INU.

Name = Shahid Nasir-
ID # 15531.
Program = BS (S.E)
Module = 3rd Semester
Teacher = Sir Latif Jan.
Assignment # = 1.

Subject = Differential

Q1 - 12:- $x^2y'' - 4xy' + 6y = 0$. $y(1) = 0.4$, $y'(1) = 0$.

Soln

Put $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$.

Put y'' in given eq....

$x^2 m(m-1) x^{m-2} - 4x m x^{m-1} + 6x^m = 0$.

$x^2 m(m-1) x^m \cancel{x} - 4x m x^m . x^{-1} + 6x^m = 0$.

Dropping common factor

$m(m-1) - 4m + 6 = 0$.

$m^2 - 5m + 6 = 0$.

Now

$m^2 - 5m + 6 = 0$.

$m_{1/2} = \frac{5 + \sqrt{(-5)^2 + 4(6)}}{2}$

$m_{1/2} = \frac{5+1}{2}$

$m = 3$, $m_2 = 2$.

This provide two real solution.

$y_1 = x^m = x^3$, $y_2 = x^{m_2} = x^2$.

$y = C_1y_1 + C_2y_2$
$= C_1x^3 + C_2x^2$

Pg#2

$y' = 3C_1x^2 + 2Cx$

determing C_1 & C_2

$$\begin{cases} 0.4 = 4(1) = C_1 \cdot 1^3 + C_2 \cdot 1^2 \\ 0 = y'(1) = 3C_1 1^2 + 2C_1 1^2 \end{cases}$$

$$\begin{cases} 0.4 = C + C_2 \\ 0 = 3C + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 - C_1 \\ 0 = 3(0.4 - C) + 2C \end{cases}$$

$$\begin{cases} 0.4 - (2 = C_1 \\ 1.2 \quad = C_2 \end{cases}$$

$$\begin{cases} 0.4 - 1.2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$\begin{cases} 0.8 = C_1 \\ 1.2 \quad C_2 \end{cases}$$

The Particular soln is

$y = (-0.8x^3) + 1.2x^2$ Ans

Q: 13:- $x^2y'' + 3xy' + 0.75y = 0$.

$y(1) = 1$

$y'(1) = 1.5$.

Sol

Put $y = x^m$

$y' = m x^{m-1}$.

$y'' = m(m-1)x^{m-2}$.

putting in eq -

$x^2 m(m-1)x^{m-2} + 3x m x^{m-1} + 0.75 x^m = 0$

$\cancel{x^2} m(m-1)x^m \cancel{x^{-2}} + 3\cancel{x} m x^m \cdot \cancel{x^{-1}} + 0.75 x^m = 0$

Dropping common factors -

$m(m-1) + 3m + 0.75 = 0$.

$m^2 + 2m + 0.75 = 0$.

$m^2 + 2m + 0.75 = 0$

$m_{1/2} = \frac{-2 \pm \sqrt{2^2 - (4)(0.75)}}{2}$.

$m_{1/2} = \frac{-2 \pm 1}{2}$

$m_1 = -1/2$, $m_2 = -3/2$.

We have 2 real soln -

$y_1 = x^{m_1} = x^{-1/2} = x^{-0}$, $y_2 = x^{m_2} = x^{-3/2}$.

The general soln is:

$y' = 0.5 c_1 x^{-1.5} - 1.5 c_2 x^{-2.5}$

To determine C_1 & C_2 Pg#4

$$\begin{cases} 1 = y(1) = C_1 \cdot 1^{0.5} + C_2 \cdot 1^{1.5} \\ 1.5 = y'(1) = -0.5 C_1 \cdot 1^{-1.5} - 1.5 C_2 \cdot 1 \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = -0.5 C_1 - 1.5 C_2 / 0.5 \end{cases}$$

$$\begin{cases} 1 = C_1 + C_1 \\ 3 = C_1 + 3 C_2 \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 2 = 2 C_2 / (2) \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

Particular soln is $y = u^{-1.5}$.

Q1-14

$u^2 y'' + u y' + a y = 0 \quad y(1) = 0$
$y'(1) = 2.5$

put $y = m u^m$ and $y'' = m(m-1)u^{m-2} = 0$

$n^2 m(m-1) u^{m-2} + m u^n + 9 n^n = 0$

$u^2 m(n-1) m \cdot h + m u + 9 n^n = 0$

Droping common factor h^m

$m(m-1) + m \, 9 = 0$

$m^2 - m + m \, 9 = 0$

$m^2 \, 9 = 0$

Findig root

Pg # 5

$m^2+9=0 \Rightarrow m^2-(3i)^2=0 \Rightarrow (m-3i)(m+3i)=0$

$m_1=3i \wedge m_2=-3i$

$u^{m_1}=u^{3i}=(e^{\ln u})^{3i}=e^{3i\ln u}$

$u^{m_2}=u^{-3i}=e^{-3i\ln u}$

$e^{a}e^{ib}=e^{a}(\cos b+i\sin b)$

$u^{m_1}=\cos(3\ln u)+i\sin(3\ln u)$

$u^{m_2}=\cos(3\ln u)-i\sin(3\ln u)$

$u^{m_1}+u^{m_2}=\cos(3\ln u)+i\sin(3\ln u)+\cos(3\ln u)-i\sin(3\ln u)=2\cos(3\ln u)$

$\frac{u^{m_1}+u^{m_2}}{2}=\cos(3\ln u)$

Subtracting 2nd equation from 1st and dividing by $2i$

$u^{m_1}-u^{m_2}=\cos(3\ln u)+i\sin(3\ln u)-\cos(3\ln u)+i\sin(3\ln u)=2i\sin(3\ln u)$

$\frac{u^{m_1}-u^{m_2}}{2i}=\sin(3\ln u)$

$y_1=\cos(3\ln u) \wedge y_2=\sin(3\ln u)$

$y=c_1y_1+c_2y_2$

$c_1\cos(3\ln u)+c_2\sin(3\ln u)$

$y'=-c_1\sin(3\ln u)\cdot(3\ln u)'+c_2\cos(3\ln u)\cdot(3\ln u)'$

$=\frac{-3c_1}{u}\sin(3\ln u)+\frac{3c_2}{u}\cos(3\ln u)$

Now to find c_1 and c_2

$0=y(1)=c_1\cos(3\ln 1)+c_2\sin(3\ln 1)$

$2.5=y'(1)=-3c_1\sin(3\ln 1)+3c_2\cos(3\ln 1)$

$C_0 = C_1 \cos(0) + C_2 (\sin)$

$\{ 2.5 = 3C_1 \sin(0) + 3C_2 (\cos) \cos 3$

$$\begin{Bmatrix} 0 = C_1 \\ 5/2 = 3C_2 / \cdot 3 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 = C_1 \\ 5/6 = C_2 \end{Bmatrix}$$

The perticular solution is
$y = 5/6 \sin(3 \ln u)$

Q1-15 $u y'' + y' + y = 0$ $\quad y(1) = 36$
$y'(1) = 0.6$

put $y = u^m, \; y'' = m(m-1)u^{m-2}$

$u^m (m-1) u^{m-2} + 3 m u^m + u^m = 0$

$u^m (m-1) u^m + 3 m u^m + u^m = 0$

Dropping Common factor

$m(m-1) + 3m + 1 = 0$

$m(m-1) + 3m + 1 = 0$

$m(m-1+1 = 0$

$m^2 + 2m + 1 = 0, \; (m+1)^2 = 0$

$m = 1$

$y_1 = u^m = u^n = 1/4$

$y'' + 3/4 \cdot y' + \frac{1}{4} \cdot y = 0$

$P(u) = \frac{3 \cdot 1}{u} \Rightarrow \int p\,du = 3 \ln(u)$

$y = u y.$

$u =$ Sudo $\wedge$ $u = 1/y_2$, e-spelor

To Pod . . . U

$\int p\,du = 3\ln|u|$ $\ln|u|)^{-3}$ -3

$e = e = (e) = u$

$U = u^{-3}\,\frac{1}{u^2} = u^{3+2} = u^{-1} = 1/u$

$h = \int \frac{du}{u} = \ln(u)$

$y_0 = uy_1 = y\ln u = \frac{1}{u}\ln u$

$y = c_1 y + c_2 y_1$

$c\frac{1}{u} + c\frac{1}{u}\ln(u)$

$\frac{1}{u}c_1\ c_2\ln u$

$y = (\frac{1}{u})(c_1 + c_2\ln u) u^{-1}(c_1 + c_2\ln u)$

$= u(c_1 + c_2\ln u) = \frac{c_1}{2}\frac{1}{u}$

$\frac{1}{u^2}(-c_1 - c_1\ln u + c_2)$

Now finding c and c_2

$\left\{\begin{array}{l} 3.6 = y(1) = \frac{1}{1}(c_1 + c_2\ln 1) \\ 3.4 = y'(1) = \frac{1}{2}(-c - c\ln 1 + c = 1) \end{array}\right\}$

$\left\{\begin{array}{l} 3.6 = c \\ 0.4 = -c + c \end{array}\right\}$

$$\left\{\begin{array}{l} 3.6 = c_1 \\ 0.4 = -3.6 + c \end{array}\right\}$$

$$\left\{\begin{array}{l} 3.6 = c \\ 4.0 = c \end{array}\right\}$$

$$y = 3.6 + 4.0.0 \ln u)^{1/2}$$

0.1-16

$$4^2 0^2 - 3n D + 4 I) = 0$$

$$y(1) = \quad , \quad y'(1) = 2\pi$$

$$4^2 y_2 - 3u y' + 4 I y = u^2 D(Dy) - 3u Dy + 4y$$

$$4^2 y'' - 3n y + 4y$$

$$4^2 y'' - 3n y' + 4y = 0$$

Put $y = u^m$ and $y'' = m(m-1)n^{m-2}, y = m u^{m-1}$

$$4^2 m(m-1) u^{m-2} - 3n m m^{m-1} + 4n^m = 0$$

$$4^2 m(m-1) \frac{4^m}{4^m} \cdot \cancel{4^2} - 3u \, m 4^m / 4 + 4 4^m = 0$$

$$m(m-1) - 3m + 4 = 0 \qquad m^2 - 4m + 4 = 0$$

Hence $y = 4^x$ is a soln.

$m^2 - 4m + 4 = 0$, $(m-1)^2 = 0$.

$y_1 = x^m = x$

general solution is.

$y = C_1 y_1 + C_2 y_2$.

$C_1 4^2 + 4^2 \ln 4$.

$4^2 (C_1 + C_2 \ln 4)$.

$y' = (x^2)' (C_1 + C_2 \ln x) + x^2 ((C_1 + C_2 \ln x))'$

$= 2x (C_1 + C_2 \ln x) + C_2 x^2 \cdot 1/x$

$= 2C_1 x + C_2 x (2 \ln 4 + 1)$.

$\left\{ \begin{array}{l} -\pi = y(1) = 1^2 (x_1 + C_2 \ln_1). \\ 2\pi = y'(1) = 2C_1 + C_2 (2 \ln_1 + 1) \end{array} \right\}$

$\left\{ \begin{array}{l} -\pi = C_2 \\ 2\pi = 2C_2 + C_2 \end{array} \right\}$

$\left\{ \begin{array}{l} -\pi = C_2 \\ 4\pi = C_2 \end{array} \right\}$

Particular solution is

$y = x^2 (-x + 4\pi \ln 4)$

Ans

$(x^2D^2 + 4D + I)y = 0 \qquad y(1) = 1, \; y'(1) = 1$

first applying given operation to the function

$x^2D^2y + xDy + Iy = x^2 D(Dy) + xDy + y$

$= x^2y'' + xy' + y$

Now

$x^2y'' + xy' + y = 0$

$y = x^m, \quad y' = mx^m, \quad y'' = m(m-1)$

$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^2} \; xm x^n \; x^{-1} + x^m = 0$

$x^{m_1} = \cos(\ln x) - i\sin(\ln x)$

$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$

$x^{m_1} + x^{m_2} = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x) - i\sin(\ln x)$

$= 2\cos(\ln x)$

$x^{m_1} - x^{m_2} = \cos(\ln x) + i\sin(\ln x) - \cos(\ln x) + i\sin(\ln x)$

$\frac{x^{m_1} - x^{m_2}}{2i} = \sin(\ln x)$

$y^1 = \cos(\ln x) \quad \& \quad y^2 = \sin(\ln x)$

$y' = -C_1 \sin(\ln x) \cdot (\ln x)' + C_2 \cos(\ln x) \cdot (\ln x)'$

$= \frac{-C_1}{x} \sin(\ln x) + \frac{C_2}{x} \cos(\ln x)$

determine C_1 & C_2

$\begin{cases} 1 = y(1) = C_1\cos(\ln 1) + C_1\sin(\ln 1) + C_2\sin(\ln 1) \\ 1 = y'(1) = C_1\sin(\ln 1) + 3C_2\cos(\ln 1) \end{cases}$

$$\begin{cases} 1 = C_1 \cos(0) + C_2 \sin(0) \\ 1 = -C_1 \sin(0) + C_2 \cos(0) \end{cases}$$

$$\begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$$

$$y = \sin(\ln x) + \cos(\ln x)$$

Ans

Eq#12

$(9x^2D^2\ 3xD + I)y = 0$ $\qquad y(1) = 1$

$\qquad y'(1) = 0$

Apply given operation to the equation

$9x^2D^2y + 3xDy + Iy = 9x^2D(Dy) + 3xx^{\ldots}$

$= 9x^2y'' + 3xy' + y$

$9x^2y'' + 3xy' + y = 0$

let $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$

$y_1 = x^m = x^{1/3}$

$y'' + \frac{1}{3x}\cdot y' + \frac{1}{9x^2}\cdot y = 0$

$p(x) = \frac{1}{3}\cdot\frac{1}{x}$

$\int p\,dx = \frac{1}{3}\ln|x|$

$y_2 = uy_1$

$u = \int U\,du$ $\qquad U = \frac{1}{y_1}e^{\int D\,dx}$

finding U.

$U = \dfrac{e^{-\int p\,dx}}{} = e^{-1/3\ln(x)} = (e^{\ln(x)})^{-1/3} = x^{-1/3}$

$U = \int \frac{du}{x} = \ln|x|$

$= y = (x^{1/3})\,(C_1 + C_2 \ln x) + x^{1/3}(C_1 + C_2 \ln x)$

$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2 \cdot \frac{1}{x}$

$= \frac{1}{3} \cdot x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2$

$\left\{ \begin{array}{l} (1) \; y(1) = 1^{1/3}(C_1 + C_2 \ln 1) \\ (1) \; y'(1) = \frac{1}{3} \cdot 1^{2/3}(C_1 + C \ln) + 1^{1/3} C_2 \end{array} \right\}$

$\left\{ \begin{array}{l} 1 = C_1 \\ 0 = C_1 + C_2 \end{array} \right\}$

$\left\{ \begin{array}{l} 1 = C \\ -\frac{1}{3} = C_2 \end{array} \right\}$ $\quad y = x^{1/3}\left(1 - \frac{1}{3}\ln x\right)$

(19) $x^2 D^2 - x D - (15)\,y = 0 \qquad y(1) = 1$

$y'(1) = 4.5$

Sol Applying given operators on the equation

$x^2 D^2 y - x D y - 15 y = x^2 D (Dy) - x D y - 15 y$

$\Rightarrow x^2 y'' - x y' - 15 y$

let $y = x^m,\; y' = m x^{m-1},\; y'' = m(m-1) x^{m-2}$

$x^2 m(m-1) x^{m-2} - x m x^{m-1} - 15 x^m = 0$

$x^2 m(m-1)x^{m} - x^2 - x\, m x^{m} x^{-1} - 15x^{m} = 0$ Dropping x^m

$= m(m-1) - m - 15 = 0$

$= m^2 - 2m - 15 = 0$

finding the roots

$m^2 - 2m - 15 = 0,$

$m_{1/2} = \dfrac{2 \pm \sqrt{-2^2 + 4 \cdot 15}}{2}$

$m_{1/2} = \dfrac{2 \pm 8}{2}$

$m_1 = 5$ & $m_2 = -3$

The two real solutions are

$y_1 = x^{m_1} = x^5$ & $y_2 = x^{m_2} = x^{-3}$

now to Determine the C_1 and C_2

$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 = 1^{-3} \\ y(1) 5 = C_1 + C_2 \end{cases}$

$\begin{cases} 0.1 - C_2 = C_1 \\ 5 = 8C_2 \quad 1:8 \end{cases}$

$\begin{cases} 0.1 - 0.625 = C_1 \\ 0.625 = C_2 \end{cases}$

$\begin{cases} -0.525 = C_1 \\ 0.625 = C_2 \end{cases}$

Particular solution

$$y = -0.525x^5 + 0.625x^3$$

Ans

Q8 2:a:- $x' = \sqrt{x}$

$$\frac{dx}{dt} = x$$

$$\frac{dx}{\sqrt{x}} = 1dt$$

$$\frac{1}{\sqrt{x}}dx = dt$$

$$\int \frac{1}{\sqrt{x}}dx = \int dt$$

$$2\sqrt{x} + c = t + c_2$$

$$2\sqrt{x} = t + c$$

$$4(x) = (t+c)^2$$

$$x = \frac{(t+c)^2}{4}$$

Ans

Qn: 1 b:- $x' = e^{-2x}$

$$\frac{dx}{dt} = e^{-2x}$$

$$\frac{dx}{e^{-2x}} = dt$$

$$\int \frac{dx}{e^{-2x}} = \int dt$$

$$\frac{e^{2x}}{2} = t + C$$

$$e^{2x} = 2(t+C)$$
$$2x = \ln 2(t+C)$$
$$x = \frac{\ln 2(t+C)}{2}$$

Ans

Qn: 1:- c:- $y' = 1 + y^2$

Soln

$$\frac{dy}{dt} = 1 + y^2$$

$$\frac{dy}{1+y^2} = dt$$

$$\int dy \frac{1}{1+y^2} = \int dt$$

$y(1) = \tan(t+c)$ Ans

Q2: 1(d):- $u' = \frac{1}{5-2u}$

Soln

$$\frac{dy}{dt} = \frac{1}{5-2u}$$

$$\frac{dy}{5-2u} = dt \; (1)$$

$$\int dy \cdot \frac{1}{5-2u} = \int dt$$

$$= -(u-5)u + c_1 = t + c_2$$

$$= -(u-5)u = t + c$$

$$u(u) = \frac{t+c}{-(u-5)}$$

Ans

Q2: 1(e):- $Q' = \frac{Q}{4+Q^2}$

Soln

$$\frac{dQ}{dt} = \frac{Q}{4+Q}$$

$$\frac{Q^2 \, dQ}{Q} = dt$$

$$\int \frac{Q^2}{Q} dQ = \int dt$$

$$3\ln|Q| + \frac{Q^2}{2} + C$$

Ay-

:1(g):- $x' = e x^2$

Soln

$$\frac{dx}{e^{x^2}} = dt$$

$$\int \frac{1}{e^{x^2}} dx = \int dt$$

$$\frac{\sqrt{\pi}\, erf(x)}{2} = 6 + C$$

$$x = \frac{2t + e}{\sqrt{\pi}\, f}$$

$$x(1) = \frac{2\cancel{f} + e}{\sqrt{\pi}}$$

As

:1(h):- $y' = r(a-y)$

Soln

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$\frac{(a-y)y}{y} = t+C$$

$$x(t) = \frac{r(t+C)}{a-y}$$

Ans

Q2-2:- Solve $y' = r(a-y)$ where r & a are constants

Sol

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$a-y = k(r)^{1/2}$$

Q 2(3):-

$$(1(t))^{1(t)} \; 2 \,, \quad x(0) = 1$$

$$x t = (t+c)^2$$

$$y(0) = 1, \quad y(t) = x(0) - 1$$

$$1 = \frac{(0+e)^2}{4}$$

$$4 = e^2 \quad \boxed{e = 2}$$

15 $\quad x(0) = 1$

$$x(t) = \frac{\ln 2(t)}{2} + c$$

$x(0) = 1$

$$1 = \frac{\ln 2(0)}{2} + c$$

$$1 = \frac{\ln(0)}{2} + c$$

$$2 = \ln e^2$$

2.4(a) 1.

$$x' = \frac{2x}{t+1}$$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\int \frac{dx}{2x} = \int \frac{1}{t+1} \, dt$$

$$\frac{x^2}{4} = \ln(t+1) + C.$$

$$\frac{x^2}{4\ln(t+1)} = C.$$

$$C = \frac{x^2}{4\ln(t+1)}.$$

Ans

Q4-4 (b):-

$$\theta' = t\sqrt{t^2+1} \quad \sec\theta$$

$$\frac{d\theta}{dt} = t\sqrt{t^2+1} \quad \sec\theta$$

$$\frac{d\theta}{\sec\theta} = t\sqrt{t^2+1}\, dt$$

$$\int \frac{d\theta}{\sec\theta} = \int t\sqrt{t^2+1} \quad dt$$

$$= \sin\theta = \frac{(t^2+1)^{3/2}}{3} + C.$$

$$C = \frac{\sin\theta \; (3)}{(t^2+1)^{3/2}}.$$

$$C = \frac{3\sin\theta}{(t^2+1)^{3/2}}$$

Ans

Q2-4(e) $(2u+1)\,u' - (t+1) = 0$.

Sol

$$(2u+1)\frac{du}{dt} = (t+1)$$

$$\int(2u+1)\,du = \int(t+1)\,dt.$$

$$u^2+u = \frac{t^2}{2} + t + C.$$

$$\frac{2(u^2+u)}{t^2} - t = C$$

$$C = \frac{2(u^2+u)}{t} - t$$

Ans

Q2-4(d) :- $R' = (t+1)(R^2+1)$.

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\frac{dR}{R^2+1} = (t+1)\,dt$$

$$\int\frac{dR}{R^2+1} = \int(t+1)\,dt$$

$$\cot(R) = \frac{t^2}{2} + t + C$$

$$C = \frac{2\cot(R) - 6}{t^2}$$

Ans