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QNo.1.)

Find PQ where P is the point in three-dimensional space with coordinates (4,1,3) and the point Q with cooridnates (1,2,4). Find the distance b/w P & Q. Find the position vector of the point dividing PQ in the ratio 1:3?

Sol:-

Given that

$$P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$$

$$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$$

Now distance between PQ,

$$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$$

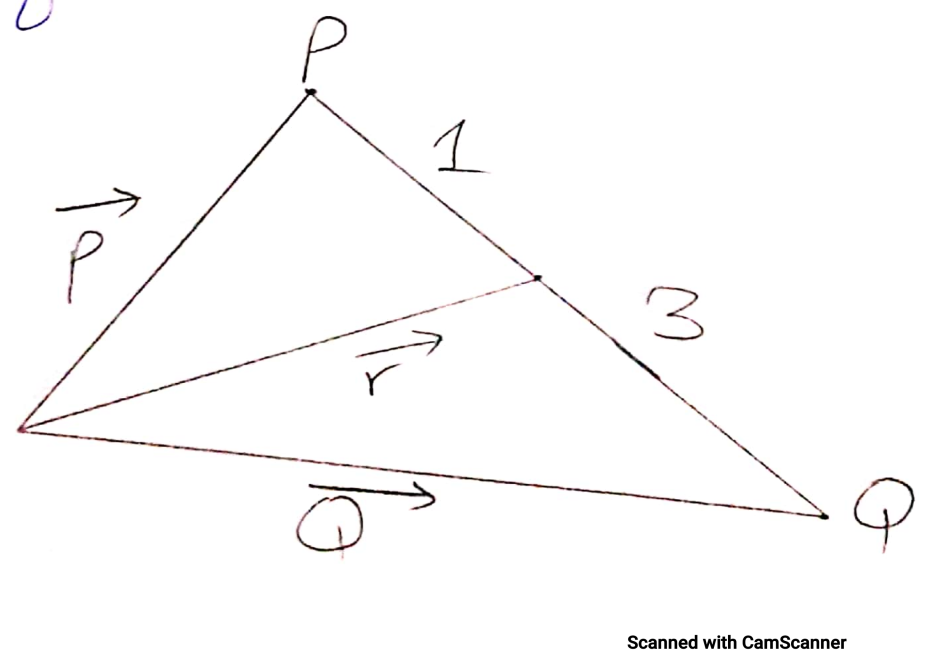
$$= \sqrt{3^2 + (-1)^2 + (-1)^2}$$

$$= \sqrt{9+1+1}$$

$$= \sqrt{11}$$

Now,

Find the position vector of the point dividing PQ in the ratio of 1:3.



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$$a : b = 1 : 3$$

$$\vec{r} = \frac{b\vec{P} + a\vec{Q}}{b+a}$$

$$= \frac{3(4\hat{i}+\hat{j}+3\hat{k})+1(\hat{i}+2\hat{j}+4\hat{k})}{3+1}$$

$$= \frac{12\hat{i}+3\hat{j}+9\hat{k}+\hat{i}+2\hat{j}+4\hat{k}}{4}$$

$$= \frac{13\hat{i}+5\hat{j}+13\hat{k}}{4}$$

$$\boxed{\vec{r} = \tfrac{13}{4}\hat{i} + \tfrac{5}{4}\hat{j} + \tfrac{13}{4}\hat{k}}$$

Ans

Q2.)

Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Sol:-

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10x+4 \\ & \pm\cancel{4x^3} \qquad\qquad \pm 2x^2 \\ \hline & -\cancel{2x^2}+10x+4 \\ & \mp\cancel{2x^2} \mp x \\ \hline & 11x+4 \end{array}$$

So,

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}$$

$$\int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 + \int \frac{11x+4}{2x^2+x} \rightarrow (i)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+\cancel{1}x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \rightarrow (ii)$$

Now,

Find $\int \frac{11x+4}{x(2x+1)} = ?$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \rightarrow (A)$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1)+Bx \rightarrow (iii)$$

put $x=0$ in (iii)

$$4 = A$$

Now,

put $x = -1/2$ in iii

$$11(-1/2)+4 = B(-1/2)$$

$$-11/2 + 4 = -B/2$$

$$\frac{-11+8}{\cancel{2}} = -\frac{B}{\cancel{2}}$$

$$-3 = -B$$

$$B = 3$$

Now,

putting the value A & B in A

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking Integral on both sides

$$\int \frac{11x+4}{x(2x+1)} dx = \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

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$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

putting these values in

(ii)

$$= x^2 - x + 4\ln|x| + 3/2 \ln|2x+1|$$

Now,

put these values in (i)

$$\boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2 \ln|2x+1| + c}$$

Ans

QNo 3.)

a.)

$$\int_0^2 x^2 e^x \, dx$$

Sol:-

$$\int_0^2 x^2 e^x \, dx$$

Now first find Integration

$$= \int x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left(x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right)$$

$$= x^2 e^x - 2 \left(x e^x - \int e^x dx \right)$$

$$= x^2 e^x - 2 x e^x + 2 e^x$$

Now put limits,

$$= \left| x^2e^x - 2xe^x + 2e^x \right|_0^2$$

$$= \left(2^2e^2 - 2(2)e^2 + 2e^2\right) - \left(0 - 0 + 2e^0\right)$$

$$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2\right)$$

$$= \boxed{2e^2 - 2.} \quad \text{Ans}$$

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

Sol:-

First find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \rightarrow (i)$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2\,dy = \frac{1}{\sqrt{x}}\,dx}$$

put in 1

$= \int \sin(y)(2dy)$

$= 2\int \sin(y)\,dy$

$= 2(-\cos y)$

$= -2\cos y$

$=$ put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

Now,

put limits,

(11)

$$= -2\left|\cos\sqrt{x}\right|_1^2$$

$$= -2\left(\cos\sqrt{2} - \cos\sqrt{1}\right)$$

$$= -2\left(\cos\sqrt{2} - \cos 1\right)$$

$$\boxed{= -2\cos\sqrt{2} + 2\cos(1)}$$

Ans

Q NO 4.)

Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three-dimensional Laplace's Equation?

Sol:-

The laplace's equation in Three - dimensional is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow A$$

So, $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

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$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial u}{\partial x} = -\{x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}]$$

$$\frac{\partial u}{\partial x} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now,

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial^2 y^2} = -\left[y\left(-3/\cancel{2}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) \right.$$
$$+ (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \rightarrow (ii)$$

Now,

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = -\left[z(-3/2)(x^2+y^2+z^2)^{-5/2}(2z) \right.$$
$$+ (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow (iii)$$

Putting (i) & (ii) & (iii) in (A)

$$= 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2} - \cancel{3x^2} + \cancel{3y^2} - \cancel{3y^2} + \cancel{3z^2} - \cancel{3z^2}\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is Solution of laplace's equation

Ans