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PAPER : DIFFERENTIAL EQUATION.

SUBMITTED TO : SIR

BSSE 3rd Sem

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Q 1: a):

solution: Homogeneous Equations..

If $g(t) = 0$, then the equation above becomes

$$y'' + p(t)y' + q(t)y = 0$$

It is called a homogeneous equation. Otherwise, the equation is nonhomogenous or (inhomogeneous).

~~Example~~

Non Homogeneous Differential equation:

We will now turn our attention to nonhomogeneous second order linear equations, equations with the standard form.

$$y'' + p(t)y' + q(t)y = g(t), \quad g(t) \neq 0$$

Each such nonhomogeneous equation has a corresponding homogeneous equation.

$$y'' + p(t)y' + q(t)y = 0$$

We will focus our attention to the simpler topic of nonhomogeneous second order linear equations with constant coefficients.

$$ay'' + by' + cy = g(t).$$

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Where a, b, and c are constant. $a \neq 0$; and $g(t) \neq 0$. It has a corresponding homogeneous equation.

$$ay'' + by' + cy = 0.$$

Example: (i): Find the general solution of

$$y'' - 5y' = 0$$

Let $u = y'$, then $u' = y''$.

$$u' - 5u = 0$$

The integration factor is $\mu = e^{-5t}$.

$$u(t) = \frac{1}{\mu(t)} \left(\int \mu(t)\, g(t)\, dt \right) = e^{5t} \left(\int 0\, dt \right) = e^{5t}(C) = Ce^{5t}$$

$$y(t) = \int u(t)\, dt = \int Ce^{5t}\, dt = \frac{C}{5} e^{5t} + C_2$$

$$= C_1 e^{5t} + C_2.$$

Example: (ii):

$$y'' - 2y' - 3y = e^{2t}$$

The corresponding homogeneous equation. $y'' - 2y' - 3y = 0$ has characteristic equation $r^2 - 2r - 3 = (r+1)(r-3) = 0$.

So the complementary solution is

$$y_c = C_1 e^{-t} + C_2 e^{3t}.$$

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Let $y = Ae^{2t}$, then $Y' = 2Ae^{2t}$, and $Y'' = 4Ae^{2t}$. Substitute them back into the origional differential equation.

$$(4Ae^{2t}) - 2(2Ae^{2t}) - 3(Ae^{2t}) = e^{2t}$$

$$-3Ae^{2t} = e^{2t}$$

$$A = -1/3$$

Hence $Y(t) = \frac{-1}{3} e^{2t}$.

Therefore, $y = y_c + Y = C_1 e^{-1} + C_2 e^{3t} - \frac{1}{3} e^{2t}$.

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Since $r = -\frac{3}{4}$ the general solution is then.

$$y(t) = c_1 e^{-3t/4} + c_2 t e^{-3t/4}$$

RESULT:

$$y(t) = c_1 e^{-3t/4} + c_2 t e^{-3t/4}$$

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b)

i) $16y'' + 24y' + 9y = 0$

Solution: Given.

$$16y'' + 24y' + 9y = 0$$

Determine the characteristic equation by replacing y'' with r^2, y' with r and y with 1 in the differential equation.

$$16r^2 + 24r + 9 = 0$$

Determine the roots:

$$r = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)} = -\frac{3}{4}$$

General equation when the characteristic equation has only 1 solution.

$$y(t) = C_1 e^{rt} + C_2 t e^{rt}$$

with r the (double) root of the characteristic equation.

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Q NO 1 B(ii)

$y'' - 4y' - 12y = 3e^{5x}$

Solution.

$y'' - 4y' - 12y = 3e^{5x} : y = c_1e^{6x} + c_2 - \frac{3}{7}e^{5x}$

Steps =

$y'' - 4y' - 12y = 3e^{5x}$

$a(x)y'' + b(x)y' + c(x)y = g(x)$

Find y_h by solving $y'' - 4y' - 12y = 0 : y = c_1e^{6x} + c_2e^{-2x}$

Find y_p that satisfies $y'' - 4y' - 12y = 3e^{5x} : y = \frac{3}{7}e^{5x}$

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Qn No 2 (A)

Solve the initial value Problem
$2y'' + 5y' + 3y = 0$, $y(0) = 3$, $y'(0) = -4$

I have come up with: $y =$
$\Rightarrow$ $C_1 e^{-\frac{3x}{2}} + C_2 e^{-x} \Rightarrow y' = -\frac{3}{2} C_1 e^{-\frac{3}{2}x} -$
$C_2 e^{-x}$

$y(0) = C_1 + C_2 = 3$

$y'(0) = -\frac{3}{2} C_1 - C = -4$

Add the two equations:
$-\frac{1}{2} C_1 = -1 \Rightarrow C_1 = 2$ ...... then: $C_2 = 1$

Q. NO #. 2 (B)

$2y'' - 5y' - 3y = 0$

$y(0) = 1,\ y'(0) = 4$

Let $y = e^{mx}$ be a solution so that the auxiliary eq is

$2m^2 - 5m - 3 = 0$

$(2m+1)(m-3) = 0$

$m = -0.5$ and $= 3$

$y = a(e^{-0.5x}) + b(e^{3x})$

$1 = a(e^{0}) + b(e^{0})$

$y' = -0.5a(e^{-0.5x}) + 3b(e^{3x})$

$4 = -0.5a(e^{0}) + 3b(e^{0})$

Solving the system of two equations

$$\boxed{\begin{array}{l} a = -2\ \ b = 1 \\ y = -2(e^{-0.5}) + e^{3x}. \end{array}}$$

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Q 2 (iii)

$y'' - 4y' - 12y = 3e^{5x}$

⇒ Solution

$y'' - 4y' - 12y = 3e^{5x}$: $y = c_1 e^{6x} + c_2 e^{-2x}$ $\frac{3}{7} e^{5x}$

⇒ $y'' - 4y' - 12y = 3e^{5x}$

⇒ General solution for $a(x)y'' + c(x)y = g(x)$

⇒ find y_h by solving $y'' - 4y' - 12y = 0$: $y = c_1 e^{6x} + c_2 e^{-2x}$

⇒ Find y_p that satisfies $y'' - 4y' - 12y = 3e^{5x}$

$y = -\frac{3}{7} e^{5x}$

$$\boxed{y = c_1 e^{6x} + c_2 e^{-2x} - \frac{3}{7} e^{5x}}$$

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Q3:

Sol: LAPLACE TRASFORMATION:-

Is a technique for solving differential equation. Here differential equation of time domain form is first transformed to algebraic equation of frequency domain form. After solving the algebraic equation in frequency domain, the result then is finally transformed to time domain form to achieve the ultimate solution of the differential equation. In other words it can be said that the Laplace transformation is nothing but a shortcut method of solving differential equation.

OR

When learning the Laplace transform, its important to understand not just the tables but the formula too.

To understand the Laplace transform formula. First let $f(t)$ be the function of t, time for all $t \geq 0$

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Laplace Transform Examples:

(i) $f t = t\cosh(3t)$

This function is not the table of Laplace transforms. However, we can use #30 in the table to compute its tranform. This will correspond #30 if we take $n=1$.

$$F(s) = L\{t g(t)\} = G'(s),$$

where, $g(t) = \cosh(3t)$

So, we than have,

$$G(s) = \frac{s}{s^2-9} \cdot \quad G'(s) = \frac{s^2+9}{(s^2-9)^2}$$

Using #30 we then have.

$$F(s) = \frac{s^2+9}{(s^2-9)^2}$$

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~~EXAMPLE (1)~~: Find the Laplace

(i) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

$$F(s) = \mathcal{L}\{f(t)\} = 6\mathcal{L}\{e^{-5t}\} + \mathcal{L}\{e^{3t}\}$$

$$+ 5\mathcal{L}\{t^3\} - 9\mathcal{L}\{1\}$$

$$= 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

(ii) $g(t) = 4\cos(4t) - 2\sin(4t) - 3\cos(8t)$

$\cancel{G(t) = \mathcal{L}\{g(t)\} = 4\mathcal{L}\{\cos\}}$

$$G(s) = \mathcal{L}\{g(t)\} = 4\mathcal{L}\{\cos(4t)\} - 2\mathcal{L}\{\sin(4t)\}$$

$$-3\mathcal{L}\{\cos(10t)\}$$

$$= 4\frac{s}{s^2+4^2} - 2\frac{4}{s^2+4^2} - 3\frac{s}{s^2+10^2}$$

$$= \frac{4s-8}{s^2+16} - \frac{3s}{s^2+100}$$

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iii) $h(t) = e^{2t} + \cos(3t) - e^{2t}\cos(3t)$

$$H(t) = \mathcal{L}\{h(t)\} = \mathcal{L}\{e^{2t}\} + \mathcal{L}\{\cos(3t)\} - \mathcal{L}\{e^{2t}\cos(3t)\}$$

$$= \frac{1}{s-2} + \frac{s}{s^2+3^2} - \frac{s-2}{(s-2)^2+3^2}$$

$$= \frac{1}{s-2} + \frac{2}{s^2+9} - \frac{s-2}{(s-2)^2+9}$$

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ii) $h(t) = t^2 \sin(2t)$

This part will also use #30 in the table. In fact, we could use #30 in one of two ways. We could use it with $n=1$.

$$H(s) = \mathcal{L}\{tf(t)\} = F'(s), \text{ Where } f(t) = t\sin(2t)$$

Or we could use it within $n=2$.

$$H(s) = \mathcal{L}\{t^2 f(t)\} = F''(s),$$
Where. $f(t) = \sin(2t)$.

Since it less work to do one derivative, lets, do it the first way. so using #9 we have.

$$F(s) = \frac{4s}{(s^2+4)^2} \quad F'(s) = \frac{12s^2 - 16}{(s^2+4)^3}$$

The transform is then.

$$H(s) \ \frac{12s^2 - 16}{(s^2+4)^3}$$

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Then the Laplace transform of $f(t)$, $F(s)$ can be defined as,

$$F(s) = \int_0^{\infty} f(t).e^{-st} dt$$

Provided that the integral exists.
Where the Laplace operator, $= s = \sigma + j\omega$, will be real or complex $j = \sqrt{(-1)}$

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1] (ii)

$$y'' + 3y' + 2y = e^{-t}$$

$y(0) = 0$

$y'(0) = 0$

Solution

~~Taking~~

Taking laplace transform of the differential equation we obtain

$$[s^2 y(s) - s(0) - 2] + 3[s\ y(s) - 0] + 2\,y(s) = \frac{1}{s+1}$$

$$s^2\ y(s) - 2 + 3s\ (s) + 2(s) = \frac{1}{s+1}$$

$$y(s)\{s^2 + 3s + 2\} = \frac{1}{s+1} + 2$$

$$y(s)\{s^2 + 3s + 2\} = \frac{1 + 2s + 2}{s+1}$$

$$\boxed{y(s) = \frac{2s+3}{[(s+1)(s^2+3s+2)]}}$$

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