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Q1:- a) Differentiat $\frac{3x^2-5x^2+5}{x^2+1}$ with respect to "x"

chang into product form

$(3x^2-5x^2+5)(x^2+1)^{-1}$

Let $y = (3x^2-5x^2+5)(x^2+1)^{-1}$

differentiat with respect to "x"

$\frac{dy}{dx} = \frac{d}{dx}\underset{\text{I}}{(3x^2-5x^2+5)}\underset{\text{II}}{(x^2+1)^{-1}}$

$= (3x^2-5x^2+5)\frac{d}{dx}(x^2+1)^{-1} + (x^2+1)^{-1}\frac{d}{dx}(3x^2-5x^2+5)$

$= (3x^2-5x^2+5)[-1(x^2+1)^{-2}\frac{d}{dx}(x^2+1) + (x^2+1)^{-1}(9x-10x)$

$= (3x^2-5x^2+5)(-x^2-1)^{-2}(2x) + (x^2+1)^{-1}(9x-10x)$

$= (3x^2-5x^2+5)(-x^2-1)^{-2}(2x) + (x^2+1)^{-1}(9x-10x)$

$= (6x^3-10x^3+10x)(-x^2-1)^{-2} + (x^2+1)^{-1}(9x-10x)$

Required Ans:-

(1) :. b

$$\frac{(x^2+1)^2}{x^2-1}$$

let $y = \dfrac{(x^2+1)^2}{x^2-1}$

chang into product form

$y = (x^2+1)^2 (x^2-1)^{-1}$

differentiat with respect to "x"

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+1)^2 (x^2-1)^{-1}$$

$$(x^2+1)^2 \frac{d}{dx}(x^2-1)^{-1} + (x^2-1)^{-1} \frac{d}{dx}(x^2+1)^2$$

$$\cancel{(x^2+1)^2 (-1 \cdot \frac{d}{dx}(x^2-1) + (x^2-1)^{-1} (2 \cdot \frac{d}{dx}(x^2+1)}$$

$$(x^2+1)^2 (-1(x^2-1)^{-2} \frac{d}{dx}(x^2-1) + (x^2-1)^{-1} 2(x^2+1) \frac{d}{dx}$$

$$(x^2+1)^2 (-x^2+1)^{-2} \frac{d}{dx}(x^2-1) + (x^2-1)^{-1} (2x^2+2)(x^2+1)$$

$$(x^2+1)^2 (-x^2+1)^{-2} (2x) + (x^2-1)^{-1} (2x^2+2)(2x) \qquad 2x$$

Reaquared Ans:.

Q 2:. a)

Find $\frac{dy}{dx}$ if $y = (1+2\sqrt{x})^3 \cdot x^{3/2}$

Solution:. $y = (1+2\sqrt{x})^3 \cdot x^{3/2}$ $\left[(1+2\sqrt{x})(x^{1/2})\right]^3$

Let $u = (1+2\sqrt{x}) \cdot x^{1/2}$ —— (i)

then $y = u^3$ —— (ii)

differentiating (ii) with respect to u, we have

$\frac{dy}{dx} = 3u^2 \quad 3\left[(1+2\sqrt{x}) \cdot x^{1/2}\right]^2 \quad 3(1+2\sqrt{x})^2 \cdot x$

differentiating (i) with respect to x, gives

$\frac{du}{dx} = (0+2 \cdot \frac{1}{2\sqrt{x}}) \cdot x^{\frac{1}{2}} + (1+2\sqrt{x})\frac{1}{2\sqrt{x}}$

$= 1 \quad \frac{1+2\sqrt{x}}{2\sqrt{x}} \quad \frac{2\sqrt{x}+1+2\sqrt{x}}{2\sqrt{x}} \quad \frac{1+4\sqrt{x}}{2\sqrt{x}}$

Using the formula $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, we have

$\frac{d}{dx}\left[(1+2\sqrt{x})^3 \cdot x^{3/2}\right] = 3(1+2\sqrt{x})^2 \cdot x \times \left(\frac{1+4\sqrt{x}}{2\sqrt{x}}\right)$

$= \frac{3}{2}(1+2\sqrt{x})^2 \sqrt{x}(1+4\sqrt{x})$

$= -(1+2\sqrt{x})(\sqrt{x}+4x)$

Q2:. b) Differentiat $\sqrt{\dfrac{a-x}{a+x}}$

Sol:- Let $y = \sqrt{\dfrac{a-x}{a+x}}$ and $u = \dfrac{a-x}{a+x}$, then $y\ u^{\frac{1}{2}}$

Now $\dfrac{dy}{du} = \dfrac{1}{2}u^{\frac{1}{2}-1} = \dfrac{1}{2}u^{-\frac{1}{2}}$

and $\dfrac{du}{dx} = \dfrac{d}{dx}\left(\dfrac{a-x}{a+x}\right) = \dfrac{\left[\dfrac{d}{dx}(a-x)\right](a+x) - (a-x)\left[\dfrac{d}{dx}(a+x)\right]}{(a+x)^2}$

$= \dfrac{(0-1)(a+x) - (a-x)(0+1)}{(a+x)^2}$

$= \dfrac{-a-\cancel{x}-a+\cancel{x}}{(a+x)^2} = \dfrac{-2a}{(a+x)^2}$

Using the formula

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\sqrt{\dfrac{a-x}{a+x}}\right) = \dfrac{1}{2}u^{-\frac{1}{2}}\left[\dfrac{-2a}{(a+x)^2}\right] = \dfrac{1}{2}\left(\dfrac{a-x}{a+x}\right)^{-\frac{1}{2}} \times \dfrac{-2a}{(a+x)^2}$

$= \dfrac{(a-x)^{-\frac{1}{2}}}{(a+x)^{-\frac{1}{2}}} \times \dfrac{-a}{(a+x)^2} = \dfrac{-a\cancel{(a-x)^{-\frac{1}{2}}}}{\cancel{(a+x)^{-\frac{1}{2}}}(a+x)^2}$

$= \dfrac{-a}{(a+x)^2}$ Ans:..

Q3:. a) Find the Integration of $\int \frac{1}{\sqrt{x^3}} dx$

Solution:. $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{\sqrt{x^3}} dx$$

$$= \int x^{3/2} dx$$

$$\cancel{= \int \frac{x^{3/2+1}}{x}}$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}$$

$$= \frac{x^{\frac{3+2}{2}}}{\frac{3+2}{2}}$$

$$= \frac{x^{5/2}}{5/2} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \frac{x^{5/2}}{5/2} + C \quad \text{Ans.}$$

(b) $\int \frac{1}{(6x+7)^6} dx$

Solution:. $\int \frac{1}{(6x+7)^6} dx$

$$\int \frac{1}{(6x+7)^6} dx = \int .1(6x+7)^{-6} dx$$

$$= \int (6x+7)^{-6} dx$$

$$= \frac{(6x+7)^{-6+1}}{-6+1}$$

$$= \frac{(6x+7)^{-5}}{-5} = \frac{1}{-5}(6x+7)^{-5}$$

$$\int \frac{1}{(6x+7)^6} dx = \frac{-1}{5}(6x+7)^{-5} + C$$

Ans.