"FINAL TERM PAPER"

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SECTION:

B

SEMESTER:

"4th"

SUBJECT:

DIFFENTIAL EQUATION

INSTRUCTOR:

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QUESTION NO : 01   PART (A)

i) $w = \sin(x+ct) + \cos(2x + 2ct)$

Solution:

$$\frac{\partial w}{\partial t} = \cdot \cos(x+ct) + (-\sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \text{---} 1$$

$$\frac{\partial w}{\partial x} = \cdot \cos(x+ct) - \sin(2x + 2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = - \sin(x+ct) - 4\cos(2x+2ct)$$

$$= [- \sin(x+ct) - 4 \cdot \cos(x+ct)]$$

$$\frac{\partial^2 w}{\partial t^2} = + c^2 [- \sin(x+ct) - 4 (\cos(2x+2ct)]$$

" Hence it is the solution of the wave equation."

$$c^2 + \frac{\partial^2 w}{\partial x^2}$$

Q: No: 01 Part (b) (ii)

$$w = \tan(ax + ct)$$

Solution:

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial t}\left(\tan(ax+c)\right)$$

$$= \sec^2(ax+ct)\cdot(c)$$

$$= c\sec^2(ax+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c\cdot\frac{\partial}{\partial t}\left(\sec^2(ax+c)\right)$$

$$= c\frac{\partial}{\partial t}\cos(ax+ct)^{-2}$$

$$= c\frac{\partial}{\partial t} -2\left(\cos(ax+ct)^{-3}(-\sin(ax+ct)\right)$$

$$= 2c^2\sec^2(ax+ct)\sin(ax+ct)$$

Now with respect to x.

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\tan(ax+ct)\right]$$

$= \sec^2(ax+ct)(a)$

$= \dfrac{\partial w}{\partial x} = a \sec^2(ax+ct)$

$\Rightarrow \dfrac{\partial^2 w}{\partial x^2} = a \dfrac{\partial}{\partial x}\left[\sec^2(ax+ct)\right]$

$= a \dfrac{\partial}{\partial x}\left[(\cos(ax+ct))^{2}\right]$

$= a(-a)\left[\cos(ax+ct)\right]^{-3}$

$\left[-\sin(ax+ct)\right](a)$

$= 6 \sec^2(x+ct) \tan(ax+ct)$

So;

$\cancel{x}' c^2 \sec^2(ax+ct) \sin(ax+ct)$

$= c^2\left[(\cancel{1}^3 \sec^2\ ax+ct) \sin(ax+ct)\right]$

$\left[\sec^2(ax+ct)(\sin(ax+ct))\right] = \left[(3\sec^2 ax+ct)\right.$

$\left.(\sin ax+ct)\right]$

" Hence it is not the solution of the wave equation."

Q:No: 03

Expand the following function in a Fourier series.

$$F(x) = x, \quad -\pi < x \leq 0$$

$$= 2x, \quad 0 \leq x \leq \pi$$

"We have to find the fourier co-efficient, a_0, a_n & b_n."

Solution:

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx.$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \pi/2} \rightarrow (i)$$

$$a_n = \frac{1}{\pi}\int_{\pi}^{\pi} F(x) \cos x \, dx.$$

$$= \frac{1}{\pi}\int_{\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x \cos nx) dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2[-(1)^n] - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So;

$$\boxed{a_n = \begin{cases} \dfrac{-2}{\pi n^2} & ; \text{ if } n \text{ is added} \\ 0 & ; \text{ if } n \text{ is even} \end{cases}} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\,\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\left[\frac{\pi\cos n\pi}{n}\right]\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

"So the required Fourier Series"

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$$

Q: No: 03

Solve the initial value problem;

$y'' - 4y' + 13y = 8\sin 3x, \quad y(0) = 1 \ \& \ y'(0) = 2$

Solution :.

$\rightarrow \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 8\sin 3x$

$= D^2y - 4Dy + 13y = 8\sin 3x$

$= (D^2 - 4D + 13)y = 8\sin 3x \rightarrow (i)$

Let $D = \dfrac{d}{dx}$ & $D^2 = \dfrac{d^2}{dx^2}$

Now general Solution of (i) is

$y = y_c + y_p \rightarrow (A)$

For y_c, characteristics eq is

$D^2 - 4D + 13 = 0$

$= D^2 - 4D + 4 + 9 = 0$

$= (D-2)^2 - 9^2 = 0$

$= (D-2)^2 - 9i^2 = 0$

$= (D-2)^2 - (3i)^2 = 0$

$= (D-2-3i)(D-2+3i) = 0$

$= D-2-3i = 0$ & $D-2+3i = 0$

$= D = 2+3i$ & $D = 2-3i$

is $D = 2 \pm 3i$

Which are imaginary roots.

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

For P.I we have

$y_p = \frac{8 \sin 3x}{D^2 - 4D + 13} = 8 \cdot \frac{\text{Im part of } e^{i3x}}{D^2 - 4D + 13}$

$$y_p = \frac{8 \cdot \text{Im part of } e^{i3x}}{(i3)^2 - 4(i3) + 13}$$

$$= 8 \cdot \frac{\text{Im part of } e^{i3x}}{-9 - 12i + 13}$$

$$= \frac{8 \text{ Im part of } e^{i3x}}{4 - 12i}$$

$$= \frac{\cancel{8}\,2 \text{ Impart of } e^{i3x}}{\cancel{4}(1 - 3i)}$$

$$= \frac{2}{(1-3i)} \cdot \frac{(1+3i)}{(1+3i)} \text{ Impart of } e^{i3x}$$

$$y_p = \frac{2 + 6i}{10} \text{ Impart of } e^{i3x}$$

$$= \frac{\cancel{2}^{1}(1 + 3i)}{\cancel{10}5} \text{ Impart } ((\cos 3x + i \sin 3x))$$

$$y_p = \frac{(3 \cos 3x + \sin 3x)}{5}$$

A $\Rightarrow$ $y = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) +$

$\frac{1}{5}(3\cos 3x + \sin 3x) \rightarrow$ 2

$y' = 2e^{2x}(C_1 \cos 3x + C_2 \sin 3x) + e^{2x}$

$(-3C_1 \sin 3x + 3C_2 \cos 3x) + 1/5(-9$

$\sin 3x + 3\cos 3x) \rightarrow$ 3

For $y(0) = 1$

2 $\Rightarrow$ $y = e^0\left[(C_1 \cos 0 + C_2 \sin 0) + 1/5(3\cos 0 + \sin 0)\right]$

$1 = C_1 + 3/5$

$\boxed{\Rightarrow C_1 = 1 - 3/5 = 2/5}$

$y'(0) = 2$

3 $\Rightarrow 2 = 2e^0[(C_1 \cos 0 + C_2 \sin 0) +$

$e^0(-3C_1 \sin 0 + 3C_2 \cos 0) +$

$(\frac{1}{5}(-9 \sin 0 + 3 \cos(0))]$

$\Rightarrow 2 = 2[(C_1 + 0) + (0 + 3C_2) +$

$1/5 (0 + 3))]$

$\Rightarrow 2 = 2C_1 + 3C_2 + 3/5$

$\Rightarrow 3_2 C = 2 - 3/5 - 2 \cdot 2/5$ | as $C_1 = 2/5$

$= C_2 = 1/3 \cdot 3/5 = 1/5 \Rightarrow C_2 = 1/5$

(2) $\Rightarrow y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{1}{5}\sin 3x\right) + \frac{1}{5}\left(3\cos 3x + \sin 3x\right)$

Ans

Q: No: 04

Solve:

$$(D^2 - DD')z = \cos x \cdot \cos 2y$$

Solution:

The can also be written as;

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = E \cos x \cdot \cos 2y$$

Now as we have;

Corresponding A.E is $m^2 - m = 0$

when $D/D' = m$ so $m = 1$, $m = 1$

$$C.F = \phi_1(y) + \phi_2(y + x)$$

Now;

$$D.I = \frac{1}{(D^2 - DD')} \cos x \cos 2y.$$

$$= \frac{1}{2} \frac{1}{D^2 - DD'} \Big[\underset{I}{\cos(x+2y)} + \underset{II}{\cos(x-2y)} \Big]$$

$$P.I = \frac{1}{2} \left[\frac{1}{-1+2} \cos(x+2y) + \frac{1}{-1-2} (\cos(x-2y)) \right]$$

(when $\cos(ax + by)$ repla D^2 by $-a^2$, D'^2
by $-b^2$ and $D.D' = -ab$)

$$P.I \quad \tfrac{1}{2} \left[\cos(x+2y) - \tfrac{1}{3} \cos(x-2y) \right]$$

The Complete Solution

$$z = \phi_1(y) + \phi_2(y+x) + \tfrac{1}{2} \cos(x+2y) - \frac{1}{6} \cos(x-2y)$$

RESULT:

$$z = \phi_1(y) + \phi_2(y+x) + \tfrac{1}{2} \cos(x+2y)$$
$$- \tfrac{1}{6} \cos(x-2y).$$