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QNo1

$x_1 - 3x_2 + x_3 = 0$ ∴ 16373

$2x_2 - 8x_3 = 8$ ∴ ID3 is "3"

$5x_1 - 5x_3 = 10.$

writ in matrix form.

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -15 & -10 & 10 \end{array}\right] R_3 - 5R_1$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 5 & 2 & -2 \end{array}\right] \begin{array}{l} 5R_2 \\ -\frac{1}{5}R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 10 & -40 & 40 \\ 0 & 5 & 2 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 0 & -44 & 44 \\ 0 & 5 & 2 & -2 \end{array}\right] R_2 - 2R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 5 & 2 & -2 \\ 0 & 0 & -44 & 44 \end{array}\right] R_2 \leftrightarrow R_3.$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 5 & 2 & -2 \\ 0 & 0 & -1 & 1 \end{array}\right] \frac{1}{44} R_3$$

∴ Lower triangle like all Zero (0).

This is perfect triangle there is Unique and consistend because

$$\boxed{\begin{array}{l} x_1 = 1 \\ x_2 = 5 \\ x_3 = -44 \end{array}}$$

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2nd Semester

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Q No 2:: Syed Junaid Ali Shah

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{bmatrix},$$ $\boxed{ID\ 4th = 7}$

Find $A^{-1}$.

First find determinant of A.

Exp by $R_1$.

$$3\begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{vmatrix},$$

$$3\begin{vmatrix} -1 & 7 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 7 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$3(-7+14) - 4(14-35) + 5(-4+5)$

$3(7) - 4(-21) + 5(1)$

$21 + 84 + 5$

$110 \neq 0$ $A^{-1}$ is Possible.

Now to find Adjoin

we have $A^{-1} = \dfrac{Adj\,A}{|A|}$

and $Adj\,A = B^t$

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where $B^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^t$

Now

$a_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} \cdot (-1\times7 - 7\times(-2)) = +(-7+14) = 7$

$A_{12} = (-1)^{1+2} (14 - 35) = -(-21) = 21$

$A_{13} = (-1)^{1+3} (-4+5) = +1$

$A_{21} = (-1)^{2+1} (28+10) = -38$

$A_{22} = (-1)^{2+2} (21-25) = -(-4) = 4$

$A_{23} = (-1)^{2+3} (-6-20) = -(-26) = 26$

$A_{31} = (-1)^{3+1} (28+5) = -(33) = -33$

$A_{32} = (-1)^{5} (21-10) = -(11) = -11$

$A_{33} = (-1)^{3+3} (-3-8) = +(-11) = -11$.

So $B^t = \begin{bmatrix} 7 & 21 & 1 \\ -38 & 4 & 26 \\ -33 & -11 & -11 \end{bmatrix}^t$

$Adj = B^t = \begin{bmatrix} 7 & -38 & -33 \\ 21 & 4 & -11 \\ 1 & 26 & -11 \end{bmatrix}$

$A^{-1} = \frac{adj\,A}{|A|}$

$A^{-1} = \frac{1}{110} \begin{bmatrix} 7 & -38 & -33 \\ 21 & 4 & -11 \\ 1 & 26 & -11 \end{bmatrix}$

Ans

Q 3.

$2x + 2y + 4z = 18$
$x + 3y + 2z = 13$
$3x + y + 3z = 14$

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Solution

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \quad R_1 = \frac{1}{2} R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - 3R_1 \end{matrix} \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad R_2 = \frac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad R_3 = R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{matrix} R_1 = R_1 - R_2 \\ R_3 = -\frac{1}{3} R_3 \end{matrix}$$

$\boxed{\text{PTO}}$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad R_1 = R_1 - 2R_3$$

$x = 1$

$y = 2$

$z = 3$

Ans

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

1) $|\lambda I - A| = \begin{vmatrix} \lambda-4 & 2 & -2 \\ -5 & \lambda-3 & 2 \\ -2 & 4 & \lambda-1 \end{vmatrix}$

$(\lambda-4)[(\lambda-3)(\lambda-1) - 8] - 2[-5(\lambda-1)+4] - 2[-20+2(\lambda-3)]$

$(\lambda-4)[\lambda^2 - \lambda - 3\lambda + 3 - 8] - 2[-5\lambda+5+4] - 2[-20+2\lambda-6]$

$(\lambda-4)[\lambda^2 - 4\lambda - 5] - 2[-5\lambda+9] - 2[2\lambda-26]$

$\lambda^3 - 4\lambda^2 - 5\lambda - 4\lambda^2 + 16\lambda + 20 + 10\lambda - 18 - 4\lambda + 52$

$\lambda^3 - 8\lambda^2 + 17\lambda + 54 = 0$

$P(\lambda) = \lambda^3 - 8\lambda^2 + 17\lambda + 54 = 0$

$P(-2) = 0$

$$\begin{array}{c|cccc} & 1 & -8 & 17 & 54 \\ 2 & 1 & 2 & 12 & 54 \\ \hline & 1 & 6 & 7 & 0 \end{array}$$

$\boxed{\lambda^2 + 6\lambda + 7 = 0}$

7

Qs. $3x_1 + 5x_2 - 4x_3 = 0$ 	ID No = 16373

$-3x_1 - 25x_2 + 4x_3 = 0$ 	Page No = 7

$6x_1 + x_2 - 8x_3 = 0$

Write in matrix form.

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix}$$

Exp by $R_1$

$3((-25)(-8) - (4)(1)) - 5(-3(-8) - (4)(6)) - 4(-3 - (6)(-25))$

$3(200 - 4) - 5(24 - 24) - 4(-3 + 150)$

$3(196) - 0 - 4(147)$

$588 - 588 = 0.$

$|A| = 0$

Since $|A| = 0 \Rightarrow$ Non-Trival Solution.

So. $\left[\begin{array}{ccc|c} 3 & 5. & -4 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \begin{matrix} \\ R_2 + R_1 \\ R_3 - 2R_1 \end{matrix}$

(3)

$x_1 =$

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$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \quad \frac{1}{9}R_3.$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -20 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 + 20R_2$$

let $\boxed{x_3 = t}$

Then from $R_2$

$0x_1 + 1x_2 + 0x_3 = 0$

$0x_1 + x_2 + 0 = 0$

$\boxed{x_2 = 0}$

from $R_1$

$3x_1 + 5x_2 - 4x_3 = 0$

$3x_1 + 5(0) - 4(t) = 0$

$3x_1 - 4t = 0$

$\frac{3x_1}{3} = \frac{4t}{3}$

$\boxed{x_1 = \frac{4}{3}t}$

Result:

$x_1 = \frac{4}{3}t$

$x_2 = 0$

$x_3 = t$

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Q 6= Reduce The matrix to Normal form and find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{bmatrix} -\frac{1}{6} R_2$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 + 3R_2$$

Since there is 2 non-zero rows

So rank is equal to "2".