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COURSE NAME: LINEAR ALGEBRA

DEPARTMENT: BS SOFTWARE ENGINEERING

SEMESTER : 2nd

SECTION: "A"

FINAL TERM.

Q.No: 1

Solution:

$$ID = 15958$$

$$ID_3 = 9$$

Putting in equation

$$x_1 - 9x_2 + x_3 = 0$$

$$2x_2 - 8x_3 = 8$$

$$5x_1 - 5x_3 = 10$$

Now changing in matrix form.

$$\left[\begin{array}{ccc|c} 1 & -9 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -9 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -10 & 3 \end{array} \right] \frac{1}{2} R_2 + 3R_3$$

$$\left[\begin{array}{ccc|c} 1 & -9 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 10 & 3 \end{array} \right] \frac{1}{2} R_3$$



This is perfect triangle

Hence consistent.

Q. 02
Solution:

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4+10 \\ 5 & -2 & 7 \end{bmatrix}$$

$$10 = 15958$$

$$10_4 = 5$$

$$\text{Let } Ab = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$$

First we find
cofactor of
every element of A.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix}$$

$$= 1 \cdot (-7 - (-10)) = 1 \cdot (-7 + 10)$$

$$\boxed{A_{11} = 3}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix}$$

$$= -1 \cdot (14 - 25)$$

$$\boxed{A_{12} = 11}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 1 \cdot (-4 - (-5)) = 1 \cdot (-4 + 5)$$

$$\boxed{A_{13} = 1}$$

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$$A_{21} = (-1)^{2+1}$$

$$\begin{bmatrix} 4 & 5 \\ -2 & 7 \end{bmatrix}$$

$$= -1 \cdot (28 - (-10)) = -1 \cdot (28 + 10)$$

$$\boxed{A_{21} = -38}$$

$$A_{22} = (-1)^{2+2}$$

$$\begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$$

$$= 1 \cdot (21 - 25)$$

$$\boxed{A_{22} = -4}$$

$$A_{23} = (-1)^{2+3}$$

$$\begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix}$$

$$= -1 \cdot (-6 - 20)$$

$$\boxed{A_{23} = 26}$$

$$A_{31} = (-1)^{3+1}$$

$$\begin{bmatrix} 4 & 5 \\ -1 & 5 \end{bmatrix}$$

$$= 1 \cdot (20 - (-5)) = 1 \cdot (20 + 5)$$

$$\boxed{A_{31} = 25}$$

$$A_{32} = (-1)^{3+2} \begin{bmatrix} 3 & 5 \\ 2 & 5 \end{bmatrix}$$

$$= -1 \cdot (15 - 10)$$

$$\boxed{A_{32} = -5}$$

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$$A_{33} = (-1)^{3+3}$$

$$\begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$$

$$= 1(-3 - 8)$$

$$\boxed{A_{33} = -11}$$

$$Ab = \begin{bmatrix} 3 & 11 & 1 \\ -38 & -4 & 26 \\ 25 & -5 & -11 \end{bmatrix}$$

$$\text{Adj } A = (Ab)^t$$

$$\text{Adj } A = \begin{bmatrix} 3 & -38 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}$$

Ans

Q.3:
Solution

Gauss-Jordan Method.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array} \right] \quad R_1 = \frac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array} \right] \times \left(\frac{1}{2}\right)$$

$$R_1/2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array} \right] \times \left(\frac{1}{2}\right)$$

$$R_2 - 1 \times R_1 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array} \right] \times (-3)$$

$$R_3 - 3 \times R_1 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array} \right]$$

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$$R_2/2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array} \right] \times (1)$$

$$R_3 - (-1) \times R_2 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & 11 \end{array} \right] \times (-1/9)$$

$$R_3 / 9 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array} \right]$$

$$R_1 - 2 \times R_3 \rightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -59/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array} \right] \times (-1)$$

ans

Q. 4
Solution

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$A = CDC^{-1}$$

$$\text{let } (A - \lambda I_3) = u$$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$= \begin{array}{c|c|c|c|c|c} 4-\lambda & 3-\lambda & 2 & -2 & -5 & 2 & -2 & -5 & 3-\lambda \\ & 4 & 1-\lambda & & -2 & 1-\lambda & & -2 & -4 \end{array}$$

$$= 4-\lambda((3-\lambda)(1-\lambda)-8) - 2(-5(1-\lambda)+4) - 2(-20+2(3-\lambda)) = 0$$

$$= 4-\lambda[3-3\lambda-\lambda+\lambda^2-8] - 2[-5+5\lambda+4] - 2[-20+6-2\lambda] = 0$$

$$= 4-\lambda[\lambda^2-4\lambda-5] - 2[5\lambda-1] - 2[-14-2\lambda] = 0$$

$$= \lambda^2 + 16\lambda - 26 - \lambda^3 + 4\lambda^2 + 5\lambda - 10\lambda + 2 + 28 + 4\lambda = 0$$

$$= -\lambda^3 + 8\lambda^2 + 15\lambda + 10 = 0$$

$$\lambda = 9.65$$

$$\lambda = 0.82$$

$$\lambda = -0.829$$

$$\text{for } \lambda = -9.65$$

$$P. 1 \dots$$

$$P-T-G$$

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$$A - \lambda I_3 = \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

For $A = -0.82$

$$A - \lambda I_3 = \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

In end only one 1 eigenspace
or 2 basis vectors are left.

So matrix A is not
diagonalizable.

$$\begin{aligned} \textcircled{1} \quad 3x_1 + 5x_2 - 4x_3 &= 0 \quad \text{---} \quad \textcircled{1} \\ -3x_1 - 25x_2 + 4x_3 &= 0 \quad \text{---} \quad \textcircled{2} \\ 6x_1 - 1x_2 - 8x_3 &= 0 \quad \text{---} \quad \textcircled{3} \end{aligned}$$

Solution

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Add or \ominus $\textcircled{1}$ and $\textcircled{2}$

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 25x_2 + 4x_3 &= 0 \end{aligned}$$

$$\begin{aligned} -20x_2 &= 0 \\ x_2 &= 0 \end{aligned}$$

Add or \ominus $\textcircled{1}$ and $\textcircled{3}$

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \end{aligned}$$

$$9x_1 + 6x_2 - 12x_3 = 0$$

$$9x_1 = 12x_3$$

$$x_1 = \frac{4}{3} x_3$$

$$\begin{bmatrix} x_1 = 4x_3 \\ x_2 = 0 \\ x_3 = \lambda \end{bmatrix}$$

Q6

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Maximum possible rank for matrix

$[A] = 0$ i) 3 if

rank no of non zero

$$[A] = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_3 - R_1$

$1 - 1 = 0$

$3 - 3 = 0$

$4 - 4 = 0$

$0 - 3 = -3$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$R_2 / 3$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$R_2 = R$

$1 - 1 = 0$

$3 - 3 = 0$

$4 - 4 = 0$

$3 - 3 = 0$