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SUBJECT :- MOS (II)

SUBMITTED TO :- Sid Saqib

Question

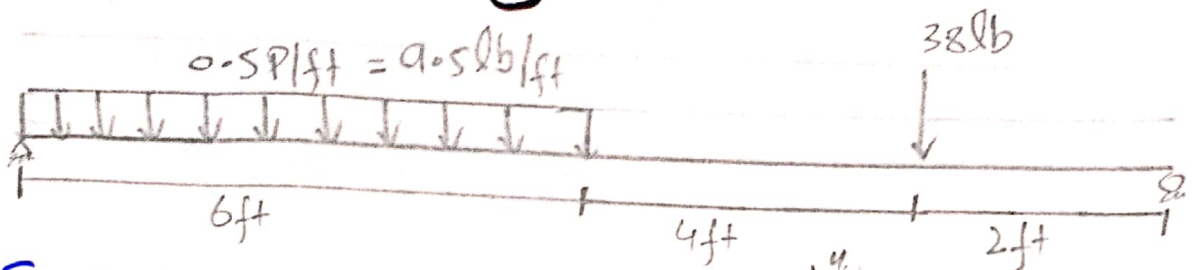
①

Construct the Mohr's Circle diagram and find the principal Stress and maximum in plane Shear Stress for the Stress State of a Point c located at the Center of uniformly distributed load and 1 inches below the top fiber of beam Cross Section Show in figure. However to Construct the Mohr's Circle it is necessary to draw the Shear stress and flexural Stress variation diagram for maximum Shear force and bending moment respectively. Compare the result obtained from the Mohr's Circle with the Stress transformation equation

Hint \Rightarrow To Calculate the Stress in the beam Cross Section the moment of inertia must be known.

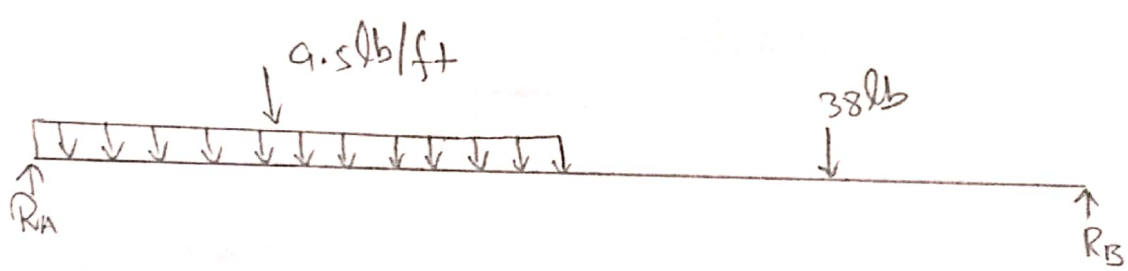
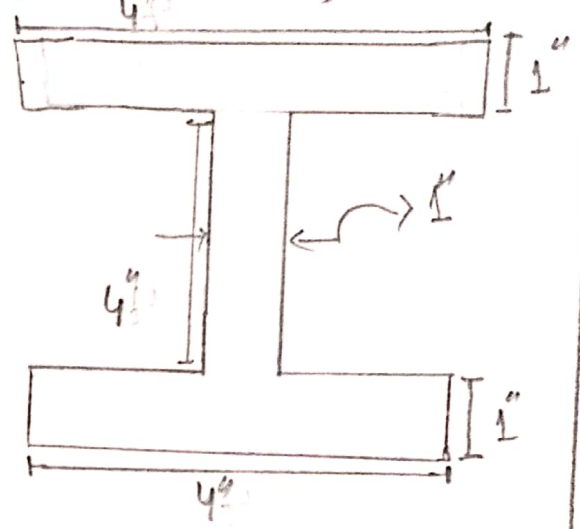
Where P is the last two digits of your class registration number in Pound.

②



Solution:

free body diagram



$$\sum F_y = 0 \uparrow + \downarrow -$$

$$R_A + R_B - 9.5 \times 6 - 38 =$$

$$\boxed{R_A + R_B = 95 \text{ lb}}$$

$$\sum M_A = 0 \leftarrow + \text{Anticlockwise positive}$$

$$R_B (12) - (38)(10) - (9.5)(6)(3) = 0$$

$$R_B (12) = 551$$

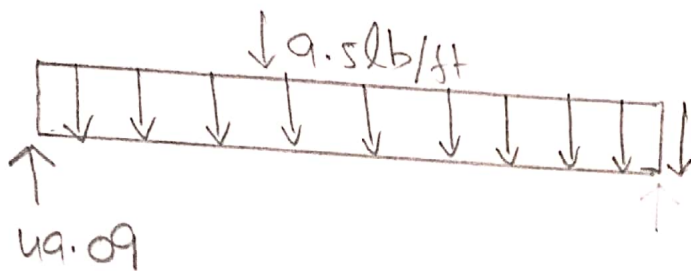
$$R_B = \boxed{45.91 \text{ lb/ft}}$$

Now

$$R_A = 95 - R_B \quad (3)$$

$$R_A = 49.09 \text{ lb}$$

Change Point of Beam

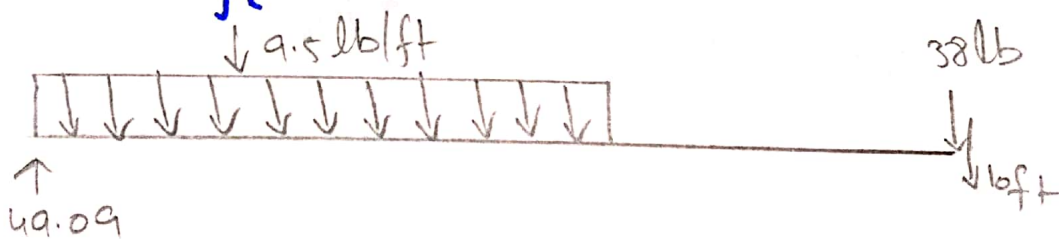


$$\sum F_y = 0 \uparrow + \downarrow -$$

$$-V_{6ft} + 49.09 - 9.5 \times 6 = 0$$

$$V_{6ft} = -7.91 \text{ lb}$$

At 10 ft

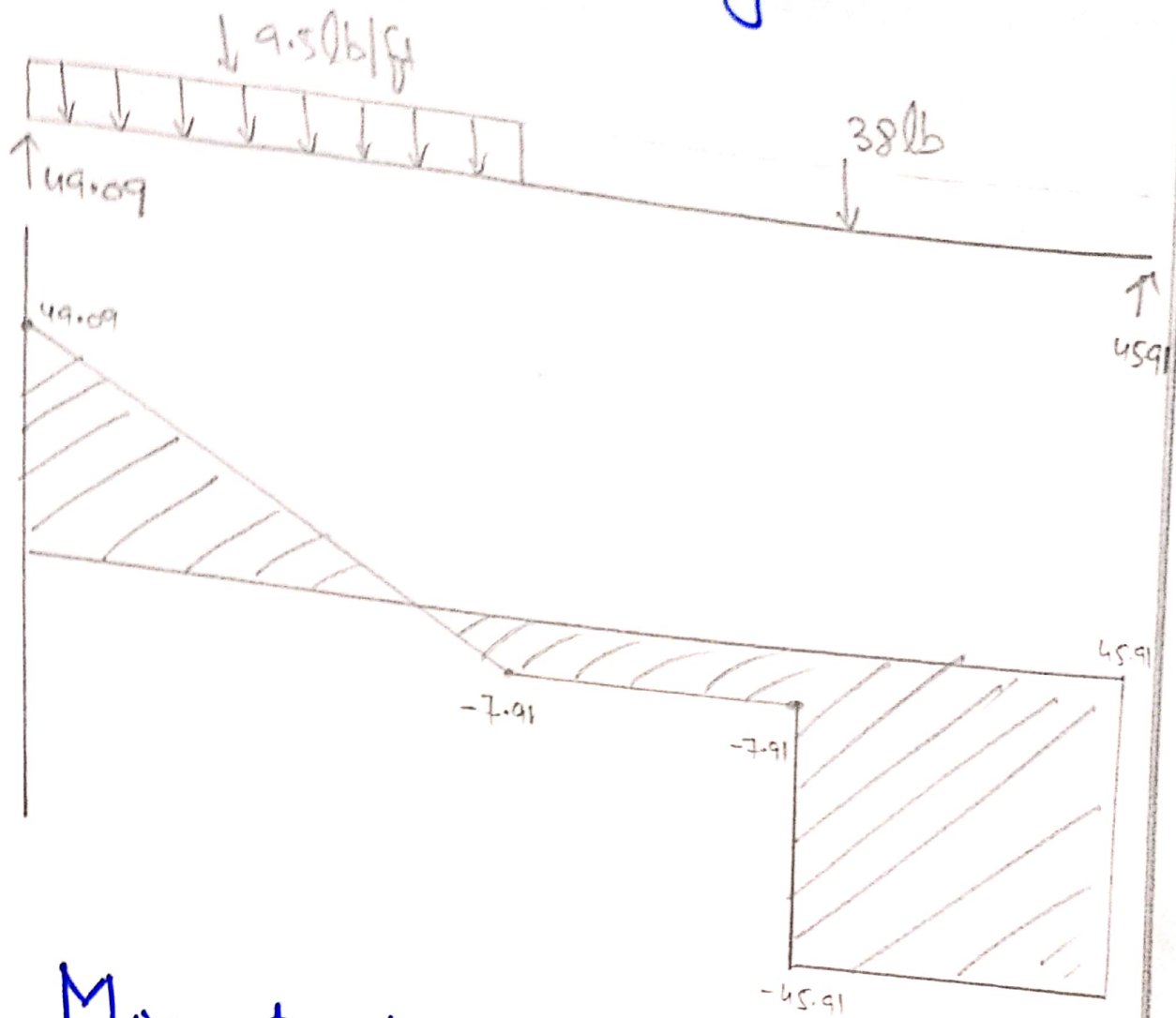


$$\sum F_y = 0 \uparrow + \downarrow -$$

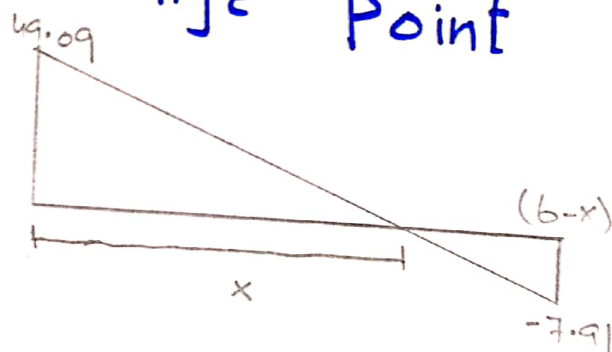
$$-V_{10ft} + 49.09 - 9.5 \times 6 - 38 = 0$$

$$V_{10ft} = -45.91$$

Shear force diagram (4)



Moment At Change Point

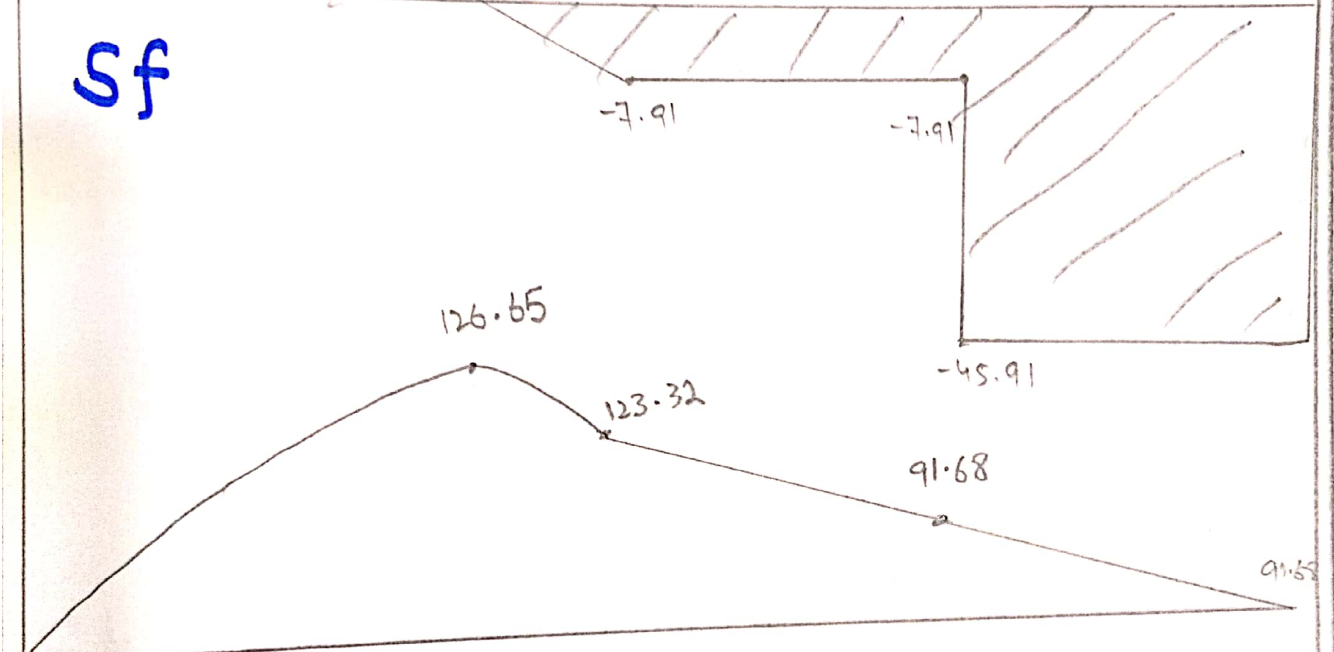


$$\frac{49.09}{x} = \frac{7.91}{(b-x)}$$

$$x = 5.16$$

Moment is max where Shear force is $= 0$

Shear force ⁽⁵⁾ and movement diagram.



Sf

Bm

⑥

Moment at 3ft from left support which is point

$$\sum M_{3ft} = 0 \leftarrow +$$

$$M_{3ft} - (49.09 \times 3) + (9.5 \times 3 \times 1.5) = 0$$

$$M_{3ft} = 104.52 \text{ lb}\cdot\text{ft}$$

$$\sum F_y = 0 \uparrow + \downarrow -$$

$$49.09 - 9.5 \times 3 = 0$$

$$V = 20.59 \text{ lb}$$

Shear Stress \Rightarrow

As per the Question the ^{shear} Stress

$T = \frac{VQ}{Ib}$ at C point occurs the

Shear Stress lies at C point is

$$20.59 \text{ lb}$$

⊕

Moment of Inertia of I section

To find the Moment of Inertia
first find the Geometric Center

$$\bar{I}_{xx} = \bar{I}_{xx_1} + \bar{I}_{xx_2} + \bar{I}_{xx_3}$$

$$\bar{I}_{xx_1} = \frac{1}{12} (4) (1^3) + (4) (2.5)^2 = 25.33$$

$$\bar{I}_{xx_2} = \frac{1}{12} (4^3) \times (1) + (4) (0) = 5.33$$

$$\bar{I}_{xx_3} = \frac{1}{12} (1^3) (4) + 4 (3-5.5)^2 = 25.33$$

$$\bar{I}_{xx} = \bar{I}_{xx_1} + \bar{I}_{xx_2} + \bar{I}_{xx_3}$$

$$\bar{I}_{xx} = 25.33 + 5.33 + 25.33$$

$$\bar{I}_{xx} = 25.33 + 5.33 + 25.33$$

$$\bar{I}_{xx} = 56 \text{ in}^4$$

$$\bar{I}_{yy} = \bar{I}_{y_1} + \bar{I}_{y_2} + \bar{I}_{y_3}$$

$$= \frac{bh^3}{12} + \frac{bh^3}{12} + \frac{bh^3}{12}$$

$$= \frac{4^3 \times 1}{12} + \frac{1^3 \times 4}{12} + \frac{1 \times 4^3}{12}$$

$$= \boxed{\bar{I}_{yy} = 11 \text{ in}^4}$$

Shear stress at ^⑧ Point C

1" below the top fibre

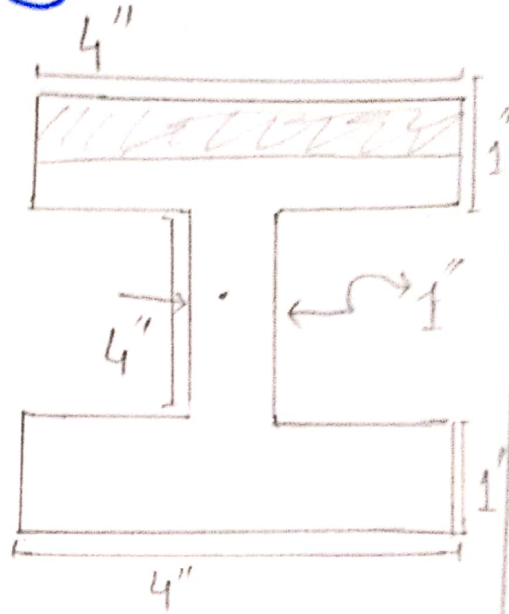
$$\tau_{xy} = \frac{VQ}{Ib}$$

$$Q = Ay$$

$$Q = 1 \times 4 \times 2.5 = 10 \text{ in}^2$$

$$I_{xy} = \frac{20.59 \times 10}{56 \times 4}$$

$$\tau_{xy} = 0.91996 \text{ lb/in}^2$$



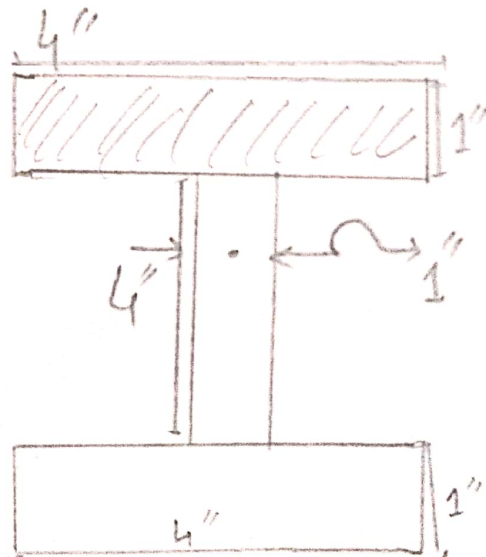
Flexure Stress at Point C

1" below from The Top fibre

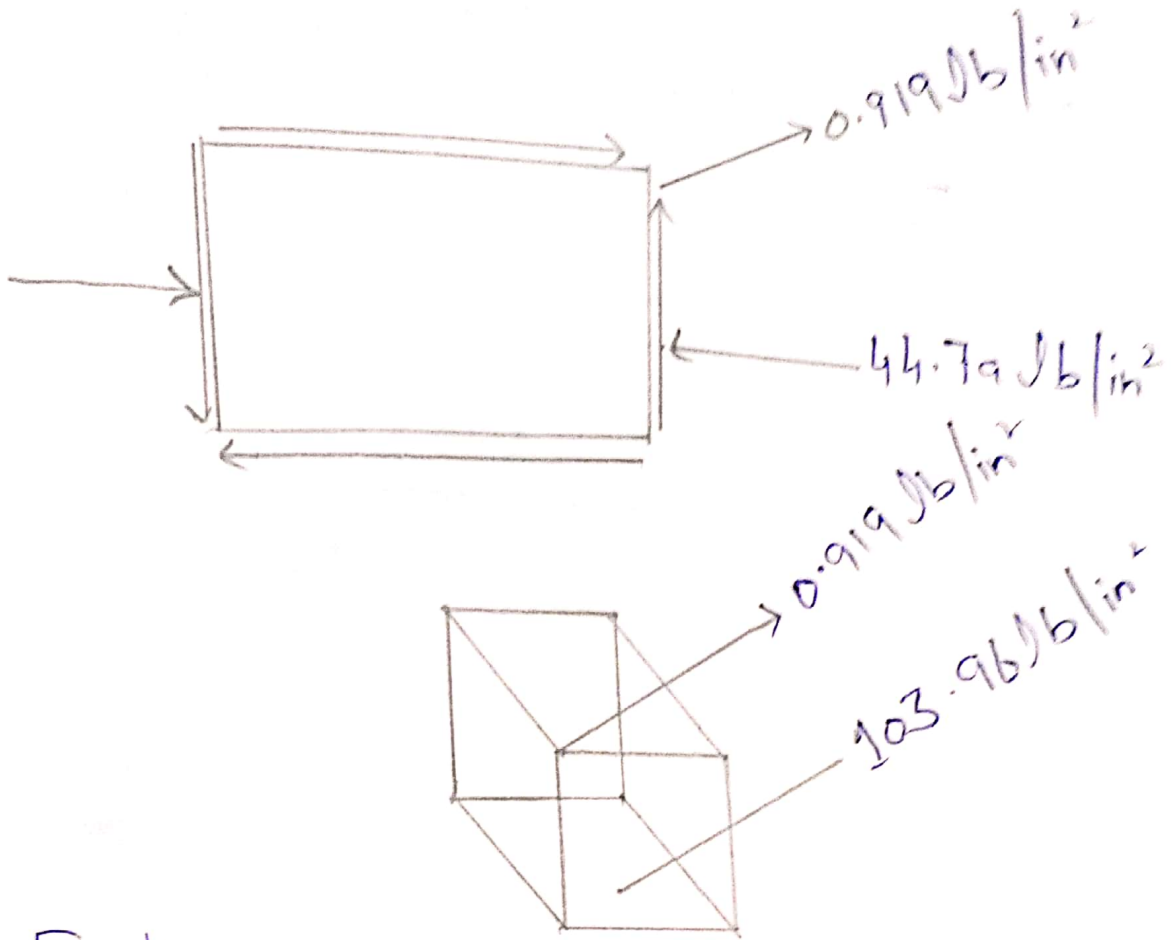
$$\sigma_x = \frac{My}{I}$$

$$= \frac{12 \times 104.52 \times 2}{56}$$

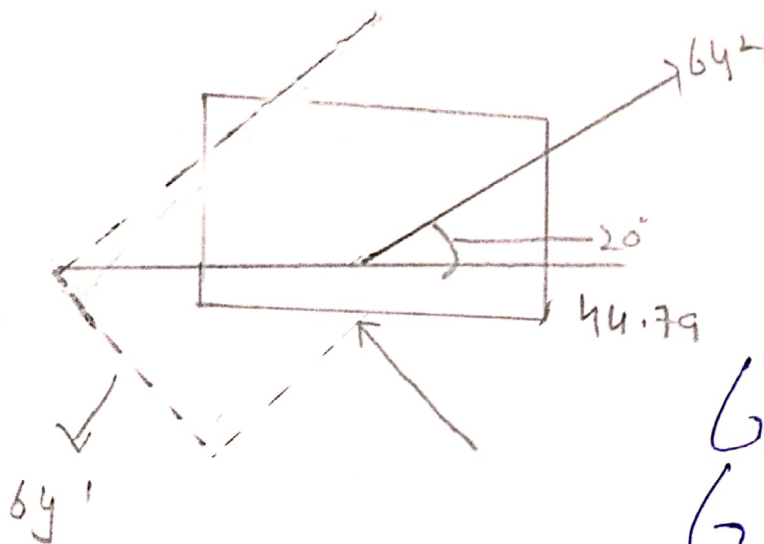
$$= 44.7 \text{ lb/in}^2$$



9



Find the Stress State Condition of Point C at Assum angle Δs 20° anticlockwise Orientation



$$\begin{aligned} \sigma_x &= -44.79 \text{ psi} \\ \sigma_y &= 0 \\ \tau_{xy} &= 0 \text{ psi} \end{aligned}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{x'} = \frac{-44.79 + 0}{2} + \frac{-44.79 - 0}{2} \cos 2(20) + 0.91 \sin 2(20)$$

$$\sigma_{x'} = -38.96 \rightarrow \text{Compressive}$$

$$\sigma_{y'} = \frac{\sigma_x - \sigma_y}{2} - \frac{\sigma_x + \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

-0.91

$$\sigma_{y'} = \frac{-44.79 + 0}{2} - \frac{-44.79 - 0}{2} \cos 2(20) - 0.91 \sin 2(20)$$

$$\sigma_{y'} = -4.65 \text{ psi} \rightarrow \text{Compressive}$$

for $\tau_{x'y'}$

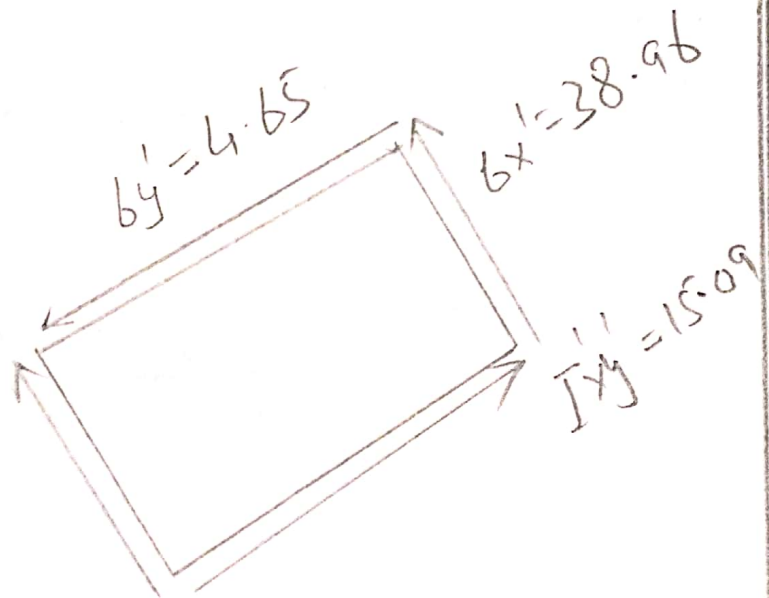
$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

(11)

$$= -\frac{-44.79 - 0 \sin 2(20) + 0.91}{2} \cos 2(20)$$

$$I_{x'y'} = 15.09$$

Now Stress state after 20° clockwise orientation is shown



Find its Principle Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + I_{x'y'}^2}$$

$$\sigma_{1,2} = \frac{-44.79 + 0}{2} \pm \sqrt{\left(\frac{-44.79 - 0}{2}\right)^2 + (0.91)^2}$$

$$\sigma_{1,2} = -22.395 \pm 22.41$$

$$\sigma_y = -22.395 + 22.41 = 0.018 \text{ psi}$$

$$\sigma_x = -22.395 - 22.395 = -44.79 \text{ Psi}$$

find $\phi_p = ?$

$$\tan 2\phi_p = \frac{I_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{0.91}{(-44.79 - 0)/2}$$

$$\phi_p = -0.081$$

Put in general Equation

$$\sigma_{x \max} = \frac{-44.79 + 0}{2} + \frac{-44.79 - 0}{2} \cos 2(-0.081) + 0.91 \sin 2(-0.081)$$

$$\sigma_{x \max} = 44.79 \text{ Psi}$$

Max in plane Shear Stress

In This Case $\Rightarrow \tan 2\phi_s = \frac{-(\sigma_x - \sigma_y)/2}{I_{xy}}$

$$\tan 2\phi_s = \frac{-(-44.79 - 0)/2}{0.91}$$

(13)

$$\theta = 49.21 \rightarrow \text{anticlockwise}$$

Put in the general Equation
for $I_{x'y'}$

$$I_{x'y'} = - \left[\frac{b_x - b_y}{2} \right] \sin 2\theta + I_{xy} \cos 2\theta$$

$$= - \left(\frac{-44.79 - 0}{2} \right) \sin 2(49.21) + 0.91 \cos 2(49.21)$$

$$I_{x'y'} = 22.17 \text{ Psi}$$

↳ Max in plane Shear
Stress

To Draw the Mohr's Circle
for the given Problem

Centre Coordinate \Rightarrow

$$(h, k) = \left[\frac{-47.79 + 0}{2}, 0 \right]$$

(14)

$$= [-22.395, 0]$$

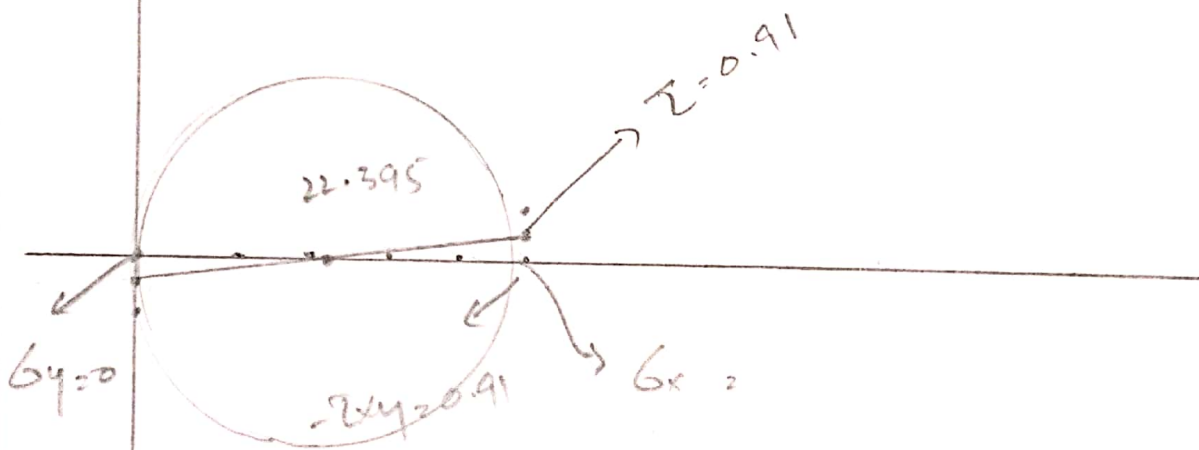
Radius of Mohr's Circle

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-47.79 - 0}{2}\right)^2 + (0.91)^2}$$

$r = 22.21 \text{ psi}$

Sketch of Mohr's Circle (15)



$$\sigma_1 = 103.67$$

$$\sigma_{xy} = 0$$

Conclusion ⁽¹⁶⁾ \Rightarrow As Shown from Mohr's Circle The value obtain that of Principle stress and maximum Shear stress are Almost Same with value obtained from transformation equations.