Name:

Hassan Zaib Khattak

ID:

7958

Section

B

Deptt:

BE (C)

Subject:

Differential

Submitted to:

Mam Shumail Mazhar

Iqra National University

Question No 01:

i) The order of matrix A is m×p and order of B is p×n Then order of matrix AB is;

The order of matrix AB

$$\boxed{AB = m \times n}$$

ii) The Nos of Non-zero rows in an Echelon form;

The Nos of non-zero rows in an Echelon form is rank of matrix.

iii) If matrix $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is Singular matrix then $a = ?$

$$B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$$

Solution:

$$B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$= \left|(1 \times a) - (4 \times 8)\right| = a - 8 = 0$$

$$\boxed{a = 8}$$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

Solution:

$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$|A| = \{(2i \times (-i)) - (i \times i)\}$

$= \{-2i^2 - i^2\}$ $\quad \because i^2 = -1$

$= \{-2(-1) - (-1)\}$

$= \{2 + 1\}$

$\boxed{|A| = 3}$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a Scalar matrix.

vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Solution:

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

Multiply dx by b.s

$$dy = y(1-2x)dx$$

Divide y by b.s

$$\frac{1}{y}dy = (1-2x)dx$$

Taking Integration

$$\int \frac{1}{y}dy = \int (1-2x)dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x-x^2+c}$$

$$\boxed{y = e^{x-x^2+c}}$$

vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$ is ?

The Order of differential equation is 1 while Degree of differential equation is 6.

viii) The order and degree of

$$\frac{d^2y}{dx^2} - 4xy = \sin\frac{d^2y}{dx^2}$$ is ?

The order of differential equation is 2 while the degree of differential equation does not exist.

ix) The differential equation

$$2\frac{dy}{dx} + x^2 y = 2x+3, \quad y(0) = 5 \text{ is?}$$

Solution:

$$2\frac{dy}{dx} + x^2 y = 2x+3, \quad y(0) = 5$$

$$2\frac{dy}{dx} = 2x + 3 - x^2 y$$

Multiply b.s by dx

$$2dy = (2x+3-x^2y)dx$$

Taking Integration

$$\int 2dy = \int (2x+3-x^2y)dx$$

$$2y = \frac{\cancel{2}x^2}{\cancel{2}} + 3x - \frac{x^3 y}{3} + c$$

$$2y + \frac{x^3 y}{3} = x^2 + 3x + c$$

$$y\left(2 + \frac{x^3}{3}\right) = x^2 + 3x + c$$

$$\left(\frac{6+x^3}{3}\right)y = x^2+3x+c$$

$$y = (x^2+3x+c) \times \frac{3}{6+x^3} \text{ ——— } \star$$

Put $x=0$, $y=5$

$$5 = 0-0+\tfrac{1}{2}c$$

$$c = 10$$

Put $c=10$ in equ $\star$

$$y = (x^2+3x+10) \times \frac{3}{6+x^3}$$

$$\boxed{y = \frac{3x^2+9x+30}{6+x^3}}$$

X_1 $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Solution:

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1-1 & b-a & b^2-a^2 \\ 1-1 & c-a & c^2-a^2 \end{vmatrix} \begin{matrix} \\ R_2-R_1 \\ R_3-R_1 \end{matrix}$$

Expand by C_1

$$= 1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

Taking $(b-a)(c-a)$ from R_1 and R_2.

$$= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)\{(c+a)-(b+a)\}$$

$$= (b-a)(c-a)\{c+\cancel{a}-b-\cancel{a}\}$$

$$\boxed{= (b-a)(c-a)(c-b)}$$

Question No 02 (i):

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the products which are linear a, b, c.

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c \\ a^3 & b^3 & c \end{vmatrix}$$

Expand by R_1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a\{b^2c^3 - c^2b^3\} - b\{a^2c^3 - c^2a^3\} + c\{a^2b^3 - b^2a^3\}$$

$$= ab^2c^3 - ac^2b^3 - ba^2c^3 + bc^2a^3 + ca^2b^3 - cb^2a^3$$

Taking Common abc

$$= (abc)\left\{(bc^2 - cb^2) - ac^2 + ca^2 + ab^2 - a^2b\right\}$$

$$= abc\left\{ bc(c-b) - ac(c-a) + ab(b-a)\right\}$$

$$= (abc)\left\{bc(c-b) - ac(c-a) + ab(b-a)\right\}$$

$$\boxed{= (abc)\left\{bc(c-b) - ac(c-a) + ab(b-a)\right\}}$$

Hence answer of Required Question.

Question no 02(ii)

Find the Eigen Value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$= (2-\lambda)\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1)\begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$+ (-1)\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + 0 = 0 \text{ --- } (1A)$$

Now

$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left\{(3-\lambda)(2-\lambda) - (-1\times -1)\right\} + 1\left\{(-1\times(2-\lambda)) - (-1\times -1)\right\}$$

$$-1\left\{(-1\times -1) - (3-\lambda)\times(-1)\right\}$$

$$= (3-\lambda)(6-5\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(5-5\lambda+\lambda^2) + (-3+\lambda) - (4-\lambda)$$

$= 15 - 15\lambda + 3\lambda^2 - 5\lambda + 5\lambda^2 - \lambda^3 - 3 + \lambda - 4 + \lambda$

$= -\lambda^3 + 8\lambda^2 - 18\lambda + 8$ —— (1)

Now

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$= (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & 3-\lambda \\ 0 & -1 \end{vmatrix}$$

$$= (-1)\left\{\left((3-\lambda)\times(2-\lambda)\right) - (-1\times -1)\right\} + \left\{\left((-1)\times(2-\lambda)\right) - 0\right\}$$

$$- \left\{(-1\times -1) - 0)\right\}$$

$= (-1)(6 - 5\lambda + \lambda^2 - 1) + (-2 + \lambda) - 1$

$= (-1)(5 - 5\lambda + \lambda^2) - 2 + \lambda - 1$

$= -5 + 5\lambda - \lambda^2 - 3 + \lambda$

$= -\lambda^2 + 6\lambda - 8$ —— (2)

Now $\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$

Expand by C_1

$$= -1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1\left\{(-1\times(2-\lambda)) - (-1\times -1)\right\} + 1\left\{(3-\lambda)\times(2-\lambda) - (-1\times -1)\right\}$$

$$= -1(-2+\lambda-1) + (6-5\lambda+\lambda^2-1)$$

$$= -1(-3+\lambda) + 5 - 5\lambda + \lambda^2$$

$$= 3-\lambda+5-5\lambda+\lambda^2$$

$$= \lambda^2 - 6\lambda + 8 \quad \text{———} \; (3)$$

Now putting equation 1, 2 and 3 in equ (1A)

$$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8) + (-\lambda^2+6\lambda-8) - (\lambda^2-6\lambda+8) = 0$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8 - \lambda^2+6\lambda-8 = 0$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+18\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda + 16-8-8 = 0$$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By Synthetic division

	1	-10	32	-32
2		2	-16	32
	1	-8	+16	0

$(\lambda - 2)(\lambda^3 - 8x^2 + 16\lambda) = 0$

$\lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$

So $\lambda = 0$, $\lambda - 2 = 0$, $\lambda^2 - 8\lambda + 16 = 0$

$\lambda = 2$

$\Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\Rightarrow \lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$\Rightarrow (\lambda - 4)(\lambda - 4) = 0$

$\Rightarrow (\lambda - 4)^2 = 0$

$\Rightarrow \sqrt{(\lambda - 4)^{\cancel{2}}} = \sqrt{0}$

$\Rightarrow \lambda - 4 = 0$

$\boxed{\lambda = 4}$

Hence

$\boxed{\lambda_1 = 0, \lambda = 2, \lambda = 4 \text{ (multiplicity)}}$

Question No 03:

The rate of change in the form of differential equation is given by $(x^2+3y^2)dx - 2xydy=0$ find the general solution at $x=2$ and $y=6$

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy = 0$$

Interchanging the term by b.s

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{\cancel{x^2}}{2\cancel{x}y} + \frac{3\cancel{y^2}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right) \longrightarrow ☆$$

Scanned with CamScanner

Comparing equ ☆ with $\frac{dy}{dx} = g(y/x)$

Equ ☆ is homogenous equ of degree 1

Put $\boxed{y/x = v}$ or $\boxed{y = vx}$

differentiate w.r.t x

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

Put in equ ☆

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$\frac{2v}{1+v^2}dv = \frac{1}{x}dx$$

Taking Integration

$$\int \frac{2v}{1+v^2}dv = \int \frac{1}{x}dx$$

$$\ln|1+v^2| = \ln x + \ln c$$

$$\cancel{\ln}|1+v^2| = \cancel{\ln}\, xc \qquad \because \ln m + \ln n = \ln m \times n$$

$$1+v^2 = xc \longrightarrow **$$

Now reput $v = y/x$ in equ **

$$1 + (y/x)^2 = xc$$

$$1 + y^2/x^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \longrightarrow ***$$

Initial Value $x=2, y=6$

$$2^2+6^2 = 2^3 c$$

$$4+36 = 8c$$

$$\boxed{c = 5}$$

Put $\boxed{c=5}$ in equ ~~***~~

$$x^2+y^2 = x^3 \cdot 5$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking Under root on b.s

$$\sqrt{y^2} = \sqrt{x^2(5x-1)}$$

$$y = \pm x\sqrt{5x-1}$$

Result:

$$\boxed{\begin{aligned} y &= +x\sqrt{5x-1} \\ y &= -x\sqrt{5x-1} \end{aligned}}$$

Hence they are required Solution.