Name:- Afaq Ahmad Shah
Department:- Software Engineering 2nd Semester
Section:- A
ID# 15715
Course:- Linear Algebra

Question No.1

Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system. where ID3 is the 3rd digit in your ID and ID-last is the last digit of your ID in inverse e.g. if your ID is 12345 then -ID_Last = -5.

Given

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\text{-}Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Solution

$ID3 = 7$, $ID\text{-}Last = 5$

$$\sim \left[\begin{array}{cccc|c} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right] \quad -3R_3+R_1$$

As we ~~[illegible]~~ ~~[illegible]~~ can see, the matrix is already in the Echelon form

$$\sim \left[\begin{array}{cccc|c} 1 & 7 & 0 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right] \quad 5R_3+R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 7 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right] \quad -7R_2+R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$$

$\therefore$ $1x_1 + 0x_2 + 0x_3 + 0x_4 = 5$

$0x_1 + 1x_2 + 0x_3 + 0x_4 = 7$

$0x_1 + 0x_2 + 1x_3 + 0x_4 = -6$

$0x_1 + 0x_2 + 0x_3 + 1x_4 = 7$

$\Rightarrow$ $\boxed{\begin{array}{l} x_1 = 5 \\ x_2 = 7 \\ x_3 = -6 \\ x_4 = 7 \end{array}}$

Question. 2
Part (a):
Given

Find the elementry row operation that transforms the first matrix into second and reverse row operation that transform the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution

Let

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \because R_3 - 2R_2$$

Let

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \because R_3 + 2R_2$$

Question. 2

Part (b):

Given below are some materices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a. Given

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

Solution

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

In not in echelon form, because a matrix to be in echelon form should contain the entries either in decimal or in fractions.

Question.2
Part (b)

b. Given

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is echelon form.

Solution

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

These are echelon form.

A matrix (A) is said to be in echelon form if;

i). The first non-zero element in each row, is called its leading entry, is 1.

ii). In any, two successive rows i and $i+1$ that do not consist entirely of zeros the leading element in the $(i+1)^{th}$ row lies to right of the leading element in i^{th} row.

iii). Any rows consist entirely of zeros lie at the bottom of the matrix.

Question-2
Part (b)

c. Given

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

It is in echelon form because it satisfies the 4rth condition that is in a column that contain the leading entry of row all the other element are zero.

d. Given

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Is in echelon form because it satisfies the 4rth condition that is in a column that contains the leading entry of row all the other elements are zero.

Question. 3
Part (a).
Given
The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.
Solution
Difference between Row Echelon form and reduce row Echelon form:-
The matrix in row echelon form meets the following requirement:
i. The first non-zero number form the left is always to the right of the first non-zero number in the row above.
ii. Rows consisting of all zero are at the bottom of the matrix.
For Example:-

$$\begin{bmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \end{bmatrix}$$

But on the other hand reduced row echelon form meets different required.
i. It is in row echelon form.
ii. The leading entry in each row is called a leading 1.
iii. Each column containing a leading 1 has zero in all its other.

For Example:-

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Practical use of Reduced Row Echelon form:-

Reduce row echelon form is a type of matrix used to solve system of linear equation. It has fair main required which we can talk before.

It is used to solve system of linear echelon form or reduced it to row echelon form, using determinants, row echelon form, using determinants, and so. be very inefficiants and on easy way to make mistakes.

Question. 3
Part (b)

Given
Find an echelon form for the below matrix using row operations. Where ID2 is 2nd digit in your ID e.g. if your ID is 12345 ID2 = 2, ID3 = 3, ID_First_Last is the first and last digit of your ID i.e. 15.

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\text{-}First\text{-}Last \end{bmatrix}$$

Solution

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix}$$

R.W

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ -7 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix} \quad -2R_1 + R_2$$

$-2(1)+2=0$
$-2(5)+8=-2$
$-2(8)-1=-17$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 1 & -4 & 15 \end{bmatrix} \quad 7R_1 + R_3$$

$7(1)+(-7)=0$
$7(5)+(0)=35$
$7(8)+(0)=56$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 0 & -9 & 7 \end{bmatrix} -1R_1+R_4$$

R.W

$-1(1)+1=0$

$-1(5)+(-4)=-9$

$-1(8)+(15)=7$

xing R_2 by $-1/2$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 35 & 56 \\ 0 & -9 & 7 \end{bmatrix}$$

$0 \times -1/2 = 0$

$-\cancel{2} \times -1/\cancel{2} = 1$

$-17 \times -1/2 = \frac{17}{2}$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & -483/2 \\ 0 & -9 & 7 \end{bmatrix} -35R_2+R_3$$

$-35(0)+0=0$

$-35(1)+35=0$

$-35(17/2)+56 = \frac{-595+112}{2}$

$= \frac{-483}{2}$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & -483/2 \\ 0 & 0 & 167/2 \end{bmatrix} 9R_2+R_4$$

$9(0)+0=0$

$9(1)+(-9)=0$

$9(17/2)+(7) = 167/2$

xing R_3 by $-2/483$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 167/2 \end{bmatrix}$$

$0 \times -2/483 = 0$

$0 \times -2/483 = 0$

$\cancel{-}\frac{\cancel{483}}{\cancel{2}} \times \frac{\cancel{-2}}{\cancel{483}} = 1$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} -\frac{167}{2} \cdot R_3+R_4$$

$-\frac{167}{2}(0)+0=0$

$-167/2\,(0)+0=0$

$-\frac{167}{2}(1)+\frac{167}{2}=0$

Result