Department of Electrical Engineering

Mid term exam

Date: 21/08/2020

Course Details

Course Title:	Complex & Multivariable Calculus	**Module:**	03
Instructor:	**mujtaba ihsan**	**Total Marks:**	30

Student Details

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Q1.	(a)	Express -3 + 4i in polar form and represent it graphically.	Marks 06+06
	(b)	Given that u (x, y) = (x3 + $3xy^2$) + i ($3x^2y$ y^3). Determine if the function is analytic or not?	CLO 1
Q2.		If 1' 5 + 3i and z, = 4 - 2î, evaluate 2 and 1/ 2	Marks 06
			CLO 1
Q3.		Given that u (x, y) = x^3 - $3xy^2$ — 5y. Determine if the function is harmonic? If so, evaluate the conjugate harmonic function of u.	Marks 08
			CLO 1
Q4.		Differentiate the following:	Marks 04
		f(z) = z^2 / (5z+2)	
	ii	f(z) = 3z° — $5z^3$ + 2z + 1	CLO 1

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Subject Complex & Multi Variable Calculus.

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Q1 (A) Express $-3+4i$ in Polar form and represent it graphically.

Sol:

$$z = -3 + 4i$$

$$z = r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{(x^2+y^2)}$$

$$r = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16}$$

$$r = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

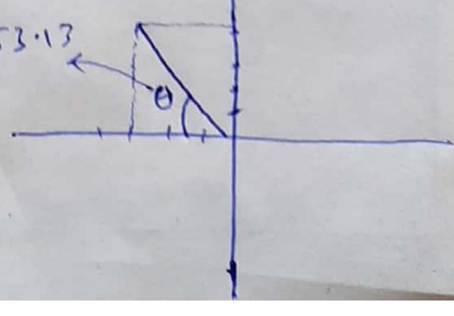
$$\theta = \tan^{-1}\left(-\frac{4}{3}\right) = \tan^{-1}(-1.33)$$

$$\theta = -53.13$$

$$z = r(\cos\theta + i\sin\theta)$$

$$= 5(\cos -53.13 + i\sin -53.13)$$

$$z = 5\cos -53.13 + i\,5\cos\sin -53.13)$$



Q1 :: (b)

Given that $u(x,y) = (x^3 + 3xy^2) + i(3x^2y - y^3)$ Determine the function is Analytic or not.

Sol:: $\frac{du}{dx} = \frac{dv}{dy}, \quad \frac{dv}{dx} = -\frac{du}{dy}$

$u(x,y) = (x^3 + 3xy^2)$ and $v = 3x^2y - y^3$

$\frac{du}{dx} = 3x^2 + 3y^2, \quad \frac{dv}{dy} = 3x^2 - 3y^2$

$\frac{du}{dx} = \frac{dv}{dy}$ only when

$$x^2 = y^2.$$

Points ~~on~~ on the lines $x^2 = y^2$ do not have nighbour hoods which these lines and hence $f(z)$ is not analytic at any Point.

So not anlytic.

Q2:-

if $Z_1 = 5+3i$ and $Z_2 = 4-Zi$

evalute $Z_1 \cdot Z_2$ and $\frac{Z_1}{Z_2}$

Sol:-

$Z_1 \cdot Z_2 = (5+3i)(4-2i)$

$Z_1 \cdot Z_2 = 20 - 10i + 12i - 6i^2$

$= 20 + 2i - 6(-1)$

$= 20 + 2i + 6$

$= 20 + 6 + 2i = 26 + 2i$

$\boxed{Z_1 \cdot Z_2 = 26 + 2i}$ Ans

(ii) $\frac{Z_1}{Z_2} = \frac{5+3i}{4-2i}$

$= \frac{5+3i}{4-2i} \times \frac{4+2i}{4+2i} = \frac{20+10i+12i+6i^2}{16+4}$

$= \frac{20-6+22i}{20} = \frac{14+22i}{20}$

$\boxed{\frac{Z_1}{Z_2} = \frac{14}{20} + \frac{22i}{20}}$ Ans

Q3:- Given that $u(x,y) = x^3 - 3xy^2 - 5y$
Determine if the function is harmonic?
if so evaluate the conjugate harmonic function of u.

Sol:-
(a) $\frac{\partial u}{\partial x} = 3x^2 - 3y^2, \quad \frac{\partial u}{\partial x^2} = 6x$

$\frac{\partial}{\partial y} = -6xy - 5$

$\frac{\partial^2 u}{\partial y^2} = -6x$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6x = 0$$

(b) evaluate the conjugate harmonic.

$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 3x^2 - 3y^2$ and

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = 6xy + 5$

Integrating g the first one

$v(xy) = 3x^2y - y^3 + h(x)$

~~and~~ ~~$\frac{dy}{dx}$~~ and $\frac{\partial v}{\partial x} = 6xy + h'(x) = 5$

~~$h(x)$~~ $h(x) = 5x + C$

Thus $v(x,y) = 3x^2y - y^3 + 5x + C$

So $v(x,y) = 3x^2y - y^3 + 5x + C$

Q4:- Differentiate the following.

(i) $f(z) = \frac{z^2}{(5z+2)}$

(ii) $f(z) = 3z^4 - 5z^3 + 2z + 1$

Sol:-

(i) $f(z) = \frac{z^2}{(5z+2)}$

use Quotient Rule.

$$\frac{d}{dz}\left[\frac{f(z)}{g(z)}\right] = \frac{gz\, f'(z) - f(z)\, g'(z)}{[\,g[z]\,]^2}$$

$$\frac{d}{dz} = \left[\frac{z^2}{(5z+2)}\right]$$

$$= \frac{5z+2\,\frac{d}{dz}(z^2) - z^2\frac{d}{dz}(5z+2)}{(5z+2)^2}$$

$$= \frac{(5z+2)(2z) - z^2(5)}{(5z+2)^2}$$

$$\boxed{= \frac{5z^2+4z}{(5z+2)^2}}$$

Q4: (iii) $f(z) = 3z^4 - 5z^3 + 2z + 1$

Sol:: use sum rule.

$$\frac{d}{dz}[f(z) + g(z)] = f'(z) + g'(z)$$

$$= \frac{d}{dz}\left[3z^4 - 5z^3 + 2z + 1\right]$$

$$= \frac{d}{dz}3z^4 - \frac{d}{dz}5z^3 + \frac{d}{dz}2z + \frac{d}{dz}1$$

$$\boxed{= 12z^3 - 15z^2 + 2}$$