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Subject — Numerical analysis

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Question No 1

Euler's Method

$$\frac{dy}{dx} = 2x \; ; \quad y(0) = 1$$

$x_0 = 0$

$y = 1 \qquad h = 0.1$

$$f(x, y) = 2x$$
$$f(x_0, y_0) = 2x_0$$
$$f(x_1, y_1) = 2x_1$$
$$f(x_2, y_2) = 2x_2$$

As we know that

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$x_{n+1} = x_n + h$$

put $n = 0$

$$x_1 = x_0 + h$$
$$x_1 = 0 + 0.1$$

$\boxed{x_0 = 0.1}$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$ $\boxed{x_3 = 0.3}$

$\boxed{x_4 = 0.4}$

Now 1st Iteration

$y_{n+1} = y_n + h f(x_n, y_n)$

$y_{0+1} = y_0 + h f(x_0, y_0)$

$y_1 = 1 + 0.1\,(2(0)$

$y_1 = 1 + 0.1\,(0)$

$\boxed{y_1 = 1}$

2nd iteration

$y_2 = y_1 + h f(x_1, y_1)$

$$y_2 = 1 + 0.1(2(0.1))$$

$$y_2 = 1 + 0.1(0.2)$$

$$\boxed{y_2 = 1.02}$$

3rd iteration

$$y_3 = 1.2 + 0.1(2(0.2))$$

$$y_3 = 1.2 + 0.1(0.4)$$

$$y_3 = 1.2 + 0.04$$

$$\boxed{y_3 = 1.24}$$

Question No 2

Runge Kutta Method

$$\frac{dy}{dx} = x^2 + x - y$$

where = $V = 0$, $x = 0$, $h = 0.2$

Solution

$$\frac{dy}{dx} = x^3 + x - y$$

$$f(x, y) = x^2 + x - y$$

$$f(x_0, y_0) = x_0^2 + x_0 - y_0$$

$$f(x_1, y_1) = x_1^2 + x_1 y_1$$

$$f(x_2, y_2) = x_2^2 + x_2 - y_2$$

As we know that

$$x_{n+1} = x_n + h$$

$$y_{n+k} = y_n + k \longrightarrow (i)$$

Ans

$$x_{n+1} = x_n + h$$

put $n = 0$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.2$$

$$\boxed{x_1 = 0.2}$$

put $n = 1$

$$x_2 = 0.2 + 0.2$$

$$\boxed{x_2 = 0.4}$$

put $n = 2$

$$x_3 = 0.4 + 0.2$$

$$\boxed{x_3 = 0.6}$$

put $n = 3$

$$x_4 = 0.6 + 0.2$$

$$x_4 = 0.8$$

we know that

$k_1 = hf(x_0, y_0)$

$k_1 = 0.2(x_0^2 + x_0 - y_0)$

$k_1 = 0.2(0+0-0)$

$\boxed{k_1 = 0}$

Now k_2

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_2 = 0.2\left(\frac{0+0.2}{2}, 0 + \frac{0.2}{2}\right)$

$k_2 = 0.2\left(\frac{0.1}{x_0}, \frac{0.1}{y_0}\right)$

$k_2 = 0.2((0.1)^2 + (0.1) - 0.1)$

$k_2 = 0.2((0.01) + 0.1 - 0.1)$

$k_2 = 0.2(0.01)$

$\boxed{k_2 = 0.002}$

Now k_3

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_3 = 0.2\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$

$k_3 = 0.2\left(0.1^2, 0.001\right)$

x_0, y_0

Now

$k_3 = 0.2\left((0.1)^2 + (0.1) - (0.001)\right)$

$k_3 = 0.2\left(0.01 + 0.1 - 0.1\right)$

$k_3 = 0.2\left(0.109\right)$

$\boxed{k_3 = 0.0218}$

Now k_4

$k_4 = hf\left(x_0 + h, y_0 + k_3\right)$

$k_4 = 0.2\left(0 + 0.2, 0 + 0.0218\right)$

$k_4 = 0.2\left(0.2, 0.0218\right)$

x_0, y_0

$k_4 = 0.2\left((0.2)^2 + (0.2) - (0.0218)\right)$

$k_4 = 0.2\left(0.4 + 0.2 - 0.0218\right)$

$k_4 = 0.2\left(0.578\right)$

$\boxed{k_4 = 1.1564}$

$$\Delta k = \frac{1}{6}(k_1 + 2k_2 + 3k_3 + k_4)$$

$$k = 1/6\ (0.2 + 2(0.002) + 3(0.0218) + 1.1564)$$

$$k = 1/6\ (0.2 + 0.004 + 0.0654 + 1.1564)$$

$$k = 1/6\ (1.4258)$$

$$\boxed{\Delta k = 0.237}$$

QUESTION NO 3

Given Data

$a = 0 \quad b = 10, \quad n = 10$

$h = \frac{b-a}{n}, \quad \frac{10-0}{10} = 1$

Sol :-

x	0	1	2	3	4	5	6	7	8
f(x)	10.1	17.2	24.2	29.2	34.6	41.2	50.9	57.8	60.3

9	10
61.2	62.1

Now using Formula

$$dx = \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \cdots f(x_9) f(x_{10}) \right.$$

$$= \frac{1}{2} (10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 62.1)$$

$$= 412.9 \text{ ft}$$

QUESTION NO 4

$$\int_2^3 \ln(x^3+1)\,dx$$

$a=2$, $b=3$

Strips = 10

$$\Delta x = \frac{b-a}{2n}$$

$$\Delta x = \frac{3-2}{2(5)} = \frac{1}{10} = 0.1$$

x	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
$f(x)$	2.19	2.32	2.45	2.57	2.69	2.81	2.92	3.02	3.23	3.23	3.33

$$A = \frac{\Delta x}{3}\left[f(x)+2\left[f(x_2)+f(x_4)+f(x_6)+f(x_8)\right]+4\left[f(x_1)+f(x_3)+f(x_5)+f(x_7)+f(x_9)+f(x_{10})\right]\right]$$

$$A = \frac{0.1}{3}\Big[2.19 + 2\big[2.43 + 2.69 + 2.92 + 3.13\big]$$

$$+ 4\Big[2.32 + 2.57 + 2.81 + 3.02 + 3.23\Big] + 3$$

$$A = 0.03\big[2.19 + 22.38 + 55.8 + 3.33\big)$$

$$A = 2.511$$

$$\int_2^3 \ln(x^3 + 1)\,dx = 2.511$$