IQRA NATIONAL UNIVERSITY PESHAWAR

DEPARTMENT OF CIVIL ENGINEERING

SUBJECT: APPLIED CALCULAS

NAME: ABDUL BASIT

ID: 7776 SECTION: C

Q.NO (01) ANSWER

QNO 01 The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t<0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

a) state any point of discountinivty.

b) Find, if they exist.

i) $\lim_{t \to 3} g$

Sol

a) To check possibility of disountinity of the funtion is $t=0$ & 4

→ First at $t=0 \Rightarrow g(t) = t^2 \Rightarrow g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2 \Rightarrow \lim_{h \to 0} 1+h^2+2h$

⇒ Apply limit.

$= 1 + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3.$

$\lim_{h \to 0} 2(1-h) + 3$

Apply limit

$= 2-2(0)+3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

$\Rightarrow$ Now at $t = 4$

$g(4) = 2(4)+3$

$= 11$

For R.H.L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$= \lim_{h\to 0} 2+2h+3$$

$\Rightarrow$ Apply limits

$= 2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim g(1-h) = 12$

$g(4) =$ R.H.L $\neq$ L.H.L

Point of discontinivty is at $t=4$

(b) Find, if they exist

i) $\lim_{t\to 3} g$

For $g(t) = t^3$

R.H.L

$\lim_{h\to 3} g(1+h) = \lim_{h\to 3} (1+h)^2$

$= \lim_{h\to 3} 1+h^2+2h$

Apply limits

$= 1+3^2+2(3) \Rightarrow 16$

L.H.L

$\lim_{h\to 3} g(1-h) = \lim_{h\to 3} 2t+3$

$= \lim_{h\to 3} 2(1-h)+3$

$= \lim_{h\to 3} 2 -2h+3$

Apply limit.

$= 2-2(3)+3$

$= 2-6+3$

$= -1$

R.H.L $\neq$ L.H.L do not exist

Since L.H.L is $\boxed{-ve}$

-------------------------END OF Q.NO 1-------------------------

Q.NO (03 PART A)   ANSWER

①

Q NO # (03) (i)

$1 + xy = x^2 + y^2$ to find y''

Diff w.r.t x

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2) \quad \text{or} \quad x\frac{dy}{dx} + y\frac{dx}{dx}$$

$$= 2x\cancel{\frac{dy}{dx}} + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{\cancel{(x-2y)}}{\cancel{(x-2y)}}\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$y' = \frac{2x-y}{x-2y}$$

again dif w.r.t x.

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(x-2y')}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' \quad \frac{\cancel{2x} - xy' - 4y^2 - \cancel{2x} + 4xy' + y - 2yy'}{(x-2y)^2}$$

$$y'' = \boxed{\frac{3xy' - 4y^2 - 2yy'}{(x-2y)^2}}$$

--------------------------END OF Q.NO 3A------------------------

Q.NO (03 PART B) ANSWER

QNO $\underline{\underline{\underline{03}}}$ (B)

$$y = x^3 (1+x)^9 \quad e^{6x}$$

$$\log y = \log \left(x^3 (1+x)^3 \quad e^{6x}\right.$$

$$\log y = \log x^3 + \log (1+x)^7 + \log e^{6x}$$

log

$$\frac{1}{y} \frac{1}{(y^2)} \frac{dy}{dx} = \frac{3x^2}{x^3} + \frac{9\cancel{(1+x)^8}}{\cancel{(1+x)^9}} + 6x$$

$$\frac{1}{y^3} \frac{dy}{dx} = \frac{3}{x^2} + 9 + 6x$$

$$\frac{1}{y^3} \frac{dy}{dx} = 3 + 9x^2 + 6x^3$$

$$\frac{1}{y^3} \frac{dy}{dx} = 6x^3 + 9x^2 + 3$$

$$\frac{dy}{dx} = y^3 (6x^3 + 9x^2 + 3)$$

-------------------------END OF Q.NO 3 B-----------------------

Q.NO (02) **ANSWER**

Q NO $\underline{02}$

$$y(x) = x^2 + \sin x$$

Since we ~~have~~ ~~know~~ know that the maclurin Series is

$$y(x) = y(x) + y'(x)(x - x_0) + \frac{y''(x_0)(x}{2!}$$

Put $\boxed{x_0 = 0}$

$$y(x) = y(0) + (x-0)\, y'(0) + \frac{(x-0)^2 y''(0)}{2!}$$

$$y(x) = y(0) + x y'(0) + \frac{x^2 y''(0)}{2!} + \cdots \text{ —}$$

Now find

$y(0) = ?$

$$y(x) = x^2 + \sin x$$

$$y(0) = 0 + \sin 0$$

$$= 0 + 0$$

$$\boxed{y(0) = 0}$$

$$y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + \cos 0$$

$$= 0 + 1$$

$$\boxed{y'(0) = 1}$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 - \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2$$

$$\boxed{y''(0) = 2}$$

Now

$y''(x) = 2 - \sin x$

$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$

$= 0 - \cos x$

$y'''(x) = 0 - \cos x$

$y'''(0) = -\cos x$

$y'''(0) = -1$

Put in eq ①

$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$

$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$

$= x + x^2 - \frac{x^3}{3!} + \cdots$

So

$y(x) = x + x^2 - \frac{x^3}{\cancel{x}3!} + \cdots$

--------------------------END OF Q.NO 2--------------------------