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Assignment	**:**	**no 1**
Subject	**:**	**Differential Equations**
teacher	**:**	**Sir Latif Jan**
Program	**:**	**BC (CS)**

Name : Irfanullah

ID No : 15431

Assignment: no 1

Subject : Differentail Equation

Teacher : Sir Latif Jan

Q1 -12

$$x^2y'' - 4xy' + 6y = 0 \qquad y(1) = 0.4$$
$$y'(1) \quad 0$$

Sol

Put $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

Put y'' in give DE quation

$$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$$

$$\cancel{x^2} m(m-1)x^m \cancel{x^{-2}} - \cancel{4x} m x^m \cdot x^{-1} + 6x^m = 0$$

Dropping Common factor (x^m)

$$m(m-1) - 4m + 6 = 0$$

$$m^2 - 5m + 6 = 0$$

New finding roots

$$m^2 - 5m + 6 = 0$$

$m_{1/2} = \frac{5 \pm 1}{2}$

$m_1 = 3$ ∧ $m_2 = 2$

This Provide two real Solution

$y_1 = x^{m_1} = x^3$ ∧ $y_2 = x^{m_2} = x^2$

General Solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^3 + C_2 x^2$

$y' = 3C_1 x^2 + 2C_2 x$

Now to determine C_1 and C_2

$$\begin{cases} 0.4 = y(1) = C_1 1^3 + C_2 \cdot 1^2 \\ 0 = y'(1) = 3C_1 \cdot 1^2 + 2C_2 \cdot 1^2 \end{cases}$$

$$\begin{cases} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 - C_1 \\ 0 = 3(0.4 - C_1) + 2C_1 \end{cases}$$

$$\begin{cases} 0.4 - C_2 = C_1 \\ 12 = C_2 \end{cases}$$

$$\left\{\begin{array}{l} 0.4 - 1.9 = C_1 \\ 1.9 = C_2 \end{array}\right\}$$

$$\left\{\begin{array}{l} 0.8 = C_1 \\ 1.9 = C_2 \end{array}\right\}$$

The Perticular Solution

$$y = (-0.8x^3) + 1.9x^2$$

Ans

Q) 1.13

$$x^2 y'' + 3x y' + 0.75 y = 0 \qquad y(1) = 1$$

$$y'(1) = -1.5$$

Sol: Put $y = x^m$

$$y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

Putting in the give DE

$$x^2 m(m-1) x^{m-2} + 3x m x^{m-1} + 0.75 x^m = 0$$

$$\cancel{x^2} m(m-1) x^m \cdot \cancel{x^{-2}} + 3\cancel{x} m x^m \cdot \cancel{x^{-1}} + 0.75 m = 0$$

Droping Common factor x^m

$$m(m-1) + 3m + 0.75 = 0$$

$$m^2 + 2m + 0.75 = 0$$

Finding root

$$m^2 + 2m + 0.75 = 0$$

$$m_{1/2} = \frac{-2 \pm \sqrt{2^2 - (4)(0.75)}}{2}$$

$$m_{1/2} = \frac{-2 \pm 1}{2}$$

$$m_1 = -\frac{1}{2} \quad \wedge \quad m_2 = -\frac{3}{2}$$

we have two real solutions

$y_1 = x^{m_1} = x^{-1/2} = x^{.5}$ / $y_2 = x^{m_2} = x^{-3/2}$

The general solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^{0.5} + C_2 x^{+5}$

$y' = 0.5\, C_1 x^{-1.5} - 1.5\, C_2 x^{-2.5}$

To determine C_1 and C_2

$$\begin{cases} 1 = y(1) = C_1 1^{0.5} + C_2 \cdot 1^{-1.5} \\ 1.5 = y'(1) = -0.5 C_1 \cdot 1^{1.5} - 1.5\, C_2 1^{-2.5} \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = 0.5\, C_1 - 1.5\, C_2 1^{5} \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = -0.5 C_1 - 1.5\, C_2 1^{5} \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 3 = C_1 + 3\, C_2 \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 2 = 2 C_2 \,/ (\div 2) \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$ Particular solution is

$y = x^{-1.5}$ Ans

Q-14 $x^2y'' + xy' + 9y = 0 \quad y(1) = 0 \quad y(1) = 0, \; y'(1) = 2.5$

Sol $y' = mx^{m-1}$ and $y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$

$\cancel{x^2} m(m-1)x^{m-\cancel{2}} + mx^m + 9x^m = 0$

Dropping Common factor x^m

$m(m-1) + m + 9 = 0$

$m^2 - \cancel{m} + \cancel{m} + 9 = 0$

$m^2 + 9 = 0$

Finding roots

$m^2 + 9 = 0 \Rightarrow m^2 - (3i)^2 = 0 \Rightarrow (m-3i)(m+3i) = 0$

$m_1 = 3i \cap m_2 = 3i$

$x^{m_2} = x^{3i} = (e^{\ln x})^{3i} = e^{3i \ln x}$

$x^{m_2} = x^{-3i} = e^{\ln x} = e^{-3i \ln x}$

$e^{z} = e^{a+ib} = e^{a}(\cos b + i\sin b)$

$e^{3im} = e^{0}(\cos(3\ln x) - i\sin(3\ln x) = \cos 3\ln x$

$$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x)$$

$$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x)$$

$$x^{m_1} + x^{m_2} = \cos(3\ln x) + \cancel{i\sin(3\ln x)} + \cos(3\ln x)$$

$$- \cancel{i\sin(3\ln x)} = 2\cos(3\ln x)$$

$$\frac{x^{m_1} + x^{m_2}}{2} = \cos(3\ln x)$$

Subtracting 2nd equation from 1st and dividing by

$$x^{m_1} - x^{m_2} = \cos(3\ln x) + i\sin(3\ln x)$$

$$- \cos(3\ln x) + i\sin(3\ln x) = 2i\sin(3\ln x)$$

$$\frac{x^{m_1} - x^{m_2}}{2i} = \sin(3\ln x)$$

$$y_1 = \cos(3\ln x) \wedge y_2 = \sin(3\ln x)$$

$$y = c_1 y_1 + c_2 y_2$$

$$c_1\cos(3\ln x) + c_2\sin(3\ln x)$$

$$y' = -c_1\sin(3\ln x)\cdot(3\ln x)' + c_2\cos(3\ln x)\cdot(3\ln x)'$$

$$= \frac{-3c_1}{x}\sin(3\ln x) + \frac{3c_2}{x}\cos(3\ln x)$$

Now to find C_1 and C_2

$$\begin{cases} 0 = y(1) = C_1 \cos(3\ln x) + C_2 (\sin)(3\ln x) \\ 2.5 = y'(1) = -3C_1 \sin(3\ln x) + 3C_2 \cos(3\ln x) \end{cases}$$

$$\begin{cases} 0 = C_1 \cos(0) + C_2 (\sin)(0) \\ 2.5 = -3C \sin(0) + 3C_2 \cos(0) \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 5/2 = 3C_1 \; / \div 3 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 5/6 = C_2 \end{cases}$$

The Particular Solution is

$$y = 5/6 \sin(3\ln x)$$

Ans

Q-15 $x^2 y'' + 3xy' + y = 0$ $\quad y(1) = 3.6$

$y'(1) = 0.4$

$3\ln x)\}$

$3\ln x)$

Put $y = x^m,\ y'' = m(m-1) = 0$

$x^m(m-1)x^{m-2} + 3mx^m + x^m = 0$

$x^2 m(m-1)x^m \cdot x^{-2} + 3mx^m + x^m = 0$

Dropping Common Factor

$m(m-1) + 3m + 1 = 0$

$m^2 - m + 3m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + 2m + 1 = 0, \quad (m+1)^2 = 0$

$m = 1$

$y_1 = x^m = x^{-1} = 1/x$

$y'' + 3/x \cdot y' + 1/x^2\, y = 0$

$P(x) = \frac{3 \cdot 1}{x} \Rightarrow \int P dx = 3\ln|x|$

$y_2 = Uy_1$

$v = \int u\,dx$, $u = \frac{1}{y_1^2} e^{-\int P\,dx}$

To find ... v

$e^{-\int P\,dx} = e^{-3\ln(x)} = (e^{\ln(x)})^{-3} = x^{-3}$

$v = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$

$v = \int \frac{dx}{x} = \ln|x|$

$y_2 = v y_1 = y_1 \ln x = \frac{1}{x} \ln x$

$y = C_1 y_1 + C_2 y_2$

$C_1 \frac{1}{x} + C_2 \frac{1}{x} \ln(x)$

$\frac{1}{x} \cdot C_1 + C_2 \ln x$

$y' = (x^{-1})' (C_1 + C_2 \ln(x)) + x^{-1} (C_1 + C_2 \ln x)'$

$= -x^{-2} (C_1 + C_2 \ln x) + \frac{1}{x} \cdot \frac{1}{x}$

$= \frac{1}{x^2} (-C_1 - C_2 \ln x + C_2)$

Now finding C_1 and C_2

$$\begin{cases} 3.6 = y(1) = \frac{1}{1}(C_1 + C_2 \ln 1) \\ 0.4 = y'(1) = \frac{1}{1^2}(-C_1 - C_2 \ln 1 + C_2) \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 0.4 = -C_1 + C_2 \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 0.4 = -3.6 + C_2 \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 4.0 = C_2 \end{cases}$$

$$y = (3.6 + 4.0 \ln x)\ \frac{1}{x}$$

Ans

Q1 -16

$(x^2D^2 - 3xD + 4I) = 0 \quad y(1) = -\pi,\ y'(1) = 2\pi$

$xD^2y - 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y$

$= x^2y'' - 3xy' + 4y$

$x^2y'' - 3xy' + 4y = 0$

Put $y = x^m$ and $y'' = m(m-1)x^{m-2}$, $y' = mx^{m-1}$

$x^2m(m-1)x^{m-2} - 3xmx^{m-1} + 4x^m = 0$

$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} - 3\cancel{x}\, mx^m\cancel{x^{-1}} + 4x^m = 0$

Dropping x^m

$m(m-1) - 3m + 4 = 0$, $m^2 - 4m + 4 = 0$

~~sum~~ hence $y = x^n$ is a solution

$y_1 = x^m = x^2$

$y'' = \frac{-3}{x}y' + \frac{4}{x^2}y = 0$

$P(x) = -3\frac{1}{x} \Rightarrow \int P\,dx = -3\ln(x)$

$y_2 = vy_1$

$u = \int U dx$ A $U = \frac{1}{y^2} e^{\int P dx}$

To find U

$e^{-\int P dx} = e^{3\ln(x)} = (e^{\ln(x)})^{-3} = x^3$

$U = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$

$U = \int \frac{dx}{x} = \ln(x)$

$y_2 = Uy_1 = y_1 \ln(x) = x^2 \ln(x)$

$y_1 \neq y_2$ ER

General Solution is

$y = c_1 y_1 + c_2 y_2$

$c_1 x^2 + x^2 \ln x$

$x^2 (c_1 + c_2 \ln x)$

$y' = (x^2)' (c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$

$= 2x (c_1 + c_2 \ln x) + c_2 x^2 \cdot \frac{1}{x}$

$= 2c_1 x + 2 c_2 x \ln x + c_2 x$

$= 2cx + c_2 x (2\ln x + 1)$

$$\begin{cases} -\pi = y(1) = 1^2\left(c_1 + c_2\, 1 \ln 1\right) \\ 2\pi = y'(1) = 2C_1 + C_2(2\ln 1 + 1) \end{cases}$$

$$\begin{cases} \pi = C_2 \\ 2\pi = 2C_1 + C_2 \end{cases}$$

$$\begin{cases} -\pi = C_1 \\ 4\pi = C_2 \end{cases}$$

Particular Solution is

$$y = x^2\left(-4 + 4\pi \ln x\right)$$

Ans

Q.17

$(x^2D^2 + xD + I)y = 0 \quad , y(1) = 1, y'(1) = 1$

First Applying given operator to the function

$x^2D^2y + xDy + Iy = x^2D(Dy) + xDy + y$

$= x^2y'' + xy' + y$

Now

$x^2y'' + xy' + y = 0$

$y = x^m, \; y' = mx^m, \; y'' = m(m-1)$

$x^2m(m-1)\,x^{m-2} + xmx^{m-1} + x^m = 0$

$\cancel{x^2}m(m-1)\,x^2 \; xmx^m x^{-1} + x^m = 0$

Dropping the Common factor of x^m

$m(m-1) + m + 1 = 0 \Rightarrow m^2 - \cancel{m} + \cancel{m} + 1 = 0 \cdot m^2 + 1 = 0$

Now finding the roots

$m^2 + 1 = 0, \; m^2 + i^2 = 0, \; (m-i)(m+i) = 0$

$m_1 = i \wedge m_1 = -i$

$x = e^{\ln x}$

$x^{m_1} \cdot x^{i} = (e^{\ln x})^{i} \cdot e^{i\ln x}$

$u^{m_2} = u^{i} \cdot (e^{\ln u})^{-i} \cdot e^{-i\ln u}$

$e^{i} \cdot e^{a+ib} = e^{a}(\cos b + i\sin b) .$ ZEC

$e^{i\ln u} = (e^{\prime}(\cos(\ln u) + i\sin(\ln u))$

$= \cos(\ln u) + i\sin(\ln u)$

$e^{-i\ln u} = (e^{\prime\prime}(\cos(\ln u) - i\sin(\ln u)$

$u^{m_1} = \cos(\ln u) + i\sin(\ln u)$

$u^{m_2} = \cos(\ln u) - i\sin(\ln u)$

$u^{m_1} + u^{m_2} = \cos(\ln u) + i\sin(\ln u) + (\ln u) - i\sin(\ln u)$

$= 2\cos(\ln u)$

$\frac{u^{m_1} + u^{m_2}}{2} = \cos(\ln(u))$

$u^{m_1} - y^{m_2} = \cos(\ln u) + i\sin(\ln u) - \cancel{\cos}(\ln u)$

$+ i\sin(\ln u)$

$\frac{u^{m_1} - u^{m_2}}{2}\ \sin(\ln u)$

$y^{1} = \cos(\ln u) \wedge y_2 = \sin(\ln u)$

$y' = -C_1 \sin(\ln x)(\ln x)' + C_2 \cos(\ln x)(\ln$

$= -\frac{C_1}{x}\sin(\ln x) + \frac{C_2}{x}\cos(\ln x)$

To determine C_1 and C_2

$$\begin{cases} 1 = y(1) = C_1 \cos(\ln 1) + C_2 \sin(\ln 1) + C_0 \sin \\ 1 = y'(1) = C_1 \sin(\ln 1) + 3C_2 \cos(\ln 1) \end{cases}$$

$$\begin{cases} 1 = C_1 \cos(0) + C_2 \sin(0) \\ 1 = -C_1 \sin(0) + C_2 \cos(0) \end{cases}$$

4)

$$\begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$$

$y = \sin(\ln x) + \cos(\ln x)$

Ans

(18) $(9x^2D^2+3xD+I)y=0$ $\quad y(1)=1$

Apply given operation to the equation $\quad y'(1)=0$

$9x^2D^2y+3xDy+Iy=9x^2D(Dy)+3xDy$

$=9x^2y''+3xy'+y=0$

Let $y=x^m$, $y'=mx^{m-1}$, $y''=m(m-1)x^{m-2}$

$9x^2m(m-1)x^{m-2}+3xmx^{m-1}+x^m=0$

$9x^2m(m-1)x^m\cancel{x^2}+3xmx^m\cancel{x}+x^m=0$

$9m(m-1)+3m+1=0$, $9m^2-9m+3m+1=0$

$=9m^2-6m+1=0$

Finding the root of equation

$m^2-4m+4=0$, $(m-2)^2=0$

$9m^2-6m+1=0$ $\quad m_{1,2}=\frac{6\pm\sqrt{6^2-4\cdot 9}}{18}$

$m_{1,2}=6/18$

$m_{1,2}=1/3$

$m=1/3$

$y_1 = x^m = x^{1/3}$

$y_1 = x^m = x^{1/3}$

$y'' + \frac{1}{3x} \cdot y' + \frac{1}{9x^2} y = 0$

$P(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int P dx = \frac{1}{3} \ln(x)$

$y_2 = v y_1$

$v = \int u \, dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int P dx}$

finding. U

$U e^{-\int P dx} = e^{-\frac{1}{3}\ln(x)} = (e^{\ln(x)})^{-\frac{1}{3}} = x^{-\frac{1}{3}}$

$U = x^{-\frac{1}{3}} \frac{1}{(x^{\frac{1}{3}})^2} = x^{-\frac{1}{3} - \frac{2}{3}} = x^{-1} = \frac{1}{x}$

$V = \int \frac{dx}{x} = \ln(x)$

$y_2 = vy_1 = y_1 \ln(x) = x^{\frac{1}{3}} \ln x$

$y = c_1 y_1 + c_2 y_2$

$c_1 x^{\frac{1}{3}} + x^{\frac{1}{3}} \ln x$

$x^{\frac{1}{3}} (c_1 + c_2 \ln x)$

$$y' = (x^{1/3})'(C_1 + C_2 \ln x) + x^{1/3}(C_1 + C_2 \ln x)'$$

$$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2 \cdot \frac{1}{x}$$

$$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2$$

$$\begin{cases} 1 = y(1) = 1^{1/3}(C_1 + C_2 \ln 1) \\ 0 = y'(1) = \frac{1}{3} \cdot 1^{-2/3}(C_1 + C_2 \ln 1) + 1^{1/3} C_2 \end{cases}$$

$$\begin{cases} 1 = C_1 \\ 0 = \frac{C_1}{3} + C_2 \end{cases}$$

$$\begin{cases} 1 = C_1 \\ -\frac{1}{3} = C_2 \end{cases}$$

$$y = x^{1/3}\left(1 - \frac{1}{3}\ln x\right)$$

Ans

Q14 $(x^2D^2 - xD - 15I)y = 0$ $y(1) = 1$

$y'(1) = 4.5$

Sol:

Applying given operator on the equation

$x^2D^2y - xDy - 15Iy = x^2D(Dy) - xDy$

$- xDy - 15y \Rightarrow x^2y'' - xy' - 15y$

let $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2m(m-1)x^{m-2} - xmx^{m-1} - 15x^m = 0$

$\cancel{x^2}m(m-1)x^m \cancel{x^{-2}} - \cancel{x}mx^m\cancel{x^{-1}} - 15x^m = 0$

Dropping x^m

$= m(m-1) - m - 15 = 0$

$= m^2 - 2m - 15 = 0$

Finding the roots

$m^2 - 2m - 15 = 0$

$m_{1,2} = \frac{2 \pm \sqrt{-2^2 + 4 \cdot 15}}{2}$

$m_{1,2} = \frac{2 \pm 8}{2}$

$m_1 = 5 \wedge m_2 = -3$

The two real solution are

$y_1 = x^{m_1} = x^5$ & $y_2 = x^{m_2} = x^{-3}$

Now the general Solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^5 + c_2 x^{-3}$

$y' = 5c_1 x^4 - 3c_2 x^{-4}$

Now to Determine the c_1 and c_2

$$\begin{cases} 0.1 = y(1) = c_1 1^5 + c_2 1^{-3} \\ -4.5 = (y')(1) = 5c_1 1^4 - 3c_2 1^{-4} \end{cases}$$

$$\begin{cases} 0.1 = c_1 + c_2 \\ 4.5 = 5c_1 + (-3c_2) \end{cases}$$

$$\begin{cases} 0.1 - c_2 = c_1 \\ -4.5 = 5(0.1 - c_2) \cdot 3c_2 \end{cases}$$

$$\begin{cases} 0.1 - c_2 = c_1 \\ 5 = 8c_2 / \div 8 \end{cases}$$

$$\begin{cases} 0.1 - 0.695 = c_1 \\ 0.695 = c_2 \end{cases}$$

$$\begin{cases} -0.595 = c_1 \\ 0.695 = c_2 \end{cases}$$

Perticuler Solution

$y = -0.595 x^5 + 0.695 x^{-3}$

Ans

Q2-1

(h)

Sol

Q2-14

Sol:

Q2-14

$$x' = \sqrt{x}$$

Sol:

$$dx = x$$

$$\frac{dx}{\sqrt{x}} = 1dt$$

$$\frac{1}{\sqrt{x}}dx = dt$$

$$\int \frac{1}{\sqrt{x}}dx = \int dt$$

$$2\sqrt{x} + c = t + c_2$$

$$2\sqrt{x} = t + c$$

$$4(x) = (t+c)^2$$

$$x = \frac{(t+c)^2}{4}$$

Ans

Q2-1 (b) $x' = e^{-2x}$

$= \frac{dx}{dt} = e^{-2x} \Rightarrow \frac{dx}{e^{-2x}} = dt$

$= \int \frac{dx}{e^{-2x}} = \int dt$

$= \frac{e^{2x}}{2} = t + c$

$e^{2x} = 2(t+c)$

$2x = \ln 2(t+c)$

$x = \frac{\ln 2(t+c)}{2}$ Ans

Q2-1 (c) $y' = 1+y^2$

Sol $\frac{dy}{dt} = 1+y^2 \Rightarrow \frac{dy}{1+y^2} = dt$

$\int dy \cdot \frac{1}{1+y^2}$

$= \int dt$

$y'(t) = \tan(t+c)$ Ans

Q2-1 (d)

Sol

Q2-1 (f)

Sol

Q2-1 (d) $u' = \frac{1}{5-2u}$

Sol

$$\frac{dy}{dt} = \frac{1}{5-2u} \Rightarrow \frac{dy}{5-2u} = dt \; (1)$$

$$= \int dy \frac{1}{5-2u} = \int dt$$

$$= -(u-5)u + C_1 = t + C_2$$

$$-(u-5)u = t + C$$

$$u(t) = \frac{t+C}{-(u-5)} \quad \text{Ans}$$

Q2-1 (f) $Q' = \frac{Q}{4+Q^2}$

Sol

$$\frac{dQ}{dt} = \frac{Q}{4+Q} \Rightarrow \frac{Q^2 dQ}{Q} = dt$$

$$= \int \frac{Q^2}{Q} dQ = \int dt$$

$$3\ln|Q| + \frac{Q^2}{2} + C$$

Ans

Q2-1
(a) $u' = e^{u^2}$

Sol

$$\frac{du}{dt} = e^{u^2}$$

$$\frac{du}{e^{u^2}} = dt$$

$$\int \frac{1}{e^{u^2}}\, du = \int dt$$

$$\frac{\sqrt{\pi}\ \operatorname{erf}(u)}{2} = t + c$$

$$u = 2\ \frac{t+c}{\sqrt{\pi}\ 2}$$

$$u(t) = \frac{2t + c}{\sqrt{\pi}}$$

Ans

Q2-1
(b) $y' = r(a-y)$

Sol

$$\frac{dy}{dt} = r\,(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$\frac{(a-y)\,u}{r} = t + c \Rightarrow u(t) = \frac{r\,(t+c)}{a-y}$$ Ans

Q2-2
Solve
Constan
Sol

Q2-3

Q2-1

(a) $u' = e^{u^2}$

Sol

$$\frac{du}{dt} = e^{u^2}$$

$$\frac{du}{e^{u^2}} = dt$$

$$\int \frac{1}{e^{u^2}}\, du = \int dt$$

$$\frac{\sqrt{\pi}\, erf(u)}{2} = t + c$$

$$u = 2\, \frac{t+c}{\sqrt{\pi}\cdot 2}$$

$$u(t) = \frac{2t+c}{\sqrt{\pi}}$$

Ans

Q2-1

(b) $y' = r(a-y)$

Sol

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$\frac{(a-y)u}{r} = t + c \Rightarrow u(t) = \frac{r(t+c)}{a-y}$$ Ans

Q2-2

Solve

Constan

Sol

Q2-3

Q2-2

Solve $y' = r(a-y)$, where r and a are Constants.

Sol

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)} \cdot dy = \int dt$$

$$a-y = K(r)^{1/2}$$

Q2-3 (a)

$(1a)7, \quad u(0)=1$

$$u(t) = \frac{(t+c)^2}{4}$$

$u(0)=1, \quad u(t) = u(0) = 1$

$$1 = \frac{(0+c)^2}{4}$$

$4 = c^2$

$c = 2$ Ans

Q2-

1b $u(0)=1$

$u(t) = \frac{\ln 2(t)+c}{2}$

$u(0)=1$

$1 = \frac{\ln 2(0)+c}{2}$

$1 = \ln\frac{(0)+c}{2}$

$2 = \ln c^2$ Ans

Q2-4 (a)

$u' = \frac{2u}{t+1}$

$\frac{du}{dt} = \frac{2u}{t+1}$

$\frac{du}{2u} = \frac{1}{t+1}\,dt$

$\frac{u^2}{4} = \ln(t+1)+c$

$\frac{u^2}{4\ln(t+u)} = c \Rightarrow c = \frac{u^2}{4\ln(t+1)}$

A

Q2-4 (b)

$$Q' = t\sqrt{t^2+1}\ \sec Q$$

$$\frac{dQ}{dt} = t\sqrt{t^2+1}\ \sec Q$$

$$\frac{dQ}{\sec Q} = t\sqrt{t^2+1}\ dt$$

$$\int \frac{dQ}{\sec Q} = \int t\sqrt{t^2+1}\ dt$$

$$= \sin Q = \frac{(t^2+1)^{\frac{3}{2}}}{3} + c$$

$$c = \frac{\sin Q\,(3)}{(t^2+1)^{\frac{3}{2}}}$$

$$c = 3\,\frac{\sin Q}{(t^2+1)^{\frac{3}{2}}}$$

M

Q2-4 (c)

$$(2v+1)v' - (t+1) = 0$$

Sol:

$$(2v+1)\frac{dv}{dt} = (t+1)$$

$$(2v+1)\,dv = (t+1)\,dt$$

$$\int(2v+1)\,dv = \int(t+1)\,dt$$

$$v^2 + v = \frac{t^2}{2} + t + c$$

$$\frac{2(v^2+v)}{t^2} - t = c$$

$$c = 2\frac{(v^2+v)}{t^2} - t$$

Ans

1 Q-4 (ii)

$$R' = (t+1)(R^2+1)$$

Sol:-

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\frac{dR}{R^2+1} = (t+1)\,dt$$

$$\int \frac{dR}{R^2+1} = \int (t+1)\,dt$$

$$\cot(R) = \frac{t^2}{2} + t + c$$

$$c = \frac{2\cot(R)}{t^2} - t$$