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seaction B.

Paper Diffrential eqluation.

Q.No 1. ①

Part A:

(a) $w = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c)$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \text{ --- (1)}$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2.$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= [-\sin(x+ct) - 4\cos(x+2ct)]$$

$$\frac{\partial^2 w}{\partial t^2} = c^2[-\sin(x+ct) - 4(\cos(2x+2ct))]$$

$$\boxed{c^2 = \partial^2 w / \partial x^2}$$ Ans

① Part B: ②

$w = \tan(2x + ct)$.

Sol: Diffrential w.r.t to t

$$\frac{\partial w^2}{\partial t^2} = c\,2\sec(2x+ct)\,\frac{\partial}{\partial t}\sec(2x+ct)$$

$$= 2c^2\sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w^2}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

Diffrentiate w.r.t to x.

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct$$

$$\frac{\partial^2 w}{\partial x^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$= 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)\ldots \tan(2x+ct)$$

So it is not possible.

Q No 2: (5)

Given ftn is

$$f(x) = \begin{cases} x, & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi . \end{cases}$$

Sol:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx .$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi} .$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{1}{\pi}\left[\frac{\pi^2}{2} - 0\right].$$

$$a_0 = -\frac{\pi}{2} + \pi = \pi/2$$

$$\boxed{a_0 = \pi/2} \longrightarrow (1)$$

Now a_n.

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (x\cos nx)\,dx + 1/\pi \int_{0}^{\pi} (2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} .$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \left\{\begin{array}{ll} \frac{-2}{\pi n^2}; & \text{if } n \text{ is odd.} \\ 0, & \text{if } n \text{ is even} \end{array}\right\} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx$$

$$+ \frac{2}{\pi}\int_0^{\pi} \sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right]$$

$=$

(5)

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the required forrier series is

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} (a_n \cos nx + b_n \sin nx$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\alpha} \frac{\cos(2n-1)x}{(2n-1)} + 3\sum_{n=1}^{\alpha} (-1)^{n+1} \frac{\sin nx}{n}$$

(b)

Q 3:-

$y'' - 4y' + 13y = 8\sin 3x \; \rightarrow (1) \quad y(0) = 1$

Associated Homogenous $\quad y'(0) = 2$

eq (1) is.

$y'' + 4y' + 13y = 0 \rightarrow (2)$

Change (2) into Auxilairy equation

Put $y = m$ in (2)

$m^2 - 4m + 13y = 0$

use Quardratic equation.

$a = 1, \; b = -4, \; c = 13$.

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

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$$m = \frac{4 \pm \sqrt{36}i}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$2 \pm 3i$$

$$m_1 = 2 + 3i$$

$$m_2 = 2 - 3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (1)$$

let $y_p = A \cos 3x + B \sin 3x \rightarrow (*)$

Diff w.r.t to x.

$$y'_p = -3A \sin 3x + 3B \cos 3x$$

Again Diff. w.r.t to x.

$$y''_p = -9A \cos 3x - 9B \sin 3x.$$

Put in (1)

$$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4 \cdot (-3A \sin 3x + 3B \cos 3x) + 13(A \cos 3x + B \sin 3x) = 8 \sin 3x$$

$$\Rightarrow (-9A \cos 3x - 12B \cos 3x + 13A \cos 3x - 9B \sin 3x + 13B \sin 3x = 8 \sin 3x.$$

$(-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x =$
$= 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$ — (i)

comparing coefficent

$\sin 3x \Rightarrow 4B + 12A = 8 \rightarrow (a)$

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$\boxed{A = 3B} \rightarrow (b)$

Put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$\boxed{B = 1/5} \rightarrow (c)$

Put (c) in (b)

$\boxed{A = 3/5} \rightarrow (d)$

Put c & d in *

$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$

The Gr. sol is

$y = y_c + y_p$.

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow C$

Now we need to find the value's of c_1 & c_2 for this

Put $x=0$ & $y=1$ in C

~~$1 = [c_1(1) + c_2(0)]$~~

$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin 3(0)) + 3/5 \cos 3(0) + 1/5 \sin 3(0)$

$1 = (c_1(1) + c_2(0)) + 3/5(1) + 1/5(0)$

$1 = c_1 + 3/5$.

$\boxed{c_1 = 2/5} \rightarrow$ **

Diff. C w.r.t x.

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{6}\sin 3x + 3/5\cos 3x \rightarrow D$

$y = y' = 2$, $x=0$ in D

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$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$$
$$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$
$$- \frac{6}{5}\sin 3x + 3/5 \cos 3x.$$

Put $y' = 2,\ x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0))$$
$$+ c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cdot\cos 3(0))$$
$$- 6/5 \sin 3(0) + 3/5 \cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + 3/5$$

$$2 = 2c_1 + 3c_2 + 3/5$$

Put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5.$$

$$2 = \frac{7}{5} + 3c_2.$$

$$3c_2 = 2 - 7/5.$$

$$\boxed{c_2 = 3/15}$$ ***

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Put ** & *** in → (c)

$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

Required General solution

Q4:

It is in symbolic form.

$(D^2 - DD')Z = \cos x \cos 2y \rightarrow (a)$

Put A.E $D^2 - DD' = 0$

As we know that

$$\frac{D}{D'} = m \quad \text{i.e } D = m, D' = 1$$

$\Rightarrow \quad m^2 - m = 0$

$m = 0, 1.$

therefore $\quad c.f = f_1(y) + f_2(y+x)$

from eq (a).

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y.$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y.$$

As $\frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y.$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$C.F = f_1(y-n) + n f_2(y-n)$$

$$P.I = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-n) + \sin(n-y)\right].$$

$$= \frac{1}{(D+D')^2}\left[2(y-n) + \sin(n-y)\right]$$

by general method.

$$m = -1,\ y - n = c.$$

$$= \frac{1}{D+D'}\int\left[2c + \sin(-c)\right]dn.$$

$$= \frac{1}{D+D'}\left[2cn - (\sin c)n\right].$$

replacing c by y-n.

$$= \frac{1}{D+D'}\left[2n(y-n) - n\sin(y-n)\right].$$

Again put $y-n=c$.

$$= \int(2nc - n\sin c)\,dn \Rightarrow cn^2 - \frac{n^2}{2}\sin c$$

Replacing c by y-n

$$= n^2(y-n) - \frac{n^2}{2}\sin(y-n) = n^2y - n^3 + \cancel{\frac{n^2}{2}\sin(n-y)}$$

$$+ \frac{n^2}{2}\sin(n-y)$$

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Hence Required sol:

$$Z = C.F + P.I = f_1(y-x) + x f_1(y-x) + x^2 y - x^3 + \frac{1}{2} x^2 \sin(x-y).$$