Q1 part i(a)

Ans) A differential equation is an equation which contains one or more items which involve the derivatives of one or more variables (i-e) dependent variable

$$\frac{dy}{dx} = f(x)$$

Here "x" is an independent variable and "y" is a dependent variable

e.g

$$\frac{dy}{dx} = 5x$$

$$\frac{dy}{dx} = 3x + 2$$

Q1 part(i)(b)

Ans) A seperable differential equation is any differential equation that we can write in the following form

$$N(y)\frac{dy}{dx} = M(x)$$

To solve this differential equation we first integrate both sides with respect to x to get

$$\int N(y)\frac{dy}{dx}\,dx = \int M(x).\,dx$$

$$u = y(x) \quad du = y'(x)dx = \frac{dy}{dx}dx$$

we get

$$\int N(u)\,du = \int M(x)\,dx$$

Subject:

Q1 part(a)

Date:

$$y' = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1$$

$$y' = \frac{x^4 y^3}{\sqrt{1+x^2}}, \quad y(0) = -1 : y = -\sqrt{\frac{2\sqrt{x^2+1}+3}{4x^2-5}}$$

$$y' = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3} y' = \frac{x}{\sqrt{1+x^2}}$$

$$N(y) \cdot y' = M(x)$$

$$N(y) = \frac{1}{y^3}, \quad M(x) = \frac{x}{\sqrt{1+x^2}}$$

Solve $\frac{1}{y^3} y' = \frac{x}{\sqrt{1+x^2}}$: $-\frac{1}{2y^2} = \sqrt{1+x^2} + c_1$

Apply initial conditions $-\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

Isolate y = $y = \sqrt{\frac{-2\sqrt{x^2+1}+3}{4x^2-5}}$, $y = -\sqrt{\frac{2\sqrt{x^2+1}+3}{4x^2-5}}$

Check solution by applying conditions

$$y = -\sqrt{\frac{-2\sqrt{x^2+1}+3}{4x^2-5}}$$

Q1 part ii (b)

$y' = e^{-y}(2x-4), \quad y(5)=0$

Solution

Multipying bothsides by e^y

$e^y \times \frac{dy}{dx} = \cancel{e^{-y}} \times \cancel{e^y} (2x-4)$

$e^y dy = (2x-4)dx$

Integrating

$\int e^y dy = \int 2x-4 \, dx$

$\varphi y = \ln(t^2+1) + f'(y)$

$\varphi y = \ln(t^2+1) - \cancel{?} = N$

$f(y) = -2, \quad f(y) = -2y$

$f \ln(t^2+1) - t^2 \, 2y = c$ — ①

initial condition of

$y(5)=6$ in ①

$\boxed{Q1}$ part ii (b)

$\therefore 0 \ln(5^2+1) - 5^2 - 2(0) = e$

$0 - 25 - 0 = c$

$$\boxed{e = -25}$$

Q2 part (i)

Ans) A first order differential equation is linear when it can be made to look like this

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x)$ and $Q(x)$ are function of x.

$$\frac{dy}{dx} = u\, dv/dx + v\, du/dx$$

1) Substitute $y = uv$; and $dy/dx = u\,dv/dx + v\,du/dx$

into

$dy/dx + P(x)y = Q(x)$ ..

Date:

Q2 part (iii)

$x' + 2x + \sin(t) = x' + 2x + \sin(t)$

$x' + 2x + \sin(t)$

couldn't simplfy further to.

$= x' + 2 + \sin(t)$

Subject: $\boxed{Q3}$ part (i)

Date:

$2xy - 9x^2 + (2y+x^2+1)\frac{dy}{dx} = 0$, $y(0) = -3$:

$y = -1 - x^2 - \frac{\sqrt{x^4+12x^3+2x^2+25}}{2}$

$2xy - 9x^2 + (2y+x^2+1)\frac{dy}{dx} = 0$

Extract differential Equation.

Substitute $\frac{dy}{dx}$ with y'

$2xy - 9x^2 + (2y+x^2+1)y' = 0$

The equation is in exact from $M(x,y) + N(x,y) \cdot y' = 0$

$\psi_x(x,y) = M(x,y) = 2xy - 9x^2$,
$\psi_y(x,y) = N(x,y) = 2y + x^2 + 1$

If the conditions are met, then $\psi_x \cdot \psi_y \cdot y' = \frac{d\psi(x,y)}{dx} = 0$

The general solution is $\psi(x,y) = C$

Verify that $\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$: True

Find $\psi(x,y)$: $\psi(x,y) = y + x^2 + y + y^2 - 3x^3 + C$

$\psi(x,y) = C_2$

Subject:

Date:

$\boxed{Q3}$ part(i) continued.

$y + x^2 y + y^2 - 3x^3 + C_1 = C_2$

combine the constants

$y + x^2 y + y^2 - 3x^3 = C_1$

Apply initial conditions : $y + x^2 y + y^2 - 3x^3 = 6$

Isolate y: $y = \dfrac{-1 - x^2 + \sqrt{x^4 + 12x^3 + 2x^2 + 25}}{2}$,

$y = \dfrac{-1 - x^2 - \sqrt{x^4 + 12x^3 + 2x^2 + 25}}{2}$

$y = \dfrac{-1 - x^2 - \sqrt{x^4 + 12x^3 + 2x^2 + 25}}{2}$

~~differential equations~~ ~~with~~

$\boxed{Q3}$ part (ii)

$((2ty/t^2+1)-2t)dt-2(2-\ln(t^2+1))dy=0$
$; y(5)=0$

$F(t,y)=(2+\ln(t^2+1))y-t^2$

I guess you know where $F(t,y)$ is coming from
$F(t,y)=2ty/t^2+1)-2t$
$F_y(t,y)=(2+\ln(t2+1))$
So general solution is $F(t,y)=C$

$F(5,0)=-25=C$
$(2+\ln(t^2+1))y-t^2+25=0$
~~youareless~~