

Subject

Advance Fluid

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Q NO. 1):

(a)

ANS: → A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion b/w body & fluid. These forces are termed as drag and lift depending on forces parallel or right angle to motion.

→ Drag force are submerged body can have 2 components.

1) Pressure Drag: It is equal to the integration of components in direction of motion of all pressure forces exerted on surface of body.

$$F_p = C_p \cdot \int \frac{v^2}{2} \cdot A$$

(Where C_p depend on shape)

2) Friction Drag: It is equal to integration of component of all shear stresses along the surface in direction of motion.

$$F_p = C_f \cdot \int \frac{v^2}{2} (B_c)$$

(C_f depends on velocity).

Friction Drag of Boundary Layer:

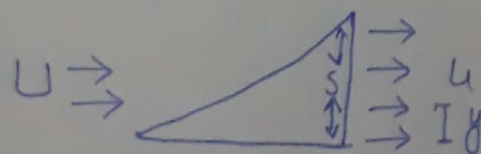
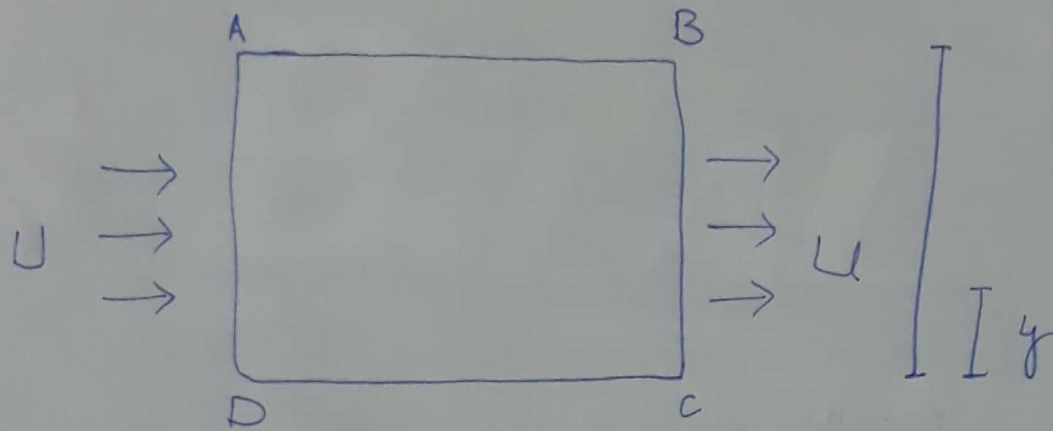


Figure shows growth of Boundary layer along one side of smooth plate

in steady flow of incompressible fluid consider volume where δ is the thickness of boundary layer and U is the undisturbed velocity.



As we have $\sum F_x = 0$

where

$$F_x = \frac{\Delta P}{\Delta t} = \frac{\Delta m v}{\Delta t} \quad \therefore m = \int v$$

$$F_x = \frac{\Delta \rho \cdot \text{vol} \cdot v}{\Delta t} = \Delta \rho Q v$$

$$\boxed{F_x = \Delta \rho Q v}$$

$-F_x = \text{rate of change of}$
 $BC + AB - AD$

$$AD = \int U (U \delta B)$$

$$BC = \int_B (u^2 dy)$$

$$AB = \int U (UB \rho) - B \cdot \int_0^{\delta} u dy$$

$$F_x = \int_B \int_0^{\delta} U (u - u) dy$$

Integration on b/s — ①

$$F_x = \int_B u^2 \delta d$$

Where a is a function of boundary layer velocity distribution

Now to find shear stress

$$\tau = \frac{F_x}{A} = \frac{d F_x}{B dx} = \frac{d F_x}{B dx}$$

$$\bar{\tau}_0 = \int_B u^2 d \frac{d\delta}{B dx}$$

$$= \int u^2 \alpha \frac{d\delta}{du}$$

$$\bar{\tau}_0 = \int u^2 \alpha \frac{d\delta}{dx}$$

Laminar Boundary Layer:

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \text{ --- (1)}$$

$$\frac{y}{\delta} = n$$

$$y = \delta n$$

$$dy = \delta dn \text{ --- (2)}$$

$$\frac{u}{U} = f(n)$$

$$du = U df(n) \text{ --- (3)}$$

For laminar flow

$$\bar{\tau}_0 = U \frac{d\tau}{dy} \text{ --- (4)}$$

$$\tau_0 = \mu \frac{d}{dx} \left(\frac{u^2}{\delta} \right)$$

$$\bar{\tau}_0 = \frac{\mu U B}{\delta} \quad \text{--- (5)}$$

As we have $\tau_0 = \mu u^2 \alpha \frac{d\delta}{dx}$

Compare both

$$\mu u^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\int d\delta = \frac{U B dx}{\int U \alpha}$$

Int on b/s

$$\frac{\delta^2}{2} = \frac{U B x}{\int U \alpha} + C$$

$$\therefore C = 0$$

$$\delta = \frac{\sqrt{2B}}{\alpha} \cdot \sqrt{\frac{\mu U}{\int U}}$$

$$B = 1.63, \quad d = 0.135 \quad \therefore R_x = \frac{U x d}{\mu}$$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \quad \text{--- (6)}$$

where (R_x) is local Reynold no.

As we have

$$\bar{\tau}_0 = \frac{\mu U B}{\delta}$$

put eq (6) in (5)

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

eq ① 2 ②

$$x=0, \quad f=0$$

$$\delta = \left(\frac{0.0287}{\alpha} \right)^{4/5} \left(\frac{\nu}{Ux} \right)^{1/5} x$$

$$\alpha = 0.0972$$

$$\delta = \frac{0.377}{(Rh)^{1/5}} \cdot x$$

$$\tau_0 = 0.0587 \rho \frac{v^2}{2} \left(\frac{\nu}{Ux} \right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 dx$$

$$F_f = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL} \right)^{1/5} BL$$

$$F_f = C_f \cdot \rho \frac{v^2}{2} BL$$

equating b/s

$$C_f = \frac{0.0735}{(R)^{1/5}}$$

$$(50,000 < R < 10^7)$$

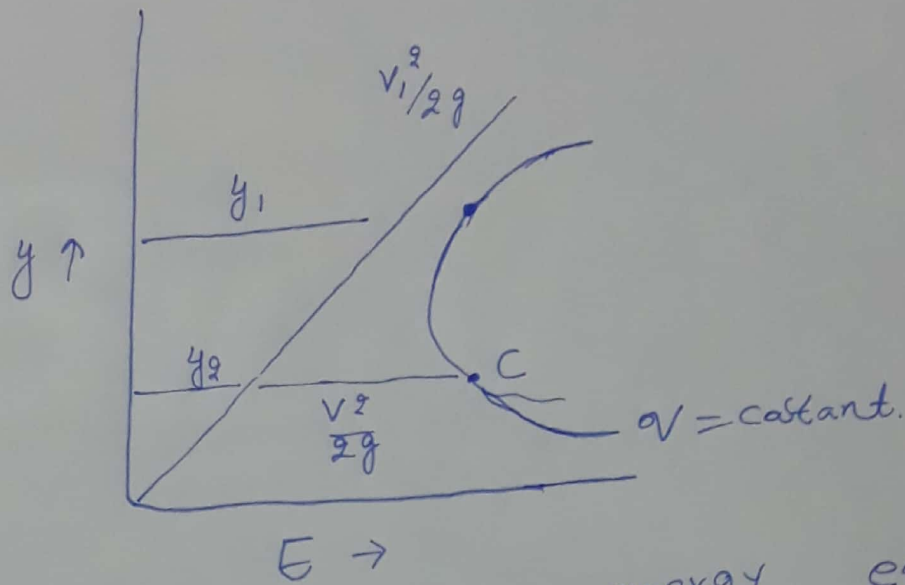
For $R > 10^7$

$$C_f = \frac{0.455}{(\log R)^{2.58}}$$

QNO. 1):

(b):

Ans:



This is specific energy equation
For particular v , there will be two
kind of possible values of y for
given E . The eqn is cubic with
three roots with third being
negative giving no values. Thus
two alternative depths represents
two totally different flow
regimes - slow & deep on upper
portion & fast and shallow on
lower portion.

Point represent dividing point
between two regima of flow.
Thus for given "v" value of E
is minimum & flow at this

Point is critical flow Depth of flow at this point is critical depth h_c and velocity at this point is critical velocity.

Thus relation of critical depth can be found as;

$$E = y + \frac{1}{2g} \left(\frac{v^2}{y^2} \right)$$

For min specific energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g} \left(\frac{v^2}{y^3} \right)$$

$$\frac{dE}{dy} = 1 - \frac{v^2}{gy^3}$$

$$1 = \frac{v^2}{gy^3}$$

$$\Rightarrow v^2 = gy^3$$

$$y_c = \left(\frac{v^2}{g} \right)^{1/3} \quad \text{critical depth}$$

$$\text{AS } v = Vy \quad , \quad v_c^2 = gy^3$$

$$\text{OR } \boxed{v_c = \sqrt{gy_c}} \quad \text{critical velocity}$$

$$y_c = \frac{v_c^2}{g}$$

Now

$$y_c = \frac{V_c^2}{2g}$$

$$E_{\min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$\frac{3}{2} y_c \quad \text{OR} \quad y_{cr} = \frac{2}{3} \quad \text{Combine both}$$

Subcritical

Critical

Super critical

Depth of Flow

$$y > y_c$$

$$y = y_c$$

$$y < y_c$$

velocity Slope

$V < V_c$
mild Slope
 $S_0 < S_c$

$V = V_c$
Critical Slope

$$V > V_c$$

Q NO. 2):

Given Data:

Depth of Rectangular Channel (d) = ?

Flow rate (Q) = $3.5 \text{ m}^3/\text{sec}$

Slop of Bed (S_0) = 0.0008

$n = 0.0219$

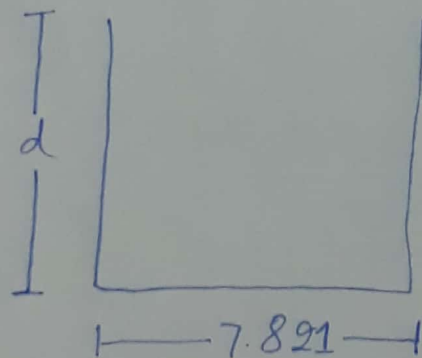
Width of bed = 7821 mm
= 7.821 m

Find:

Critical depth = ?

Flow subcritical or supercritical = ?

Solution:



$$\text{Area} = 7.821 \times d$$

$$= 7.821 d$$

$$\text{Parameter} = d + 7.821 + d$$

$$= 7.821 + 2d$$

$$\text{Hydraulic Radius (} R_h \text{)} = A/p$$

minimum velocity
 $C_0 = 0.332$

$$A/P = \frac{7.821d}{7.821 + 2d}$$

By using Manning eqn

$$Q = \frac{1}{n} ARh^{2/3} (S_0)^{1/2} \quad - \star$$

putting values

$$3.5 = \frac{0}{0.0219} \times 7.821d \times \frac{(7.821d)}{2d + 7.821} \times (0.008)^{1/2}$$

$$d = 0.55 \text{ m}$$

$$\begin{aligned} \text{Area} &= 7.821(0.55) \\ &= 4.30 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 7.821 + 2(0.55) \\ &= 8.921 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Hydraulic Radius (Rh)} &= \frac{4.37}{8.921} \\ &= 0.489 \text{ m} \end{aligned}$$

AS critical Depth

$$y_{cr} = \left(\frac{Q^2}{g} \right)^{1/3}$$

$$\text{AS } v = Q/B$$

$$= 3.5 / 7.821$$

$$= 0.447 \text{ m}^2/\text{sec}$$

$$y_{c2} = \left(\frac{(0.447)^3}{9.81} \right)^{1/3}$$

$$= 0.27$$

$y > y_{c2}$ so flow is
sub critical

$$0.55 > 0.27$$

Q No. 3):

Solution:

Friction Drag (F_D) = ?

Width (B) = 200 mm = 0.2 m

Length (L) = 800 mm = 0.8 m

undisturbed velocity (v) = 5 m/sec

specific gravity (S_f) = 0.89

kinematic viscosity (ν) = $0.98 \times 10^{-4} \text{ m}^2/\text{sec}$

Check whether Flow is laminar
or not By Reynold number

$$R = \frac{DV}{\nu}$$

For smooth plate

$$D = L, v = U$$

$$\therefore R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.98 \times 10^{-4}} = 43010$$

43010 < 500,000 \rightarrow Laminar

By using formula

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}}$$

$$C_f = 0.0064$$

$$S = \frac{\rho_{\text{soil}}}{\rho_{\text{water}}}$$

$$0.89 = \frac{\rho_{\text{soil}}}{1000}$$

$$\rho_{\text{soil}} = 0.89 \times 1000$$

$$\rho_{\text{soil}} = 890 \text{ kg/m}^3$$

$$F_f = C_f \times \rho \times \frac{U^2}{2} \times B \times L$$

$$F_f = 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$F_f = 11.39 \text{ N}$$