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Question 1 (i)

Orthogonal matrix:

An orthogonal matrix is a square matrix in which all of the vector that make up the matrix are orthonormal to each other. This must hold in terms of geometry, orthogonal mean that two vector are Perpendicular to each other.

Example:

Also its eigenvector would be orthogonal Eg Prove $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ orthogonal matrix.

Question 1 (ii)

Vector Space:

Let $(F, +)$ be a given field and V be a non empty set with two Composition (an V. one is Internal called addition and denoted by +, + and other is External called Scalar matrix Then set V is called vector sp...

If =0 v is abelion group under addition $(v, +)$

Example:

The callection $\{i, j\}$ is a basis for R^2 Since it Span R^2 and vector i and j are linearly independent. This is called stadord basis R^2.

$\{ e = (1, 0, 0 \cdots 0) = (0, 1, 0, 0\} \{0, 0, 1, 0\}$

Question 1 (iii)

Span set of vector:

In linear algebra the linear Span are also called the linear hull or just Span) of a set S of vector in a vector space is the smallest linear Subspace that contain the set. It can be characterized either as the intersection of all linear subspace that contain S, or as the set of linear combination of element of S. The linear Space of a set of vector is therefore a vector space. Span can be generalized matroids and modules.

For expressingin that a vector space V is a span of a set S, one commaunly uses the following phrases: S spans V; S generates V; V is spanned by S; V is generated by S; S is a spanning set of V; S is a generating set of V.

$$\operatorname{span}(S) = \left\{ \sum_{i=1}^{k} \lambda_i v_i \quad k \in N, \; v_i \in S, \lambda \right.$$

Question 1 (iv)

Eigenvector:

An eigenvector is a vector whose direction remains unchanged when a linear transformation is applied to it. Consider the image below in which three vector are shown. This unique determine relation is exactly the reason that those vector are called eigenvector

Question1(v)

Dimension of vector space:

In mathematics, the dimension of a vector space V is the cardinality of a basis of V over its base field. It is sometimes called Hamel dimension or algebraic dimension to distinguish it form other type of dimension.

The vector space R^3

Example: $\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$

Question1(vi)

Subspace of vector space:

If V is a vector space over a field K and if W is a subset of V, then W is a subspace of V if under the operation of V, W is a vector space over K. Equivalently, a nonempty subset W is a subspace of V if, whenever w_1, w_2 are element of W and α, β are element of K, it follow $\alpha w_1 + \beta w_2$ is in W.

Example: $V_1 V_2 \geq 0$ The $U+V = W$

Question 1(vii)

Kernel of a linear transformation:

The Kernel related to 1-1 linear transformation is the idea of the kernel of a linear transformation

The kernel of a linear transformation L is the set of all vector V such that $L(V) = 0$

$$L(H) = L(V)$$

implies $H = V$

Example: $L((x, y)) = xt^2 + yt$

Question 1(viii)

Nullity of linear transformation:

The nullity of T is the dimension of its kernel while the rank of T is the dimenision of its images. These are denoted nullity (T) and rank (T), respectively. Given coordinate system for V and W, so that every linear transformation (T) can be described by a matrix A so that $T(x) = Ax$ ... $\dim(V) = \text{rank}(T) + \text{nullity}(T)$.

Quesion1(ix)

image of linear transformation.

The images of a linear transformation or matrix is the span of the vector of the linear transformation. Think of it as what vector you can get from appling the linear transformation or multipliny the matrix by a vector. It can be written as Im(A)

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

Question1(x)

Rank of linear transformation:

The rank of a linear transformation L is the dimension of its image, written rankL. The nullity of a linear transformation is the dimension of the kernel, writtenL. Theorem (Dimension formula). Let $L (V \to W)$ be a linear transformation, with V a finite - dimension vector space 2.

Question1 (xi)

Characteristic Polynomial of square matrix

In linear algebra the characteristic Polynomial of a square matrix is a Polynomial which is invariant under matrix similarity and has the eigenvalues as roots. It has the determine and the trace of the square matrix as coefficient. The characteristic Polynomial of a graph is the characteristic Polynomial of its adjiancy matrix. It is a graph invariant thought It is not complete.

Question1 (xii)

Equivalence relation:

A relation b/w element of a set which is reflexive, symmetric, and transitive and which defines exclusive classes whose member bear the relation to each other and not to those in other classes.

Question 1 (xiii)

Homegenoues Solution to linear system of equation.

If we write a linear system as a matrix equation, letting A be the coefficient matrix, x the variable vector, and b the known vector of constants then the equation $Ax = b$ is said to be homogenous if b is zero vector.

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \begin{aligned} 3x+2y&=0 \\ x-5y+z&=0 \end{aligned}$$

Question 1 (xiv)

Particular saluation of linear system of equation:

General Solution to a Nonhomogenous linear equation A Solution $y_p(x)$ of a differential equation that contain no arbitrary constant is called a Particular Solution to the equation.

Question 1 (xv)

General Solution to a linear System of equation.

A general Solution of a System of linear equation is a formula which gives all Solution for different Values of Parameter.

Example:

Consider the System

$$x + y = 72x + 4y = 18$$

Question 1 (xvi)

Direct sum of Pair of subspace of vector space.

The direct sum of two vector space V_1, V_2 is a vector space $V_1 \times V_2$ with operation defined as you wrote. The connection between a direct and ordinary sum is that if $V_1, V_2 \subset V$ and V_1 and V_2 are linearly independent then $V_1 + V_2$ is isomorphic to $V_1 + V_2$

Question 1 (xvii)

Orthogonal complement to a subspace of vector space.

In the mathematical fields of linear algebra and functional analysis, the orthogonal complement of a subspace W of a vector space V equipped with a bilinear form B is the set $W^{\perp}$ of all vector in V that are orthogonal to every vector in W. It is a subspace of V.

Example

In the case that W is the subspace of $V = R^2$

$$\begin{pmatrix} 1 & 0 & \underline{2} & 3 & 5 \\ 0 & 1 & \underline{6} & \underline{4} & \underline{3} \end{pmatrix}$$

its orthogonal complement $W^{\perp}$ is spanned the three row vecter of

$$\left(\begin{array}{c|c|ccc} -2 & -6 & 1 & 0 & 0 \\ -3 & -9 & 0 & 1 & 0 \\ -5 & -3 & 0 & 0 & 1 \end{array}\right)$$

Question # 2

Consider the system of equation

$$x + y + z + w = 1$$
$$x + 2y + 2z + 2w = 1$$
$$x + 2y + 3z + 3w = 1$$

find solution decomposition for matrix $MX = V$

Sol:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}$$

$$R_2 - R_1 \to R_2$$

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}$$

$R_3 - R_1 \to R_3$

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

$R_3 - R_1 \to R_3$

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

we know

$A = LU$

$Ax = B$ $\qquad Ux = Y$

$LUx = B$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\boxed{y_1 = 1}$

$-y_1 + y_2 = 1$

$\boxed{y_2 = 2}$

$-y_1 - y_2 + y_3 = 1$

$-1 - 2 + y_3 = 1$

$\boxed{y_3 = 4}$

$-y_1 - y_2 + y_3 + y_4 = 1$

Question : 3

Determiants:

i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= 4 - 6$$

$$\boxed{= -2}$$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$= 1\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} - 2\begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + 3\begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$$

$$= 1(45-48) - 2(36-42) + 3(32-35)$$

$$= -3 + 12 - 9$$

$$\boxed{= 0}$$

Determinant

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

Sol:

$R_2 - 5R_1$

$R_3 - 9R_1$

$R_4 - 13R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -20 & -24 \\ 0 & -12 & -24 & -46 \end{bmatrix}$$

exPan

$$= \begin{bmatrix} -4 & -8 & -12 \\ -8 & -20 & -24 \\ -12 & -24 & -46 \end{bmatrix}$$

$$= \begin{vmatrix} -4 & -8 & -12 \\ -8 & -20 & -24 \\ -12 & -24 & -46 \end{vmatrix}$$

$$= -4\begin{bmatrix} -20 & -24 \\ -24 & -46 \end{bmatrix} + 8\begin{bmatrix} -8 & -24 \\ -12 & -46 \end{bmatrix} - 12\begin{bmatrix} -8 & -20 \\ -12 & -24 \end{bmatrix}$$

$$= -4(920 - 576) + 8(368 - 288) - 12(192 - 240)$$

$$= -4(344) + 8(80) - 12(-48)$$

$$= -1376 + 640 + 576$$

$$= -1376 + 1216$$

$$\boxed{= -160}$$

Determinant

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$$

Sol
=

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$$

$R_2 - 6R_1$

$R_3 - 11R_1$

$R_4 - 16R_1$

$R_5 - 21R_1$

$$|A| = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -25 & -20 \\ 0 & -10 & -20 & -38 & -40 \\ 0 & -15 & -30 & -45 & -60 \\ 0 & -20 & -40 & -60 & -80 \end{bmatrix}$$

$$|A| = \begin{bmatrix} -5 & -10 & -25 & -20 \\ -10 & -20 & -38 & -40 \\ -15 & -30 & -45 & -60 \\ -20 & -40 & -60 & -80 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 3R_1$

$R_4 - 4R_1$

$$|A| = \begin{bmatrix} -5 & -10 & -25 & -20 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 40 & 0 \end{bmatrix}$$

$$|A| = \quad 0$$

This is undefined Detetemiant becaule c_1, c_2, c_3 is zero.

Determinant

$$\begin{bmatrix} 1 & 2 & 3 & n \\ n+1 & n+2 & n+3 & 2n \\ 2n+1 & 2n+2 & 2n+3 & 3n \\ n^2-n+1 & n^2-n+2 & n^2-n+3 & n^2 \end{bmatrix}$$

Sol:

$$A = \begin{bmatrix} 1 & 2 & 3 & n \\ n+1 & n+2 & n+3 & 2n \\ 2n+1 & 2n+2 & 2n+3 & 3n \\ n^2-n+1 & n^2-n+2 & n^2-n+3 & n^2 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 3R_1$

$R_4 - (R_1)^2$

$$|A| = \begin{vmatrix} 1 & 2 & 3 & n \\ n-1 & n-2 & n-3 & 0 \\ 2n-2 & 2n-4 & 2n-6 & 0 \\ n^2-n & n^2-n-2 & n^2-n-6 & 0 \end{vmatrix}$$

$$|A| = \begin{vmatrix} n-1 & n-2 & n-3 \\ 2n-2 & 2n-4 & 2n-6 \\ n^2-n & n^2-n-2 & n^2-n-6 \end{vmatrix}$$

$= (n-1)\left[(2n-4)(n^2-n-6) - (n^2-n-2)(2n-6)\right]$

$\cancel{(2n-2)} - (n-2)\left[(2n-2)(n^2-n-6) - (n^2-n)(2n-6)\right]$

$+(n-3)\left[(2n-2)(n^2-n-2) - (n^2-n)(2n-4)\right]$

$= n-1\left[2n^3-6n^2-8n+24\right] - \left[2n^3-8n^2+2n+12\right]$

$= -n-2(2n^3-4n^2-10n+12) - (2n^3-8n^2+6n)$

$+n-3(2n^3-4n^2-2n+4) - (2n^3-6n^2+4n)$

$= n-1(\cancel{2n^3}-6n^2-8n+24-\cancel{2n^3}+8n^2-2n-12)$

$-(n-2)(\cancel{2n^3}-4n^2-10n+12-\cancel{2n^3}+8n^2-6n)$

$+(n-3)(\cancel{2n^3}-4n^2-2n+4-\cancel{2n^3}+6n^2-4n)$

$= (n-1)(2n^2-10n+12) - (n-2)(4n^2-16n+12)$

$+(n-3)(2n^2-6n+4)$

$= n^3-11n^2+\cancel{22n}-\cancel{12}-4n^3+24n^2-\cancel{44n}+\cancel{24}$

$+2n^3-12n^2+\cancel{22n}-\cancel{12}$

$\boxed{=-n^3+n^2}$ Ans