3}$
$\frac{3+4}{12}$
$= 7/12$

The ROC is $\frac{1}{4} < |Z| < 3$.
The sketch is as under.



Q3:

(ii) $$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n & n \ge 0 \\ 0 & \text{Elsewhere} \end{cases}$$

using z- transform pair equation here.

i.e. $x[n] = \alpha^n u(n) \longleftrightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$ RoC $|z| > |\alpha|$ Eq.(3).

By using z- transform.

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 0.3^n z^{-n}$$

$$\Rightarrow \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

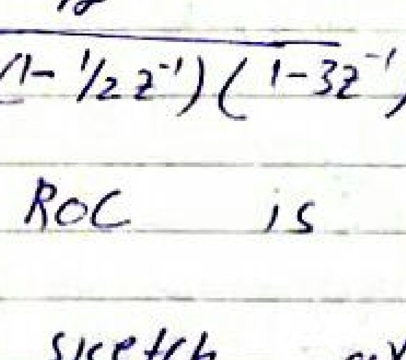
By cross multiplication.

$$\frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$\frac{-\frac{5}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

The RoC is $|z| > 3$.

The sketch are.



—— X —— X —— X.