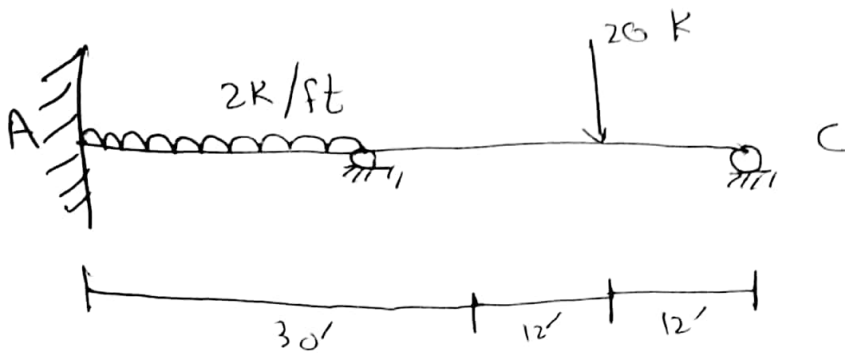


# Mid - term

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Date:-	21/8/2020
Sub Ject	Structure Analysis II

# Question # 1

①



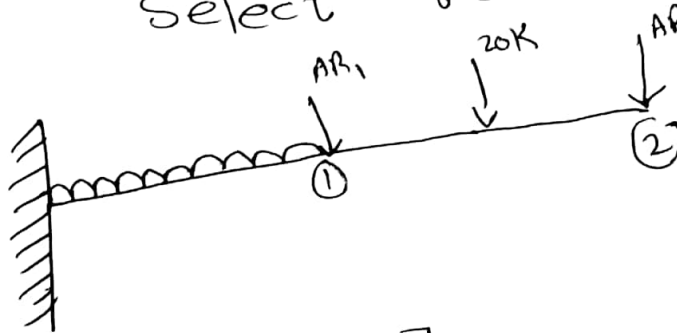
$EI = \text{Constant}$

Solution:

Structural indeterminacy = 2

Step # 1

Select redundant actions

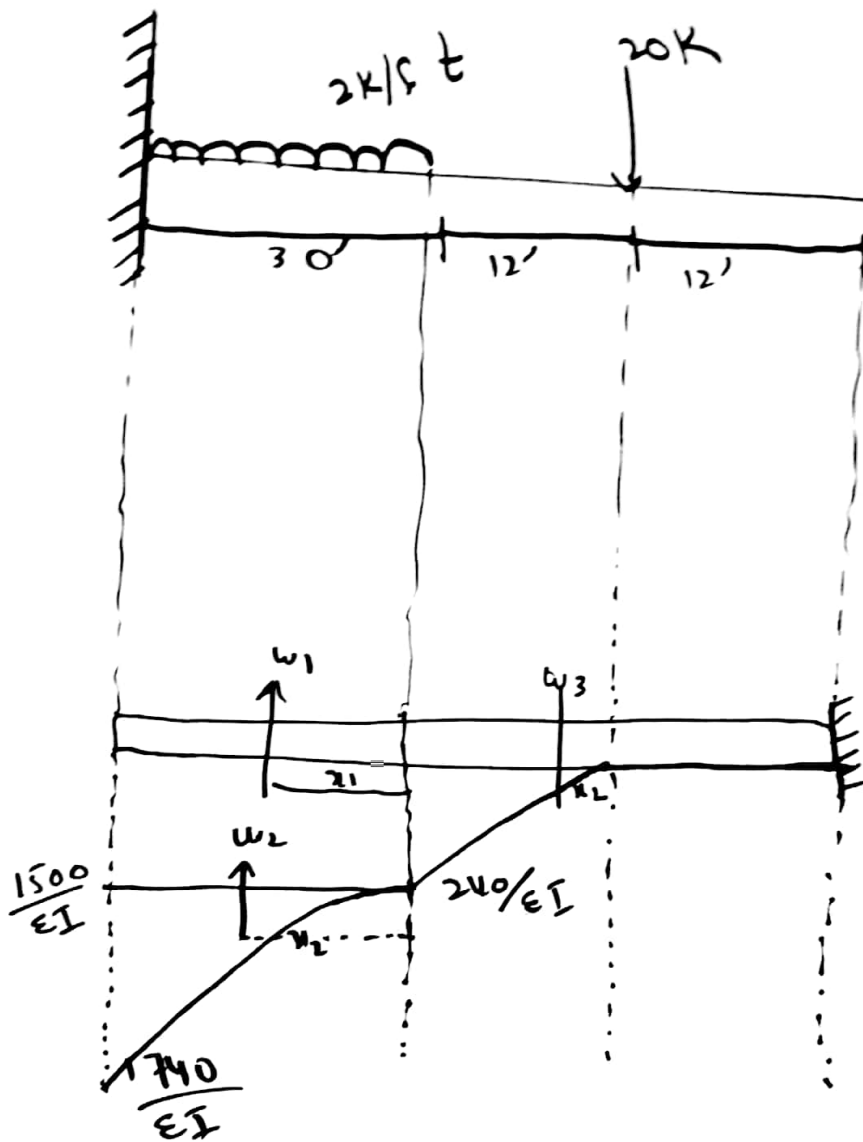


$$\begin{bmatrix} DR_1 \\ DR_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL] + [F] \times [AR]$$

Step # 2

Compute the value of  $[DRL]$  2



$$w_1 = 1500 \times 30 = 4500$$

$$w_2 = \frac{1}{3} \times 30 \times 2400 = 2400$$

$$w_3 = \frac{1}{2} \times 12 \times 240 = 1440$$

$$20 \times 12 = 240$$

$$20 \times (12 + 30) =$$

$$2 \times 30 \times 15 = 1740$$

$$x_1 = \frac{b}{2} = \frac{30}{2} = 15'$$

$$x_2 = \frac{3}{n+2} \times l = \frac{3}{2+2} \times 30 = 22.5'$$

$$x_3 = \frac{2}{3} \times l = \frac{2}{3} \times 12 = 8'$$

Now find DRL

$$DRL_2 = w_1 (x_1 \times 24) + w_2 (x_2 \times 24) + w_3 (x_3 \times 12)$$

$$= 45000 (15 \times 24) + 2400 (22.5 \times 24) + 1440 (8 \times 12)$$

$$DRL = \boxed{1895400 EI}$$

$$DRL_1 = 45000 (15) + 2400 (22.5)$$

$$= 675000 + 54000$$

$$= \boxed{729000}$$

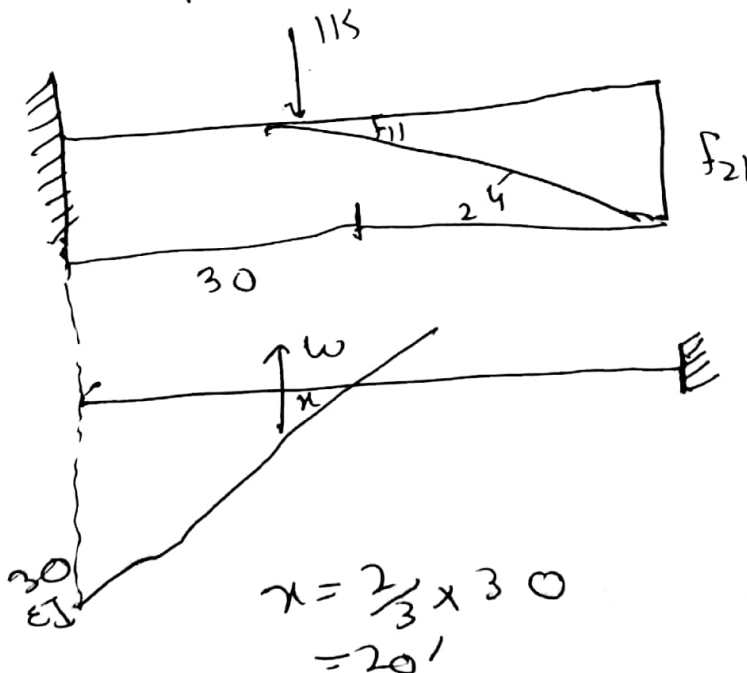
so

$$DRL = \frac{1}{EI} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

Step # 3 flexibility method.

$$F_{2 \times 2} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

(a) applying unit load on AR<sub>1</sub>



$$w = \frac{1}{2} \left( \frac{30}{EI} \times 30 \right)$$

$$= 450/EI$$

$$w = \frac{2}{3} \times 30$$

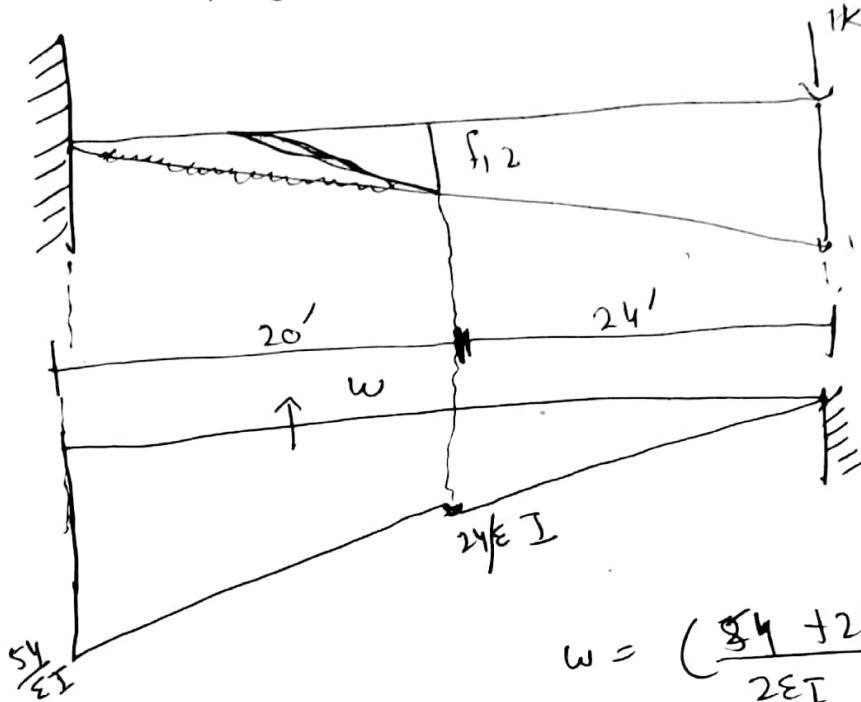
$$= 20'$$

So,

$$F_{11} = \frac{450}{EI} (20) = \frac{9000}{EI}$$

$$F_{21} = \frac{450}{EI} (20 + 24) = \frac{19800}{EI}$$

(B) Now apply unit load on AR<sub>2</sub>



$$w = \left( \frac{54 + 24}{2EI} \right) \times 30$$

$$= 1170 EI$$

Now the distance

$$x = \frac{L}{3} \left[ \frac{b + 2(a)}{a+b} \right] = \frac{30}{3} \left[ \frac{24 + 2(54)}{54 + 24} \right] = 16.92'$$

$$F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$F_{22} = \frac{1170}{EI} \times (16.92 \times 24) = \frac{47876.4}{EI}$$

Hence

$$F_{2 \times 2} = \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} \frac{1}{EI}$$

Step # 3 compute the value of AR

$$[AR] = [DRS - DR2] \times [F]^{-1}$$

$$[FJ]^{-1} = \frac{1}{|F|} \times \text{Adj } F$$

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$$= \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 198000 & 47876.4 \end{vmatrix}} \times \text{Adj} \begin{bmatrix} 9000 & 19796.4 \\ 198000 & 47876.4 \end{bmatrix}$$

$$|F| = (9000 \times 47876.4 - 19796.4 \times 198000)$$

$$= (430887600 - 391968720)$$

$$\boxed{|F| = 38918880}$$

$$\text{Adj } A = \begin{bmatrix} 47876.4 & -19796.4 \\ -198000 & 9000 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 & -729000 \\ 0 & -1895400 \end{bmatrix} \times \frac{1}{3} I \times \frac{1}{38918880} \begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \times \frac{1}{3} I \times \frac{\begin{bmatrix} 47876.4 & -19796.4 \\ -198000 & 9000 \end{bmatrix}}{38918880}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$

There are two main methods of Structural analysis using the matrix approach.

Force method

Displacement method

### Force Method

It is also known as flexibility matrix method or compatibility method

In force method the unknowns are taken as reaction & force

In this the no of redundants =  $D_s$

In this the forces are found by compatibility equation of displacement

In this the type of indeterminacy is static indeterminacy

It is suitable when  $D_s < D_k$

### Displacement Method

It is also called equilibrium method or stiffness matrix method.

In displacement method the unknowns are taken as joint displacement ( $\delta, \Delta$ ).

In this the no of redundants =  $D_k$

In this the displacements are found by equilibrium equation of force.

In this the type of indeterminacy is kinematic indeterminacy

It is suitable when  $D_s > D_k$ .

# Suitable method

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Ans

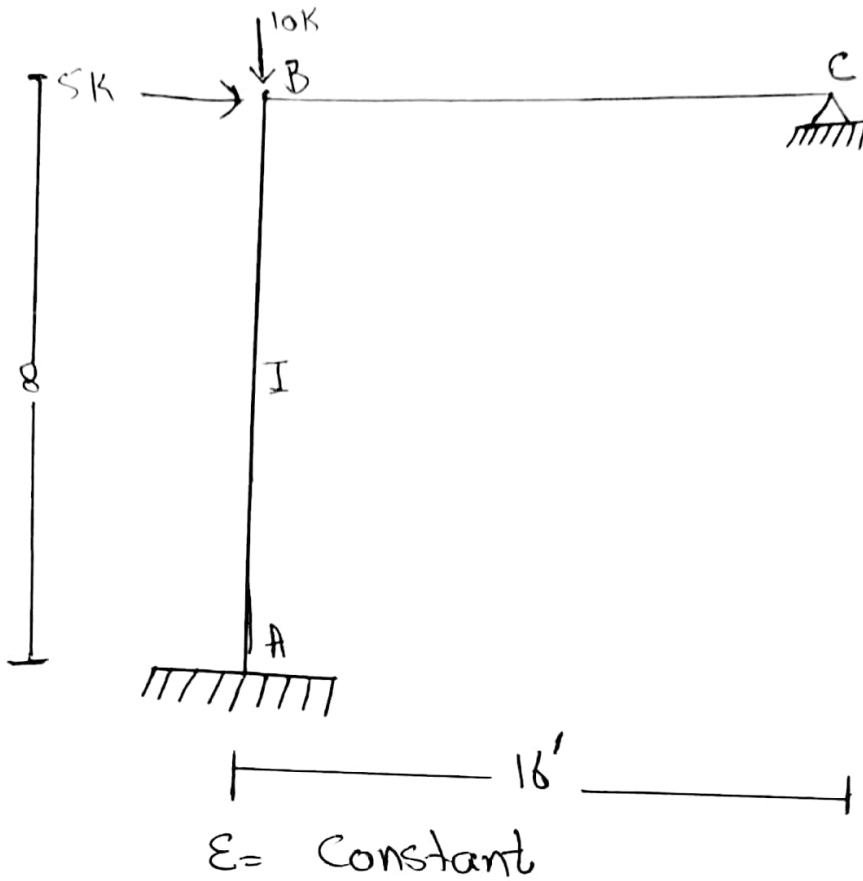
For Analysis of structure of matrix approach both the force method or displacement method can be used depend upon situation.

- ① When the degree of static indeterminacy is less than the degree of kinematic indeterminacy i.e.  $D_s < D_k$ , then it is suggested to use force method of Analysis.
- ② When the degree of static indeterminacy is more than kinematic indeterminacy i.e.  $D_k > D_s$  then it is suggested to use displacement method of Analysis.

① # 3



Q # 3



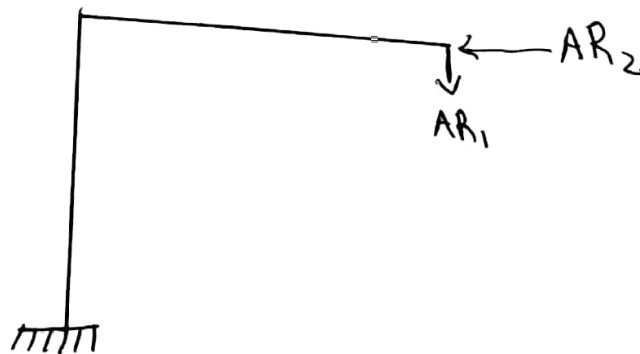
Solution:

Total statical interdependency

$$R - 3 = 5 - 3 = 2$$

Step # 1

Identifying Redundant action



$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} DR_1 \\ DR_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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## Step # 2

compute

value of  $[DRL]$

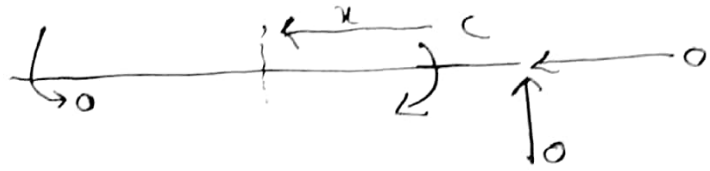
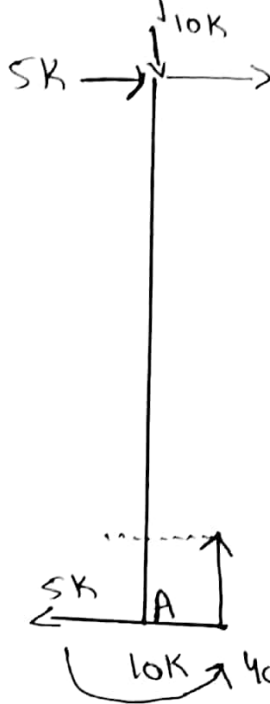


fig AMR value  
M value

## Step # 3

$[F]$  or  $[AMR]$

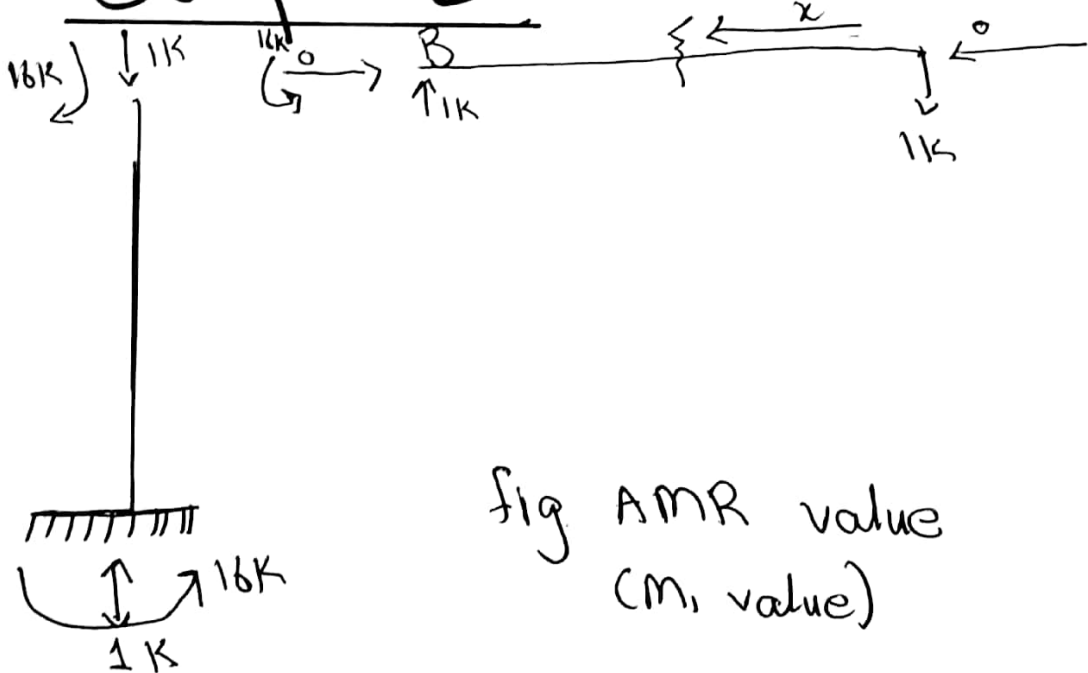


fig AMR value  
(M<sub>i</sub> value)

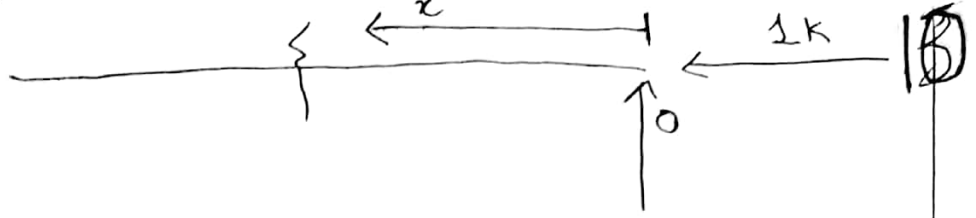
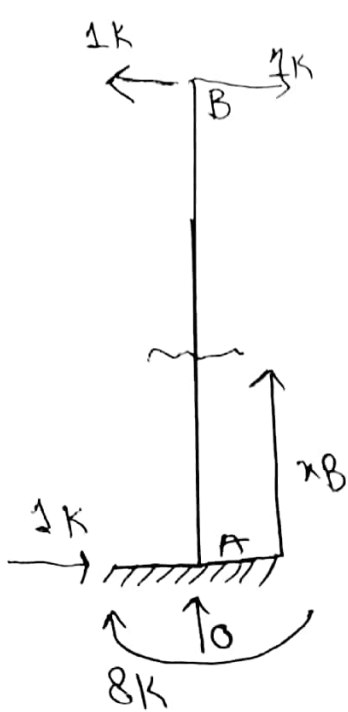


fig AMB value  
 $[M_2$  value

Member	AB	BC
origin	A	C
limit	0-8	0-16
I	I	21
M	$5x - 40$	0
$M_1$	-16	-x
$M_2$	$8 - x$	0

→ find value of DRL

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot m_1(AB)}{EI} + \int_0^{16} \frac{M_{BC} \cdot m_2(BC)}{EI}$$

$$= \int_0^8 \frac{(5x - 40)(-16) dx}{EI} + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx$$

$$\boxed{DRL_1 = \frac{2560}{EI}}$$

$$DRL_2 = \int_0^8 \frac{(5x - 40)(8 - x) dx}{EI} + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx$$

$$\boxed{DRL_2 = \frac{-853.33}{EI}}$$

→ Compute flexibility matrix:

$$f_{2 \times 2} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$\begin{aligned} \Rightarrow f_{11} &= \int_0^8 m_1^2(AB) + \int_0^{16} \frac{m_1^2(BC)}{EI} \\ &= \int_0^8 \frac{(-16)^2 dx}{EI} + \int_0^{16} \frac{x^2 dx}{E(2I)} \end{aligned}$$

$$F_{11} = \frac{2730.67}{EI}$$

12 (12)

$$\begin{aligned} \rightarrow f_{12} = f_{21} &= \int_0^8 m_1(AB) \cdot m_2(AB) + \int_0^{16} m_1(BC) \cdot m_2(BC) \\ &= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{E(2I)} dx \end{aligned}$$

$$F_{12} = F_{21} = -\frac{512}{EI}$$

$$\Rightarrow f_{22} = \int_0^8 (m_1)^2 AB dx + \int_0^{16} (m_2)^2 BC dx$$

$$\int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{E(2I)} dx$$

$$F_{22} = 170.67$$

As we know that  
 $[DRS] = [DRL] + [AR] \times [F]$

$$[AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$[AR] = [F]^{-1} \times [DRS - DRL]$$

B ④

$$= \Sigma I \begin{bmatrix} 2730.67 & -512 \\ -512 & 176.67 \end{bmatrix} \times \begin{bmatrix} 0 & -0.256 \\ 0 & +8533 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -6.0005 \\ 4.97 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$