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Section B

Subject D. Equation

Question No 1

Part a

$W = \sin(x+ct) + \cos(2x+2ct)$

Solution:

$W = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{dw}{dt} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{d^2w}{d^2t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct)$$

$$* 4c^2 \text{ ...... } \underline{1}$$

$$\frac{dw}{dx} = \cos(x+ct) - \sin(2x+2ct) \times 2$$

$$\frac{d^2w}{dx^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4 \cdot \cos(x+2ct)\right]$$

$$\frac{d^2w}{dt^2} = +c\left[-\sin(x+ct) - 4(\cos+2ct)\right]$$

$$c^2 \cdot \frac{\partial^2 w}{\partial x^2}$$

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Part B

$W = \tan(2x + ct)$

Solution

$$\frac{dw}{dx} = \frac{1}{1+(2x+ct} \times 2 = \frac{2}{1+4x^2+4xct+c^2t^2}$$

$$\frac{d^2w}{dx^2} = -2(1+4x^2+4xct+c^2t^2)^{-2}(8x+4ct)$$

and

$$\frac{dw}{dt} = \frac{1}{1+(2x+ct)} \times c = \frac{c}{1+4x^2+4xct+c^2t^2}$$

$$\frac{d^2w}{dt^2} = -c/(1-4x^2+4xct+c^2t^2) \times (4x+2c^2t)$$

$$\frac{d^2w^2}{dt^2} = \frac{-c(4x+2c^2t)}{(1+4x^2+4xct+c^2t^2)^2}$$

$$\frac{d^2w}{dt^2} = \frac{d_2w}{dx_2^2} \times c^2/_4 \text{ Proved}$$

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Question No 3

$$y'' - 4y' + 13y = 8\sin 3x \quad — (1)$$

$$y(0) = 1$$

$$y'(0) = 2$$

Solution:

Homogenous eq of (1) is

$$y'' - 4y' + 13y = 0 \longrightarrow (2)$$

Change eq (2) into Auxilary equation

Put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

use Quadratic formula

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

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$$m = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$m_1 = 2+3i$

$m_2 = 2-3i$

$y_c = e^{2x}(c_1\cos 3x + c_2 \sin 3x) \to (i)$

$y_P = A\cos 3x + B\sin 3x$ — (x)

D.ff w.r.t "x)

$y'P = -3A\sin 3x + 3B\cos 3x$

Again Diff wit "x"

$y''_p = -9A\cos 3x - 9B\sin 3x$

Put in 1

$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x$

$+ 3B\cos 3x) + 13(A\cos 3x + B\sin 3x)$

$= 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x =$

$8\sin 3x$

Comparing coefficients

$\sin 3x \Rightarrow 4B + 12A = 8$

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$A = 3B \longrightarrow$ b

Put B in "a"

$4B + 12(3B) = 8$

$4B + 36(B) = 8$

$40B = 8$

$B = 1/5$ — c

Put c in b

$A = 3/5$ — (d)

Put "c" and "d" in (x)

$y_p = 3/5 \cos 3x + 1/5 \cdot \sin 3x$ → (B)

The Gernal solution is

$y = y_c + y_p$

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + 3/5 \cos + 1/5 \sin 3x$ — (C)

Need to find value of c_1 & c_2 for the Put $x = 0$, $y = 1$ in "c"

$1 = (c_1(1) + 2(0) + 3/5(1) + 1/5(0))$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5$

$c_1 = 2/5$ — (x_2)

Diff c w.r.t "x"

$y' = c_1(2e^{2x} \cos 3x - 3e^{2x} \sin 3x) + c_2(2e^{2x}$

$\sin 3x + 3e^{2x} \cos 3x - 6/5 \sin 3x + 3/5 - \cos 3x$ → (D)

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Put $y' = 2, x = 0$ in "D"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - 6/5 \sin 3x + 3/5 \cos 3x$

Put $y' = 2, x = 0$

$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - 6/5 \sin 3(0) + 3/5 \cos 3(0)$

$2 = c_1(2) + 2(3) - 0, + 3/5$

$2 = 2c_1 + 3c_2 + 3/5$

Put $c_1 = 2/5$

$2 = 4/5 + 3c_2 + 3/5$

$2 = 7/5 + 3c_2$

$3c_2 = 2 - 7/5$

$3c_2 = 3/5$

$c_2 = 3/15$ — x_3

Put x_2 & x_3 in "C"

(9)

$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}\cdot e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

. Required General formula.

Question No 4

$(D^2 - DD')z = \cos x \cos 2y$

Solution:

The given PDE can be rewrit as:

$$D(D-D') = \cos x \cos 2y$$

in CF is give by:

$$CF = \phi_1(y) + \phi_2(y+x)$$

While its PI is given by

$$PI = \frac{1}{(D^2 - DD')}, \ 1/2 \left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2}\cos(x+2y) + \frac{1}{(1-2}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - 1/6 \cos(x-2y)$$

Hence the complete solution of the given PDE is give by

$u. = \phi_1(y_1) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y)$

$- \frac{1}{6}\cos(x-2y)$ Ans-

Question No 2 : Solution :-

Given function is

$$F(x) \begin{cases} x ; & -\pi < x \leq 0 \\ 2x ; & 0 \leq x \leq \pi \end{cases}$$

we have to find the fourier co-efficient.

Now

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi} \int_{-\pi}^{0} x\,dx + \frac{1}{\pi} \int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2}\right]_{-\pi}^{0} + 2\left[\frac{x^2}{2}\right]^{\pi}$$

$$= \frac{1}{\cancel{\pi}} \left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}} \left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi + \pi}{2} = \pi/2} \quad (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx\,dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos x\,dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos x)\,dx$$

$$= \frac{1}{\pi} \left[x\left(\frac{\sin x}{n}\right) - \left(-\frac{\cos x}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(\frac{\sin x}{n}\right)-\left(-\frac{\cos x}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{1-(-1)^n+2(-1)^n-n}{n^2}\right]$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin x\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}$$

$$\int_0^{\pi} x\sin x\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos x}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}$$

$$\left[x\left(-\frac{\cos n^2}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\pi\frac{\cos n\pi}{n}\right] = \frac{-3\cos nx}{n}$$

$$= \frac{3(-1)^{n+1}}{n}$$

So the required fourier series is

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)}{2n-1} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

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