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SEMESTER: 2TH

SECTION: COMPUTER SCIENCE

PAPER: LINEAR ALGEBRAA



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Paper: Algebra.

Question No: 1

$$\begin{bmatrix} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & -9 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Solution:-

~~R₁ + 7R₂~~ $R_1 + 7R_2$ (Add 7Row2 to Row1)

$$= \begin{bmatrix} 1+0 & 7-7 & 3+63 & 0+0 & 5-49 \\ 0 & 1 & -9 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 66 & 0 & -44 \\ 0 & 1 & -9 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$R + (-66R_3)$$

$$= \begin{bmatrix} 1+0 & 0+0 & 66-66 & 0 & -44+396 \\ 0 & 1 & -9 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$



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$$= \begin{bmatrix} 1 & 0 & 0 & 352 \\ 0 & 1 & -9 & 7 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= R_2 + 9R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 352 \\ 0+0 & 1+0 & -9+9 & 7+54 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 352 \\ 0 & 1 & 0 & -47 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

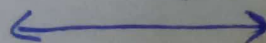
This is the final linear system

$$x_1 = 352, \quad x_2 = -47, \quad x_3 = -6, \quad x_4 = 1$$

Verification:

$$\begin{aligned} 1) \quad & \cancel{2x_1 + 7x_2 + 3x_3} = 5 \\ & x_1 + 7x_2 + 3x_3 = 5 \\ & 352 + 7(-47) + 3(-6) = 5 \\ & 352 - 329 - 18 = 5 \\ & 23 - 18 = 5 \\ & 5 = 5 \quad \{ \text{True} \} \end{aligned}$$

$$\begin{aligned} 2) \quad & x_2 - 9x_3 = 7 \\ & -47 - 9(-6) = 7 \\ & -47 + 54 = 7 \\ & 7 = 7 \quad \{ \text{True} \} \end{aligned}$$





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Question No : 2

Part A:

$$\begin{array}{c} \text{Matrix 1} \\ \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \end{array}, \begin{array}{c} \text{Matrix 2} \\ \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \end{array}$$

Matrix 1 \otimes Matrix 2

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

 $R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2-2 & -5+8 & -4-1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \Rightarrow \text{This is } \overset{\text{now}}{\text{matrix (2)}}$$

Matrix 2 to matrix (1)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

 $R_3 - 2R_2$



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$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0+2 & 3+(-8) & -5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$



So this matrix (1)



Question 2 part A Finish.



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Q2: part B:

A)
$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$
 is in echelon form

Answer: It is reduced echelon form because all the element without diagonal is zero.

B)
$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 is in echelon form.

Answer: It is row echelon form because the below element of diagonal is zero.

C)
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$
 is in reduced row echelon form.

Answer: It is row echelon form because the below element of diagonal is not zero.



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$$a: \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Answer: It is row echelon form because the below number of element is diagonal is zero.

←————→ (Question 2) Finish :-

Question 3:

part A

Difference B/w Row Echelon and Reduce row echelon form.

Solution:

Row Echelon Form:

- * All nonzero rows are above any rows of all zeros.
- * Each leading entry of a row is in a column to the right of the leading entry of the row above it.
- * All entries in a column below a leading entry are zero.

Examples:

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



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Reduced Echelon form.

- * The leading entry in each non-zero row is 1.
- * Each leading 1 is the only non-zero entry in its column.

Examples:

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Practical USE OF Reduced row Echelon form?

In an earlier paper published in the Journal of Natural Sciences Research in 2015 on how to balance chemical equations using ~~matrix to echelon form~~ using elementary ~~row operations~~ matrix algebra, Gabriel and Onwuka showed how to reduce the resulting matrix to echelon form using elementary row operations. However, they didn't show how elementary row operations can be used in reducing the resulting echelon matrix to row reduced echelon form. we show that the



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Solution obtained is actually the nullspace of the matrix. Hence the solution can be infinitely many. In addition, we show that instead of manually using row operations to reduce the matrix to row reduced echelon form, software environments like octave or Matlab can be used to reduce the matrix directly. In all the examples presented in this paper, we reduced all matrices to row reduced echelon form showing all row operations, which was not clearly stated in the Gabriel and Onwuka paper. Most importantly, with the availability of mathematical software, we show we ~~we~~ don't need to carry out these row operations by brute force.



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Question No: 3
part: B

Solution:

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & -4 & 19 \end{bmatrix}$$

⇒ $R_3 + 2R_1$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 56 & -7 \\ 1 & -4 & 19 \end{bmatrix}$$

⇒ $2R_4 - R_2$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 56 & -7 \\ 0 & 0 & -37 \end{bmatrix}$$

⇒ $R_2 - 2R_1$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & 56 & -7 \\ 0 & 0 & -37 \end{bmatrix}$$

⇒ $R_3 + 14R_2$

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & -4 & -17 \\ 0 & 0 & 245 \\ 0 & 0 & -37 \end{bmatrix}$$



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$$\Rightarrow 2R_1 + 3R_2$$

$$\begin{bmatrix} 2 & 0 & -35 \\ 0 & -4 & -17 \\ 0 & 0 & 245 \\ 0 & 0 & 38 \end{bmatrix}$$

$$\Rightarrow R_1 - 1$$

$$\begin{bmatrix} 1 & 0 & -35 \\ 0 & -4 & -17 \\ 0 & 0 & 245 \\ 0 & 0 & 38 \end{bmatrix}$$

this is required echelon form.

Finish Question 3