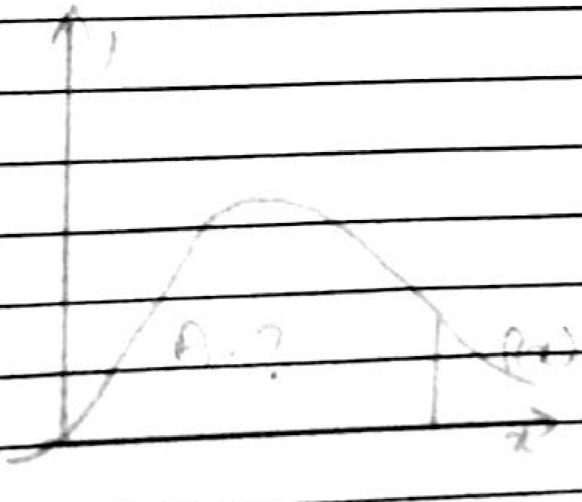
Ques 1:-

Review of integration Concept:

Intoduction to Integration:-

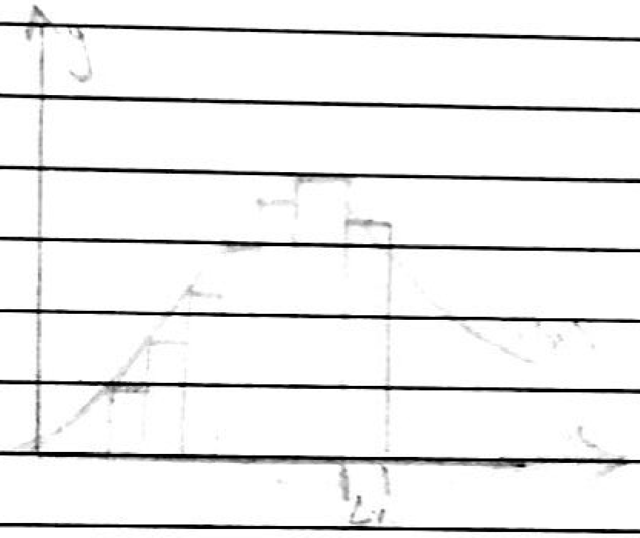
Integration is a way of adding slices to find the whole.

Integration can be used to find areas, volumes, centeral points and many usefull things. But it is easiest to start with finding the area under the curve of a function like this.

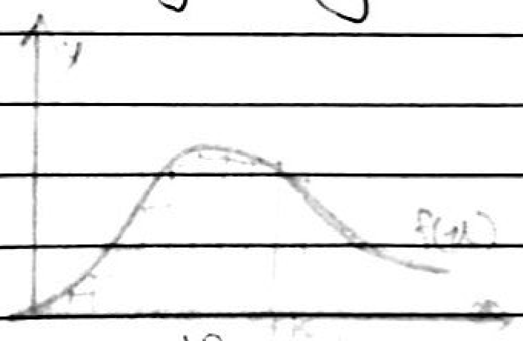


What is the area under $y = f(x)$?

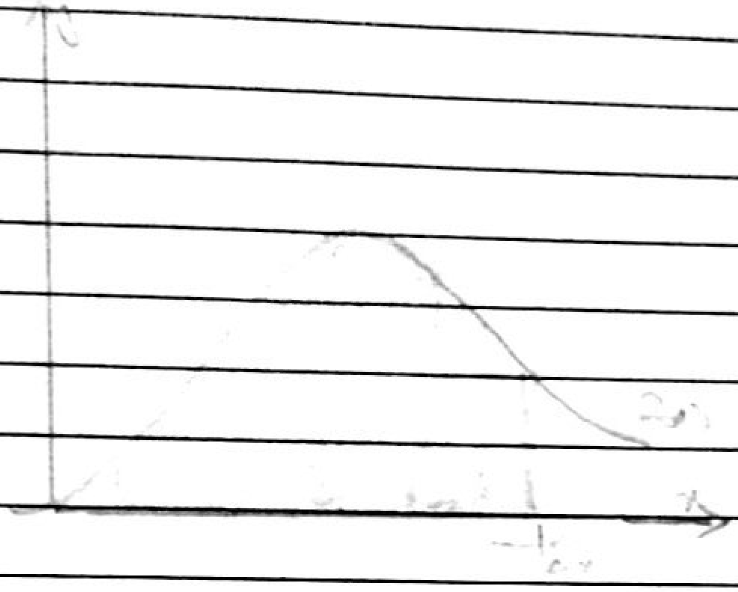
We could calculate the function at a few points and add up slices of width Δx like this (but the answer won't be very acute).



We can make Δx a lot smaller and add up many small slices (answer is getting better)



An as the slices approach zero in width the answer approches the true answer. we now write dx to mean Δx slices are approching zero.



Question 2:-

Application of Trapezoidal rule and Simpson's rule in Engineering

Trapezoidal Rule:-

We know from previous lesson that we can use Riemann Sums to evalute definite Integral $\int_{a}^{b} f(x)\,dx$

Riemanns Sums use rectangle to approximate the area under curve.

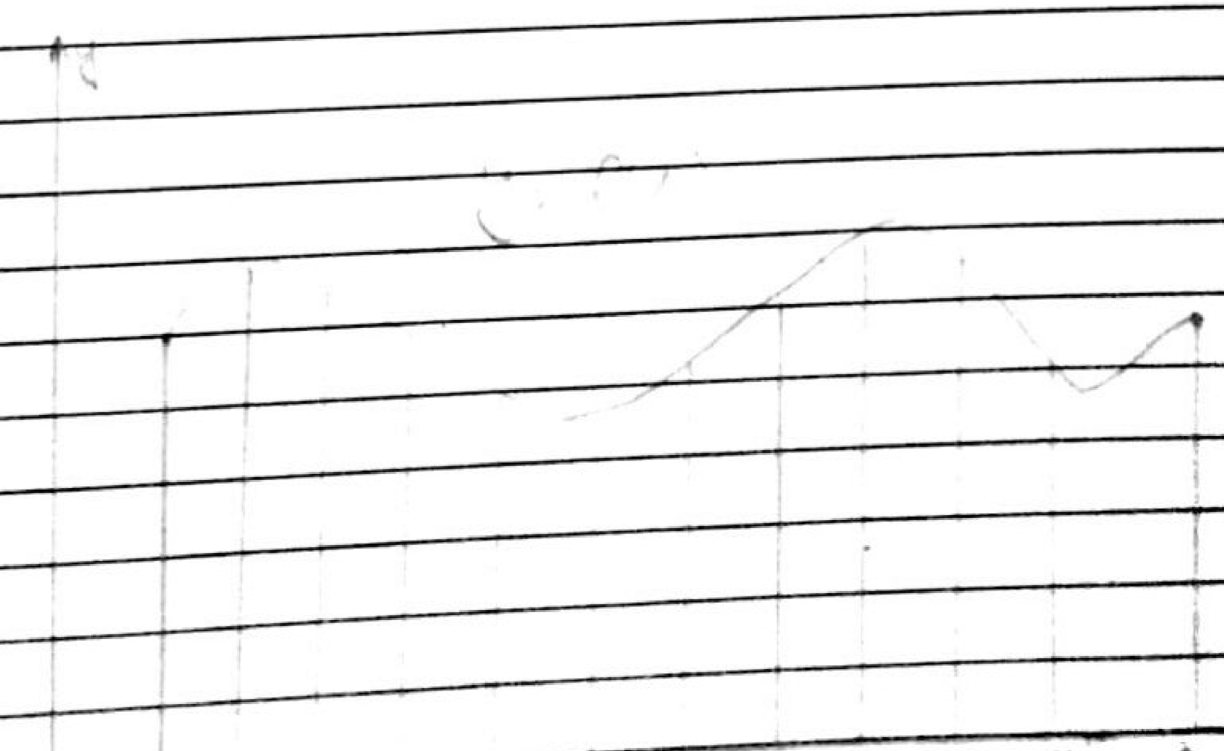
Another usefull integration rule is the Trapezoidal rule. Under this rule, the area under a curve is evaluated by dividing the total area into little trapezoids rather than rectangles.

Let $f(x)$ be continous on $[a, b]$. we partition the interval $[a, b]$ into n equal sub intervals, each of width.

$$\Delta x = \frac{b-a}{n}$$

Such that

$$a = x_0 < x_1 < x_2 < \dots < x_n = b$$



The trapezoidal rule for approximating $\int_a^b f(x)\, dx$ is given by

$$\int_a^b f(x)\, dx \approx T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1)] +$$
$$2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)],$$

where

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

As $n \to \infty$ the right-hand side of the expression approaches the definite integral

$$\int_a^b f(x)\, dx$$

Simpson Rule :-

It is a numerical method that approximate the value of definite integral by using "quadratic function"

This method is named after the English mathematician Thomas Simpson (1710 - 1761)

Simpson's rules is based on the fact that given three points, we can find the equation of a quadratic through those points.

To obtain an approximation of the definite integral $\int^{b} f(x)\,dx$ using Simpson's rule, we partition the interval $[a, b]$ into an even number n of subintervals, each of width

$$\Delta x = \frac{b-a}{n}.$$

One each pair of consecutive subintervals $[x_{i-1}, x_i]$, $[x_i, x_{i+1}]$, we consider a quadratic function $y = ax^2 + bx + c$ such that it passes through the points $(x_{i-1}, f(x_{i-1}))$, $(x_i, f(x_i))$, $(x_{i+1}, f(x_{i+1}))$

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If the function $f(x)$ is continous on $[a,b]$, Then

$$\int_a^b f(x)\,dx \approx \Delta x/3 \,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)$$
$$+ 2f(x_4) \ldots + 4f(x_{n-1}) + f(x_n)]$$

The coefficients in Simpson's rule have the following pattern

$$\underbrace{1, 4, 2, 4, 2, \ldots, 4, 2, 4, 1}_{n+1 \text{ points}}$$

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