SUMMER EXAMINATION
MID TERM

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GD NO ⇒ 7965

SUBJECT ⇒ NUMERICAL ANALYSIS

SUBMITTED TO ⇒ MAM SHUMAILA

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Question :- 01

Newton Raphson Method :-

Formula :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Required :-

find the root of the eq.

Given :-

$$f(x) = x^3 + 3.993 \times 10^4 = 0.165x^2$$

$$x_0 = 0.02$$

Solution :-

$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^4 = 0$$

$$x_0 = 0.02$$

$$f(x_0) = (0.02)^3 - 0.165(0.02)^2 + 3.993 \times 10^4$$
$$= 0.000008 - 0.000066 + 3.993 \times 10^4$$
$$= 8 \times 10^{-6} - 6.6 \times 10^{-5} + 3.993 \times 10^4$$
$$= 39929.999$$

$$f'(x) = \frac{d}{dx}\{x^3 - 0.165x^2 + 3.993 \times 10^4\}$$

$$= 3x^2 - 0.33x + 0$$

$$= 3x^2 - 0.33x$$

1st Iteration :-

put $n=0$

then,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

put $x_0 = 0.02$

$$x_1 = 0.02 - \frac{f(0.02)}{f'(0.02)}$$

$$= 0.02 - \frac{39929.999}{-5.4\times10^{-3}}$$

$$= 0.02 - (-7.394444)$$

$$= 7.4144444$$

$\therefore f(0.02) =$

$(0.02)^3 - 0.165(0.02)^2 + 3.993\times10^4$

$= 8\times10^{-6} - 6.6\times10^{-5} + 3.993\times10^4$

$= 39929.999$

$\therefore f'(0.02) =$

$= 3(0.02)^2 - 0.33(0.02)$

$= 1.2\times10^{-3} - 6.6\times10^{-3}$

$= -5.4\times10^{-3}$

2nd Iteration :-

put $n=1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

put $x_1 = 7.4144$

$$x_2 = (7.4144) - \frac{f(7.4144)}{f'(7.4144)}$$

$\therefore f(7.4144) =$

$(7.4144)^3 - 0.165(7.4144)^2 + 3.993\times10^4$

$= 407.594 - 9.07059 + 3.993\times10^4$

$= 40328.523$

$x_2 = 7.4144 - \frac{40328.523}{162.47315}$ ∴ $f'(7.4144)$

$= 7.4144 - 248.2165$ $= 3(7.4144)^2 - 0.33(7.4144)$

$= -240.8021$ $= 164.9199 - 2.44675$

$= 162.47315$

3rd Iteration :-

put $n = 2$ ∴ $f(-240.8021)$

$= (-240.8021)^3 - 0.165(-240.8021)^2 + 3.993 \times 10^4$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ $= -13963066.62 - 9567.6324 + 3.993 \times 10^4$

put $x_2 = -240.8021$ $= -13932704.25$

$x_3 = -240.8021 - \frac{f(-240.8021)}{f'(-240.8021)}$

$= -240.8021 - \frac{(-13932704.25)}{174036.4188}$

$= -240.8021 + 80.05625$ ∴ $f'(-240.8021)$

$x_3 = -160.74585$ $= 3(-240.8021)^2 - 0.33(-240.8021)$

$= 173956.9541 + 79.46469$

$= 174036.4188$

Question :- 02

LAGRANGE INTERPOLATION METHOD :-

FORMULA :-

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$\ldots\ldots \frac{(x-x_4)}{(x_1-x_4)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$$

..... and so on .

Required :-

To Find lagrange interpolation polynomial.

Solution:-

$x_0 = 2, \quad x_1 = 2.75, \quad x_2 = 4$

Eq we have

$$f(x) = \frac{1}{x}$$

for y_0,

$$y_0 = f(2) = \frac{1}{2}$$

$$= 0.5$$

for $y_1 = f(2.75) = \frac{1}{2.75} = 0.3636$

for $y_2 = f(4) = \frac{1}{4} = 0.25$

$y_0 = 0.5, \quad y_1 = 0.3636, \quad y_2 = 0.25$

where $x = 3$

putting above values in formula,

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} \times 0.5$$

$$+ \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.3636$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} \times 0.25$$

$$= \frac{(0.25)(-1)}{(0.75)(-2)} \times 0.5 + \frac{(1)(-1)}{(0.75)(-1.25)} \times 0.3636$$

$$+ \frac{(1)(0.25)}{(2)(1.25)} \times 0.25$$

$$= \frac{-0.25}{1.5} \times 0.5 + \frac{-1}{+0.9375} \times 0.3636$$

$$+ \frac{0.25}{2.5} \times 0.25$$

$$= -0.08333 + 0.38784 + 0.025$$

$$\boxed{y = 0.32951}$$

QUESTION :- 03

Given :-

$x = 1.0, 1.3, 1.6, 1.9, 2.2$

$y = 0.7651977, 0.6200860, 0.4554022, 0.2818186$ and 0.1103623

Required :-

To construct the interpolating polynomial.

Solution :-

our 5 data points table would be,

x	F(x)	First D.D	2nd D.D	3rd D.D	4th D.D
1.0	0.7651977				
		-0.483705			
1.3	0.6200860		-0.108735		
		-0.548946		0.06588	
1.6	0.4554022		-0.049443		0.0018234
		-0.578612		0.0680681	
1.9	0.2818186		0.0118183		
		-0.571521			
2.2	0.1103623				

First Divided Difference :-

$$F(x_0, x_1) = \frac{F(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{0.6200860 - 0.7651977}{1.3 - 1.0}$$

$$= \frac{-0.1451117}{0.3}$$

$$= -0.4837705$$

$$F(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$$

$$= \frac{-0.1646838}{0.3}$$

$$= -0.548946$$

$$F(x_2, x_3) = \frac{F(x_3) - f(x_2)}{x_3 - x_2}$$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$= -0.578612$$

$$f(x_3,x_4) = \frac{f(x_4)-f(x_3)}{x_4-x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2-1.9}$$

$$= -\frac{0.1714563}{0.3}$$

$$= -0.571521$$

2nd Divided Difference :-

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$

$$= \frac{-0.548946-(-0.483705)}{1.6-1.0}$$

$$= \frac{-0.065241}{0.6}$$

$$= -0.108735$$

$$f(x_1,x_2,x_3) = \frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1}$$

$$= \frac{-0.578612-(-0.548946)}{1.9-1.3}$$

$$= -\frac{0.029666}{0.6}$$

$$= -0.049443$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= \frac{0.007091}{0.6}$$

$$= 0.0118183$$

3rd Divided Difference :-

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.049443 - (-0.108735)}{1.9 - 1.0}$$

$$= \frac{0.059292}{0.9}$$

$$= 0.06588$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.0118183 - (-0.049443)}{2.2 - 1.3}$$

$$= \frac{0.0612613}{0.9}$$

$$= 0.0680681$$

4th Divided Difference :-

$$F(x_0, x_1, x_2, x_3, x_4) = \frac{F(x_1, x_2, x_3, x_4) - F(x_0, x_1, x_2, x_3)}{x_4 - x_0}$$

$$= \frac{0.0680681 - 0.06588}{2.2 - 1.0}$$

$$= \frac{0.0021881}{1.2}$$

$$= 0.0018234166$$

The 5 coefficients of the Newton's Interpolating polynomial are :-

a_0	$= F(x_0)$	$= 0.7651977$
a_1	$= F(x_0, x_1)$	$= -0.4837705$
a_2	$= F(x_0, x_1, x_2)$	$= -0.108735$
a_3	$= f(x_0, x_1, x_2, x_3)$	$= 0.06588$
a_4	$= F(x_0, x_1, x_2, x_3, x_4)$	$= 0.0018234166$