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Differential Equation

Q1

(a) Definition: A differential equation is an equation that relates one or more functions and their derivatives. ~~represent~~ in applications, the functions generally represent physical quantity. the derivatives represent their rate of change.

Exmple 1: Find the velocity for the following expression $S = 16t^2$

Sol:- $S = 16t^2$

for velocity we take different equation w.r.t t

$$\frac{ds}{dt} = 16t^2$$

$$v = 2(16t)$$

$$\boxed{v = 32t}$$

Exmple 2: $F(x) = 2x^2 + 4y$

derivate w.r.t dx

P.T.O

Sols- $f(x) = 2x^2 + 4y$

now derivative w.r.t dx

$$\frac{dy}{dx} = \frac{dy}{dx}(2x^2 + 4y)$$

$$\frac{dy}{dx} = 4y\left(\frac{d}{dx}2x^2\right)$$

$$\frac{dy}{dx} = 4y\left([4x]\right)$$

$$\frac{dy}{dx} = 4y(4x)$$

$$\boxed{\frac{dy}{dx} = 16xy}$$

(b) Separable Differential Equation:

If any differential equation that we can write in the following form

$$n(y)\frac{dy}{dx} = m(x)$$

Q.#1)

b) i

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \quad y(0) = -1$$

First Seperate and then Integrate both Sides

$$y^{-3}\,dy = x(1+x^2)^{-1/2}\,dx$$

$$\int y^{-3}\,dy = \int x(1+x^2)^{-1/2}\,dx$$

$$-\frac{1}{2y^2} = \sqrt{1+x^2} + c$$

Apply Initial condition to get value of c.

$$-\frac{1}{2} = \sqrt{1} + c \quad c = -\frac{3}{2}$$

The Implicit Solontion is

P.T.O

$$-\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

Now let's Solve for $y(x)$

$$\frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$$

$$y^2 = \frac{1}{3-2\sqrt{1+x^2}}$$

$$y(x) = \pm \frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

Reapplying the Initial Condition Shows us that "−" is the corred Sign. The Explicit Soluti-on is then,

$$y(x) = -\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

$$3 - 2\sqrt{1+x^2} > 0$$

$$3 > 2\sqrt{1+x^2}$$

$$9 > 4(1+x^2)$$

$$\frac{9}{4} > 1+x^2$$

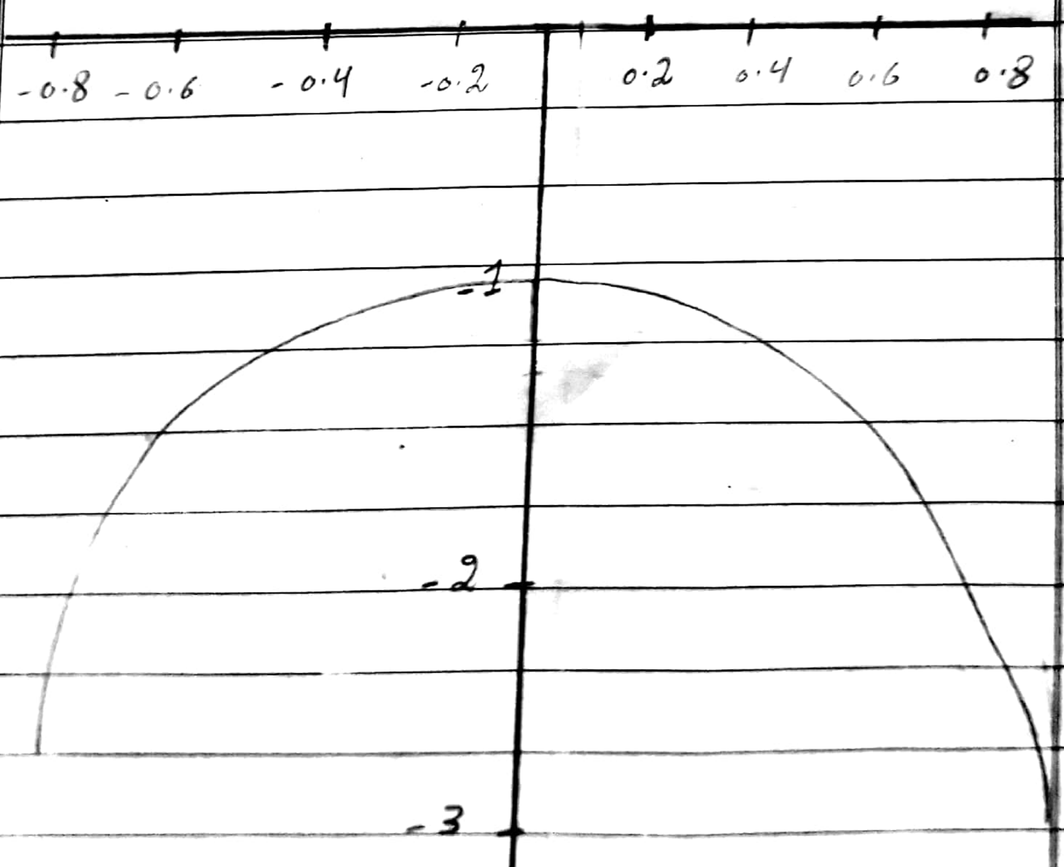
$$\frac{5}{4} > x^2$$

Now we are able to Square both Sides of the Inequality because both Sides of the Inequality guaranted to be Positive in this Case. Finally Solving for x's that will not give division by Zero or Square root of negative number will be

$$-\frac{\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$$

and nicely enough this also contains the Initial condition $x = 0$ This Interval is therefore our Interval of validity

Here is a graph of the Solution.



Q 1)

(ii) $\frac{d}{dx}\left(\frac{1}{x}\right)$

Soln:

$$\frac{d}{dx}\left(\frac{\tan(x)-1}{\sec(x)}\right)$$

$$\frac{d}{dx}\left(\frac{\tan(2x)}{1-\cot(2x)}\right)$$

$$\frac{d}{dx}\left(\frac{\tan(3x)}{x^2}\right)$$

$$\frac{d}{dx}\left(\frac{\tan(x)-3}{\sec(x)}\right)$$

$$\frac{d}{dx}\left(\frac{\tan(x)}{1+\cos(x)}\right)$$

Ans

Q#2

i) $\cos(x)y' + \sin(x)y = 2\cot(x)\sin(x) - 1$

$\because y(\pi/4) = 3\sqrt{2}$

$\because 0 \le x \le \frac{\pi}{2}$

Solution:

$$\frac{y' + \sin(x)y = 2\cos^2(x)\sin(x) - \frac{1}{\cos x}}{\cos(x)}$$

$y' + \tan(x)y = 2\cos^2(x)\sin x - \sec x$

$v(t) = e^{\tan(x)dx} = e^{\ln(\sec(x))} = e^{\ln\sec x}$

$\Rightarrow \sec(x)y^3 + \sec(x)\tan(x)(y) = 2\sec x$
$\cos^2 x\sin(x) - \sec^2(x)$

$\Rightarrow (\sec(x)y)^2 = 2\cos(x)\sin(x) - \sec^2(x)$

$\Rightarrow \int (\sec(x)y(x)\,dx = \int 2\cos(x)\sin x - \sec^2(x)$

P.T.O

$$\Rightarrow \sec x\, y(x) = -\frac{1}{2}\cos 2x - \tan x + c$$

$$\Rightarrow y(x) = -\frac{1}{2}(\cos x)(\cos 2x) - \cos(x)$$

$\tan(x) + \cos x$

$$y(x) = -\frac{1}{2}\cos x \cos 2x - \sin(x) + \cos(x)$$

put $y = 3\sqrt{2}$ and $x = \frac{\pi}{4}$

$$3\sqrt{2} = y\left(\frac{\pi}{4}\right) = -\frac{1}{2}\cos\left(\frac{\pi}{4}\right)$$

$$\cos\frac{\pi}{2} - \sin\frac{\pi}{2}$$

$$3\sqrt{2} = \frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2}$$

$$c = 7$$

How

$$y(x) = -\frac{1}{2}\cos(x)\cos(2x) - \sin x$$

Best Quality

P.T.O

$+7 \cos x$

~~Hence~~

Hence

$$\boxed{y(x) = -\frac{1}{2}\cos(x)\cos(2x - \sin x + 7\cos x}$$

Ans

Q#2

ii) $x' + 2x = Sint$

$u(t) = e^{\int 2dt} = e^{2t}$

$(xe^{2t})' = e^{2t} \, Sint$

Integrating Both Sides

$xe^{2t} = \int e^{2t} \, Sint \, dt + c^{-2t}$

$x(t) = e^{-2t}\left[e^{2t}\left(\frac{2}{5} sint - \frac{1}{5} cost\right)\right]$

$+ Ce^{-2t}$

$$\boxed{= \frac{2}{5} Sint - \frac{1}{5} Cost + Ce^{-2t}}$$

Ans

Q#03:

i) $2xy - 9x^2 + (2y + x^2 + 1)\dfrac{dy}{dx} = 0$

$y(0) = 3$

Soln:

$M = 2xy - 9x^2 \qquad N = 2y + x^2 + 1$

Now,

$$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}\left(2xy - 9x^2\right)$$

$$\boxed{\frac{\partial M}{\partial y} = 2x}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(2y + x^2 + 1\right)$$

$$\boxed{\frac{\partial N}{\partial x} = 2x}$$

So,

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ Exact

P.T.O

Therefore

Integrating

$$\int (2xy - 9x^2)\,dx + \int (2y+1)\,dy = 0$$

$$\int 2xy\,dx - \int 9x^2\,dx + \int 2y\,dy + \int 1\,dy = 0$$

$$2y\frac{x^2}{2} - \frac{9x^3}{3} + \frac{2y^2}{2} + y + c = 0$$

$$\cancel{2}y\frac{x^2}{\cancel{2}} - \frac{{}^{3}\cancel{9}x^3}{\cancel{3}} + \cancel{2}\frac{y^2}{\cancel{2}} + y + c = 0 \quad (A)$$

put Initial values of $x = 0$

and $y = 3$ in (A) we get

$$yx^2 - 3x^3 + y^2 + y + c = 0 \quad \text{— (B)}$$

put $x = 0$ and $y = 3$ in (B)

P.T.O

$3(0)^2 - 3(0)^3 + 3^2 + C = 0$

$9 + 3 + C = 0$

$\boxed{C = -12}$

put $C = -12$ in (B) we get

$\boxed{y(x^2) - 3x^3 + y^2 + y - 12 = 0}$ Ans

Q#03)

ii) $\frac{2ty}{t^2+1} - 2t - \left(2 - \ln(t^2+1)\right)\frac{dy}{dt} = 0$

$y(5) = 0$

Soln.

$M = \frac{2ty}{t^2+1}$ $\quad N = -2 + \ln(t^2+1)$

It is given that equation is Exact so,

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}} \text{ Exact}$$

Integrating

$$\int \left(\frac{2ty}{t^2+1} - 2t\right)dt + \int -2\, dy = 0$$

P.T.O

$$\int \frac{2tY}{t^2+1}\,dt - 2\int t\,dt - 2\int y\,dy + C = 0$$

$$2Y\ln(t^2+1) - 2\frac{t^2}{2} - 2y + C = 0$$

$$2Y\ln(t^2+1) - t^2 - 2Y + C = 0$$

Put $Y=0$ and $t=5$

$$2(0)\ln(0+1) - (5)^2 - 2(0) + C = 0$$

$$\boxed{C = 25}$$

So

A Becomes

$$\boxed{2Y\ln(t^2+1) - t^2 - 2Y + 25 = 0}$$ Ans

The End