Q No # 1 :-

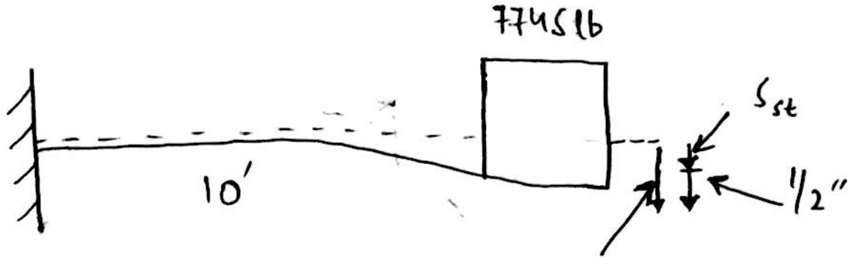
A) :- GIVEN DATA :-

$E = 29000$ ksi

$I = 150\ in^4$

$\delta_{st} = 7745\ lb$

A) DIAGRAM :-



Sol :-

The general EOM for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped $(c=0)$ undergoing free vibration $(P(t)=0)$

Hence general EOM become $ku + m\ddot{u} = 0$ —(i)

$$k = \frac{3EI}{l^3} = \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$= \underline{7.55\ k/in}$$

In order to eliminate the chances of mistake during calculation, It is more appropriate to use fundamental units like lb, ft sec or kg, m, Sec.

$$k = 7.55\ k/in = \underline{90625\ lb/ft}$$

$$m = \frac{7745\ lb\ sec^2}{32.2\ ft} = \underline{240.5\ slug.}$$

$$\Rightarrow \quad W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{240.5}} = \underline{19.41\ rad/sec}$$

$$\Rightarrow \quad T_n = \frac{2\pi}{W_n} = \frac{2\pi}{19.41} = \underline{0.323\ sec.}$$

substituting the corresponding values in eq(i)

$$90625u + 240.5\ddot{u} = 0$$

where 'k' is in lb/ft and 'm' is in lb sec/ft²

General solution to the EOM for undamped vibration is

$$u(t) = u(0)\cos(w_n t) + \dot{u}(0)/w_n \sin(W_n t)$$

$$u(0) = 1/2'' = 1/24\ ft \text{ and } \dot{u}(0) = 0$$

$$u(t) = (1/24) \times \cos(19.41t) + 0 - (1/24) \times \cos(19.41t)$$

Equivalent static force at any time 't' is:-

$$f_s(t) = k.u(t) = \frac{90625 \times \cos(19.41t)}{24}$$

$$f_s(t) = 3776 \cos(19.41t)$$

Amplitude of dynamic displacement, u_o for undamped vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

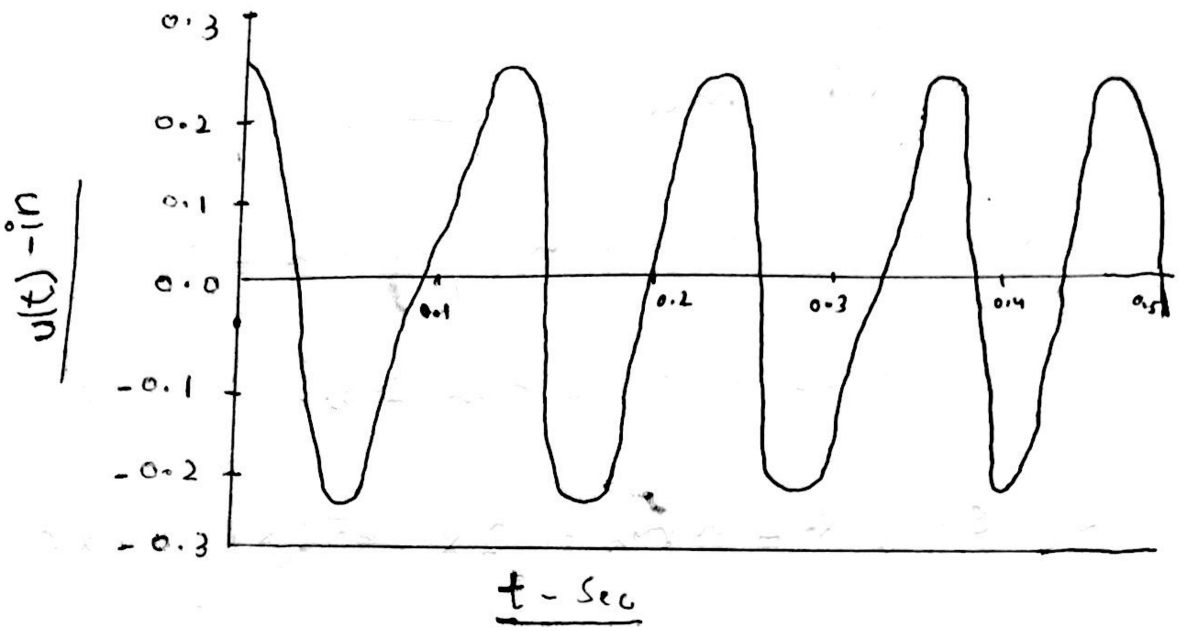
$$u_o = \sqrt{\left[(1/24)^2 + 0\right)} = 1/24 \text{ ft}$$

Amplitude for equivalent static force is;

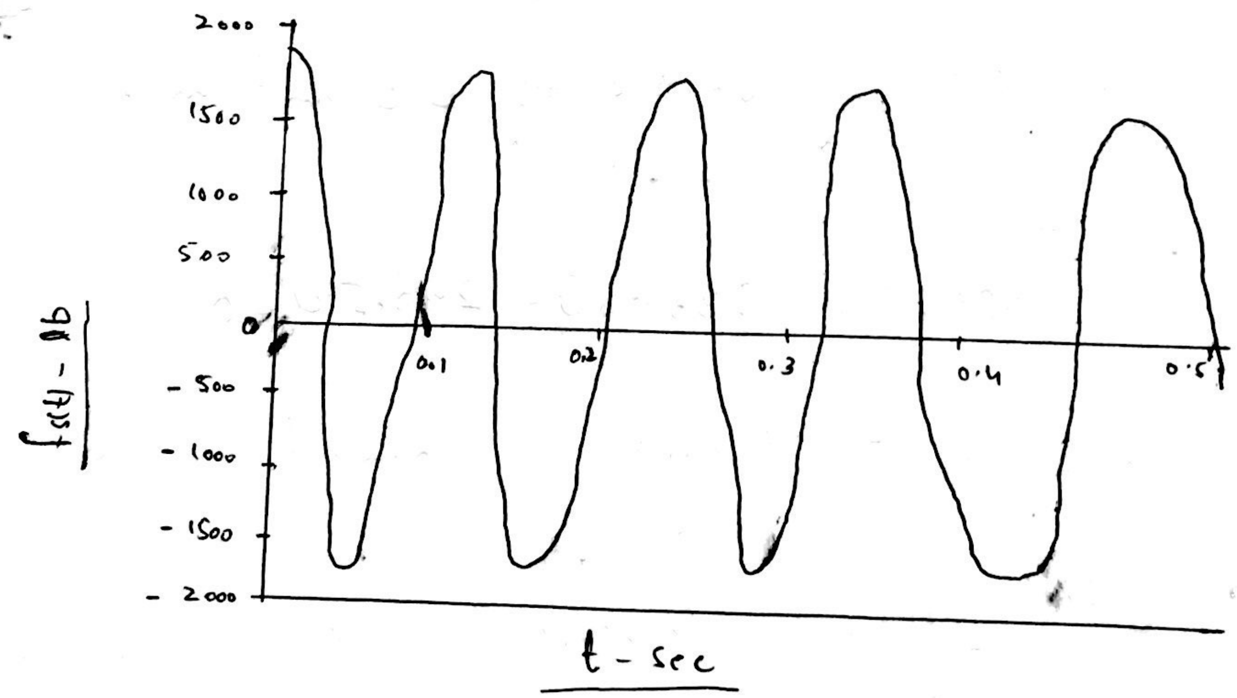
$$ku_o = 90625 \times 1/24 = 3776 \text{ lb}$$

*) :- GRAPH :-

*) :- UNDAMPED FREE VIBRATION :-



Variation of displacement with time



Variation of equivalent satetic force with time

Q NO # 2 :-

Solution :-

E.O.M for damped vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{---(i)}$$

It is known from question 1 that

$k = 90625$ lb/ft and $m = 240.5$ lb. sec^2/ft

$$c = \zeta \times 2m\omega_n = 2 \times 240.5 \times 19.41 \times 0.025$$

$$c = 233.40 \ \text{lb}\frac{\text{sec}}{\text{ft}}$$

By substituting values of k, c and m in (i) we get

$$90625u + 233.40\dot{u} + 240.5\ddot{u} = 0$$

solution to the E.O.M for damped free vibration

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}[\dot{u}(0) + u(0)\zeta\omega_n].\sin(\omega_D t)\right]$$

$\omega_D = 19.41$ rad/sec

$$u(t) = e^{-0.025 \times 19.41 t}\left[\frac{1}{24} \times \cos(19.41t) + \frac{1}{19.41} \times \left[0 + \frac{1}{24} \times 0.025 \times 19.41 \times \sin(19.41t)\right.\right.$$

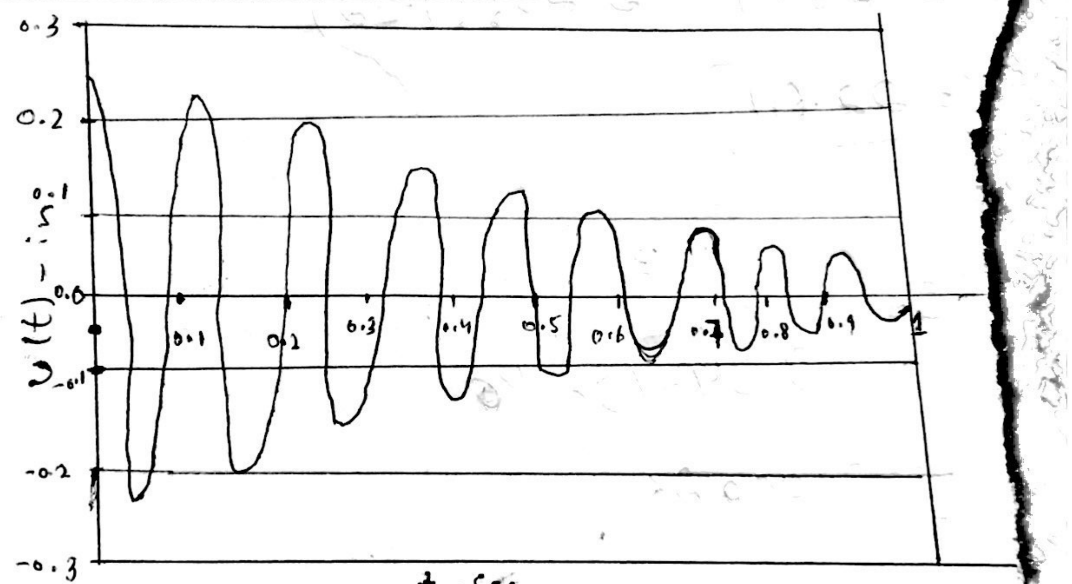
$$u(t) = e^{-0.485t}\left[0.0416 \times \cos(19.41t) + 0.0515 \times \sin(19.41t)\right]$$

$$f_s(t) = k \cdot u(t) = 90625 \times u(t)$$

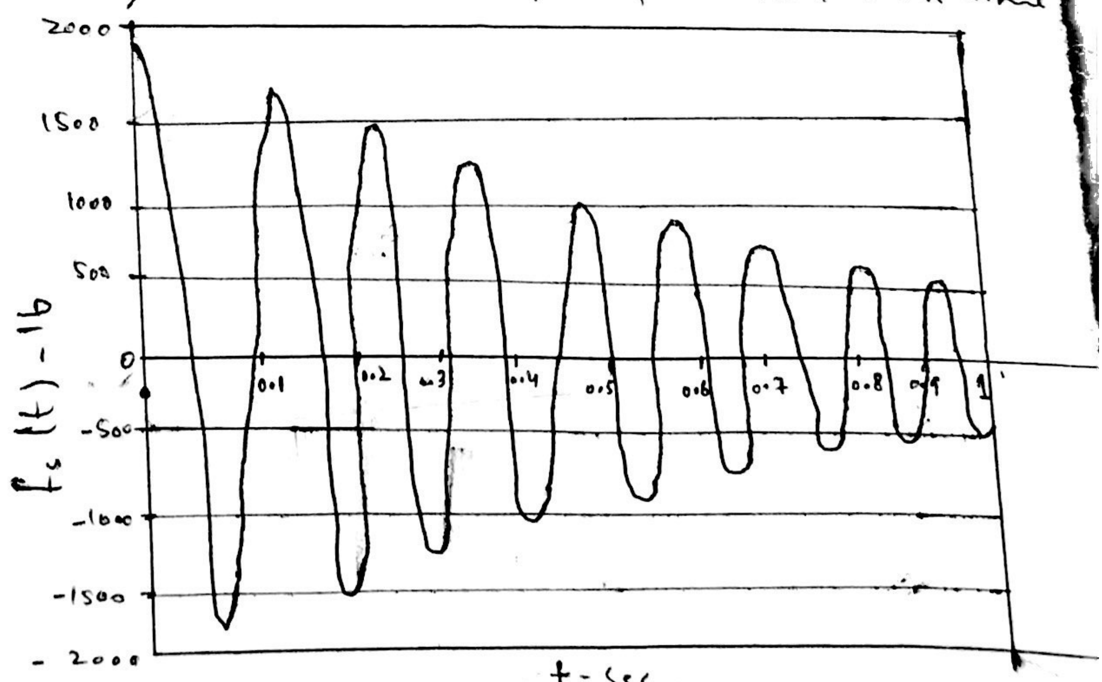
$$f_s(t) = e^{-0.483t}\left[3776 \cos(19.41t) + 47.13 \times \sin(19.41t)\right]$$

*) DIAGRAM :-

*) :DAMPED FREE VIBRATION:-



variation of displacement with time



variation of Equivalent static forces with time

Q No # 3 :-

*): Given Data :-

*) Force = 60 kips

*) $u_1 = \frac{7745}{1000} = 7.745$ in

*) After; j = 7 (cycles)

*) completed = 3.57 sec

*) u_{J+1} = 2.286 cm = 0.9 in

*) Ignore the vertical vibration

*) Required :-

(a) Damping ratios

(b) Natural period of undamped vibration

(c) stiffness of structures

(d) weight of tank

(e) Damping co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5".

4) Solution :-

(a)

As ;

$$j = \frac{1}{2\pi \zeta} \ln \left[\frac{u_1}{u_{j+1}} \right]$$

By putting values;

$$7 = \frac{1}{2(3.14)\zeta} \ln \left(\frac{7.745}{0.9} \right)$$

$$\zeta = (7 \times 2 \times 3.14) = 2.152$$

$$\zeta = (43.96) = 2.152$$

$$\zeta = \frac{2.152}{43.96}$$

$$= 0.0489$$

$$\boxed{\zeta = 4.89\%}$$

(b) :- T_n = ?

As seven cycles are completed in "3.57" sec.

Thus time required to complete 1 cycle = 7/3.57 = 1.96 sec

$$T_D = 1.96 \text{ sec}$$

Now;

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n} \left(\sqrt{1-\zeta^2} \right)$$

As;

$$T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

(9)

$\Rightarrow T_n = T_D \left(\sqrt{1-\xi^2}\right) \Rightarrow 1.96\left(\sqrt{1-(0.0489)^2}\right)$

$\left(\underline{T_n = 1.957 \text{ sec}}\right) \rightarrow$ (Natural period of undamped vibration)

(c) stiffness of structure = k = ?

As;

$$k = \frac{F.\cos\theta}{2} \Rightarrow \frac{60.\cos(60^\circ)}{2} = 15k/in \quad \left(\begin{matrix} F = 60kips \\ \theta = 60^\circ \end{matrix}\right)$$

$$\left(\underline{k = 18000 lb/ft}\right)$$

(d) weight of tank:- w = ?

As;

$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{(w/g)}} = \sqrt{\frac{k.g}{w}}$$

$$\Rightarrow w_n^2 = \frac{k.g}{w} \Rightarrow \left(w = \frac{k.g}{w_n^2}\right)$$

By putting values of $w_n = 2\pi/T_n$

$$w = \frac{k.g}{(4\pi^2/T_n^2)} = k.g\left(\frac{T_n^2}{4\pi^2}\right)$$

$$w = 18000 lb/ft \cdot 32.2 ft/sec^2 \left(\frac{(1.957)^2}{4(3.14)^2}\right)$$

$$\left(w = 56284.75 lb = \underline{56.284 k lb}\right)$$

(e) Damping Co-efficient; "C" = ?

It is know that; $\zeta = \dfrac{c}{2m W_n}$

$$\Rightarrow C = \zeta(2m w_n) = \zeta\left(2m\left(2\pi/T_n\right)\right)$$

By Putting values;

$$C = 0.0489\left(2\left(\frac{56284}{32.2}\right)\left(2\times\frac{3.14}{1.957}\right)\right)$$

$$\left(\underline{c = 548.575 \text{ lb.sec/ft}}\right)$$

(f) No. of cycles to reduce displacement altitude from "7.745 into 0.5 in":-

$J = ?$

$$J = \frac{1}{2\pi\zeta}\ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0489)}\ln\left(\frac{7.745}{0.9}\right)$$

$$= 3.256 \times 2.15$$

$$= 7.0004 \Rightarrow \text{ok}$$

$$\left(\underline{j = 7 \text{ cycles}}\right)$$