DIFFERENTIAL EQUATIONS
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Q1. Define differential equation along with 2 examples

ANS. A differential equation is an equation that relates one or more function and their derivates in application, the functions generally represent physical quantities the derivatives represent the rate of change and differential equation define relation between the two

$$dy/dx = f(x)$$

here 'x' is an independent variable
and 'y' is dependent variable

e.g $dy/dx = 5x$
$dy/dx = 7x$

Define a seperable differential equation.

Seperable differential equation is used when the differential equation can be written in form $(dy/dx = f(y)\,g(x)$ where 'f' is the function of 'y' only 'g' is the function of x only. Taking on initial condition.

e.g

$$\int \frac{1}{12000-s} \frac{ds}{dt}\, dt = \int \frac{3}{200}\, dt$$

$$\Rightarrow \int \frac{1}{12000-s}\, ds = \int \frac{3}{200}\, dt$$

Solution of IVP

$$y' = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1$$

Sol:

$$x = 0, \quad y = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}}\, dx$$

Taking integration on b/s

$$\Rightarrow \int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\, dx$$ → Solving R.H.S by subtractin method

Let $y = 1+x^2$

$$\Rightarrow \frac{y^{-3+1}}{-3+1} =$$

$dy = 2x\,dx$

$$\frac{dy}{2} = x\,dx$$

$$\Rightarrow \frac{y^{-2}}{-2} = \int \frac{1}{\sqrt{y}} = \frac{dy}{2} = \int \frac{1}{2\sqrt{y}} = \sqrt{y} = \sqrt{1+x^2} + c$$

$$= \frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

(ii) SD equation.

$$\frac{dx}{dt} = \frac{t}{x}$$

Sol:

$$\frac{dx}{dt} = \frac{t}{x}$$

$$\Rightarrow \quad x\,dx = t\,dt$$

Taking integral on b/s:

$$\int x\,dx = \int t\,dt$$

$$\frac{x^2}{2} = \frac{t^2}{2} + c$$

$$\Rightarrow \quad x^2 = t^2 + 2c$$

$$\boxed{x^2 = t^2 + c \quad \text{or} \quad x = \sqrt{t^2+c}}$$

Q_2 (a) 1^{st} step is to convert D.E into its standard form. i.e $Y' + P(x)y = Q(x)$.

(b) then we will find the integrating factor by $e^{\int P(x)\,dx}$.

(c) Multiply b/s with the integrating factor.

(d) Simplify the relation.

(e) take integral on both sides.

(f) put the value of x and y.

(g) find c.

(h) put value of 'c' in seperable equation.

(i) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin x - 1$

Sol:

divide by $\cos(x)$ on b/s we get

$$\Rightarrow y' + \frac{\sin x}{\cos x}y = 2\frac{\cos^3 x}{\cos x}\sin x - \frac{1}{\cos x}$$

$$\Rightarrow y' + \tan(x)y = 2\cos^2 x \sin x - \sec x.$$

$$\Rightarrow \frac{dy}{dx} + \tan(x)y = 2\cos^2 x \sin x - \sec x. \rightarrow (a)$$

Now finding integrating factor.

$$e^{\int \tan x\,dx}.$$

$$\Rightarrow e^{\ln \sec x + c}$$

Now multiply I.F on b/s of eq (a) (4)

$\Rightarrow \sec x \frac{dy}{dx} + \sec(x)\tan(x)\, y = 2\cos^2 x \sin x \sec x - \sec^2 x$

Above eq can be written as

$\Rightarrow \frac{d}{dx}(\sec(x)\, y) = 2\cos x \sin x - \sec^2 x$

Now taking integral

$\Rightarrow \int \frac{d}{dx}(\sec(x)\, y) = \int 2\cos x \sin x\, dx - \int \sec^2 x\, dx$

$\Rightarrow \sec(x)\, y = \cancel{2}\left(\frac{1}{\cancel{2}}\sin^2 x\right) - \tan x + C$

$\Rightarrow \sec(x)\, y = \sin^2 x - \tan x + C$

$\Rightarrow y = \frac{\sin^2 x}{\sec x} - \frac{\tan x}{\sec x} + C$

$\Rightarrow y = \sin^2 x \cos x - \sin x + C$ — (b)

Now $y = 3\sqrt{2}$ and $x = \frac{\pi}{4}$ so

$\Rightarrow 3/\sqrt{2} = \sin^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - \sin \pi/4 + C$

$\Rightarrow 3/\sqrt{2} = \left(\frac{1}{\sqrt{2}}\right)^2 \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= 3/\cancel{\sqrt{2}} = \frac{1}{2}\frac{1}{\cancel{\sqrt{2}}} - \frac{1}{\cancel{\sqrt{2}}} + C$

$\Rightarrow \quad 3 = -\frac{1}{2} + c$

$= \quad 3 + \frac{1}{2} = c$

$\Rightarrow \quad \frac{7}{2} = C$

Now eq (b) becomes.

$$\boxed{\Rightarrow \quad y = \sin^2 x \cos x - \sin x + 7/2} \quad \text{Ans}$$

(ii) $\quad x' + 2x = \sin t$

Sol:

$\frac{dx}{dt} + 2x = \sin t$ —— (a)

Find I.F

$e^{\int 2dx} = e^{2x} + c$

multiply eq(a) by I.F on b/s.

$e^{2x} \cdot \frac{dx}{dt} + e^{2x} \cdot 2x = e^{2x} \cdot \sin t$

$\frac{d}{dt}\left(e^{2x} \cdot x\right) = e^{2x} \cdot \sin t.$

Taking integral on b/s

$\int \frac{d}{dt}\left(e^{2x} \cdot x\right) = e^{2x} \cdot \sin t$

$\Rightarrow \cancel{e} \cdot x = \cancel{e^{2x}} \sin t$

$$\boxed{\Rightarrow x = \sin t}$$

Answer

Q(3)(i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, y(0) = -3$

Sol:

In this equation we ~~have~~ can take $M = (2xy - 9x^2)$ & $N = 2y + x^2 + 1$.

Lets take a function ψ and integrate N w.r.t. to y.

$\psi = \int N(x, y)\, dy$.

$= \int (2y + x^2 + 1)\, dy$.

$= \cancel{\int (y^2 + x^2 y + y)}$ $h(x)$

$= (2y + x^2 y + y) + c$

$\psi - c = 2y + x^2 y + y$

$\Rightarrow C = 2y + x^2 y + y \rightarrow$ (a) $\boxed{-9 = 2y + x^2 y + y}$ Ans

Now put $x = 0$ and $y = -3$ we get

$\Rightarrow c = 2(-3) + (-3) \Rightarrow c = -9$ Now put this in

(ii) $\frac{2+y}{t^2+1} - 2t - \left((2 - \ln(t^2+1)\right)y' = 0$, $y(5)=0$ ⑦

Sol: $M =$ $N =$

Taking derivation of N w.r.t to y

$\int (2 - \ln(t^2+1)dy$

$\Rightarrow \int 2\,dy - \int \ln(t^2+1)\,dy$.

$=$ $2y -$ solving by substitution we get

$- \quad 2y = \ln(1+t^2) - 2(t - \text{cotan}(t)) + c$ (a)

Now put $y = 0$ and $t = 5$

$\Rightarrow \quad 0 = \ln(1+(5)^2) - 2(5 - \text{cotan}(5)) + c$

$C = \ln(1+(5)^2) - 2(5 - \text{cotan}(5))$

put value of c in eq (a).