Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	Marks 7 CLO 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	Marks 7 CLO 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6 CLO 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	Marks 6 CLO 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\left\lvert H\left(\frac{\pi}{4}\right)\right\rvert^2 = \frac{1}{2}$.	Marks 6 CLO 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	**Marks** **6** **CLO** **3**
Q 4	(a)	A finite duration sequence of Length L is given as $x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$ Determine the N- point DFT of this sequence for N ≥ L	**Marks** **6** **CLO** **2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $x_1(n) = \left\{\underset{\uparrow}{2}, 1, 2, 1\right\}$ $x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$	**Marks** **6** **CLO** **2**

Owais Afridi ID #13686

Question # 01

Part(a) $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

To the input $x(n) = (-1)^n u(n)$. intial Conditions are $y(-1) = y(-2) = 0$

Solution:

The characteristics equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \text{ hence}$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular Solution is

Substituting this solution into the difference eq,

we obtain

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2$

$$K(9) = 2$$

$$K = \frac{2}{9}$$

The total Solution is

$$y[n] = [C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n] u(n)$$

From the initial condition we obtain
$y(0) = 1$, $y(1) = 2$ then

$$c_1 - \frac{2}{9} = 1$$

$$c_1 = \frac{2}{9} - 1$$

$$= \frac{9-2}{9}$$

$$\boxed{c_1 = 7/9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$2\left(\frac{7}{9}\right) + 2c_2 = 2 + \frac{2}{9}$$

$$+2c_2 = \frac{20}{9} - \frac{14}{9}$$

$$2c_2 = \frac{\cancel{6}\,2}{\cancel{9}\,3}$$

$$c_2 = \frac{2}{3} \times 2$$

$$\boxed{c_2 = 4/3}$$

3

Question # 1

Part (B): Determine the impulse response and unit step response

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution: The characteristics is

$$\lambda^{n} - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0.$$

$$\lambda^{2-n}[\lambda^2 - 0.7\lambda + 0.1] = 0.$$

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda^2 - 0.5\lambda - 0.2\lambda + 0.1 = 0.$$

$$\lambda(\lambda - 0.5) - 0.1(\lambda - 0.5) = 0.$$

$$(\lambda - 0.5)(\lambda - 0.1) = 0$$

$$\lambda = 0.5, \quad \lambda = 0.1.$$

General form of the solution to be Homogenous equation

$$y_n(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

$$y_n(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{5}\right)^n$$

$$y(0) = 2$$

$$y_2(1) = 0.7y(0) = 0$$

$$y(1) = 1.4$$

(4)

Hence $c_1 + c_2 = 2$ and impluse response:

$$\frac{1}{2}c_1 + \frac{1}{5}c_2 = 1.4$$

$$= \frac{7}{5}$$

$$c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

These equation yield

$$c_1 = \frac{10}{3}$$

$$c_2 = -4/3$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

Step Response is:

$$S(n) = \sum_{k=0}^{n} h(n-k) \Rightarrow \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-1} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}.$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left[\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k\right.$$

$$= \frac{10}{3}\left[\left[\frac{1}{2}\right]^n (2^{n+1} - 1)\right] u(n) - \frac{1}{3}\left(\frac{1}{5}\right]^n (5^{n+1} - 1)\, u(n$$

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Question# 02

Part (a) Determine the Causal Signal ($x(n)$) having z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution: $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

= By partial fraction method:

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{—(1)}$$

Put $z = 1$

$$1 = A(1-1)^2 + B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$\boxed{C = -1}$$

⑥

Put $z = 2$ in eq ①

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$$\boxed{A = 4}$$

Put $z = 3$ in eq ①

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B(1/3)(2/3) + C(1/3)(1/3)$$

$$1 = \frac{4}{9}A + \frac{2}{9}B + \frac{1}{9}C$$

$$1 = \frac{4}{9}(4) + \frac{2}{9}B - \frac{1}{9}$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$-\frac{6}{9} \times \frac{9}{2} = B$$

$$\boxed{-3 = B}$$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$.

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Question # 02

Part(b): Evaluate the inverse z-transform

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Solution:

We have

$$x(n) = \frac{1}{2\pi j}\oint_c \frac{z^{n-1}}{1-az^{-1}}\,dz$$

$$= \frac{1}{2\pi j}\oint_c \frac{z^n\,dz}{z-a}$$

where C is a Circle at radius greater than $|a|$. We shall evaluate this integral using

$$\frac{1}{2\pi j}\oint_c \frac{f(z)}{z-z_0}\,dz = \begin{cases} \text{if } z_0 \text{ is incide } C \\ 0, \text{ if } z_0 \text{ is outside } C \end{cases}$$

with $f(z) = z^n$, We distinguish two cases.

1. If $n \geq 0$, $f(z)$ has only zeros and hence no poles incide C. The ohny pole inside C is $z = a$, Hence

$$x(n) = f(z_0) = a^n \qquad n \geq 0$$

a- If $n<0$, $f(z)=z^n$ has an nth-order pole at $z=0$, which is also inside c. Thus there are contributions from both poles.

For $n=-1$, we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz$$

$$= \frac{1}{z-a}\bigg|_{z=0} + \frac{1}{z}\bigg|_{z=a}$$

$$= \frac{1}{-a} + \frac{1}{a}$$

$$= 0$$

If $n=-2$, we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz$$

$$= \frac{d}{dz}\left(\frac{1}{(z-a)}\right)\bigg|_{z=0} + \frac{1}{z^2}\bigg|_{z=a}$$

$$= \frac{1}{-a^2} + \frac{1}{a^2}$$

$$= 0$$

By continuing in the same way we can show that $x(n)=0$ for $n<0$. Thus

$$x(n) = a^n u(n).$$

Question # 03

Part (a) A two-pole low pass filter has the System response

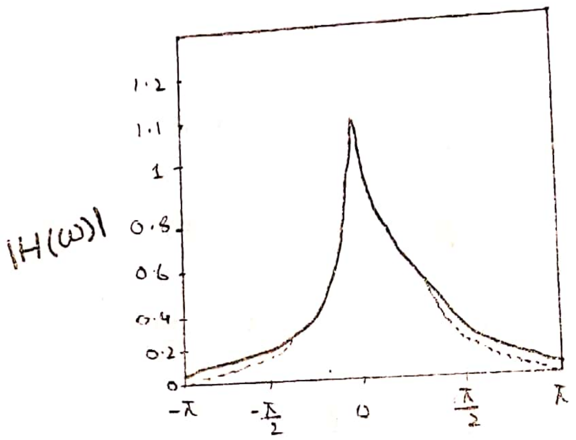
$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

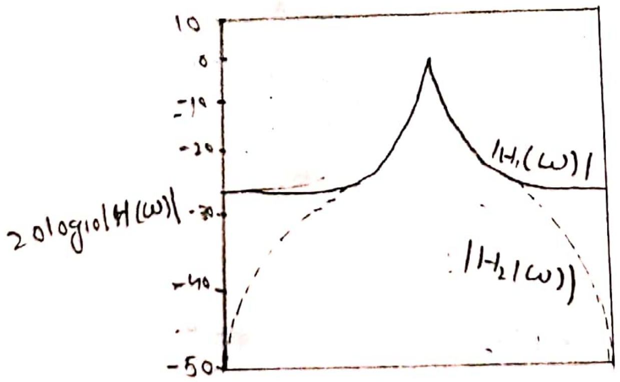
Determine the value of b_0 & P, Condition

$$H(0) = 1 \quad \& \quad \left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$$

Solution:

Linear Time-Invariant Systems as frequency-Selective filters





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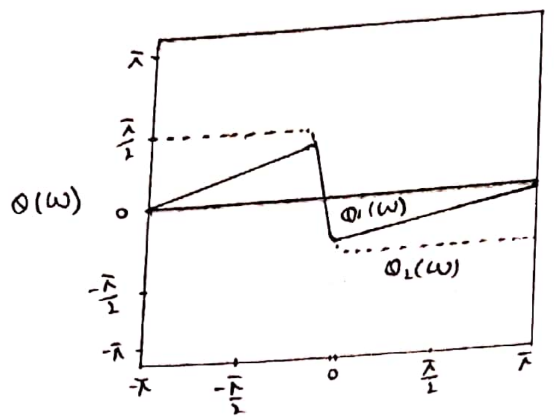


Magnitude of and phase response of (1) a sigle-pole filter and (2) a one-pole, one-zero filter:

$H_1(z) = (1-a)/(1-az^{-1})$

$H_2(z) = \{(1-a)/2\}[1+z^{-1}/(1-az^{-1})]$

$a = 0.9$

Now we have to determine the value of b_0 & p. Such that the frequency response $H(\omega)$ Satifies the Conditions.

$$H(0) = 1 \quad \text{and} \quad |H(\tfrac{\pi}{4})|^2 = \tfrac{1}{2}$$

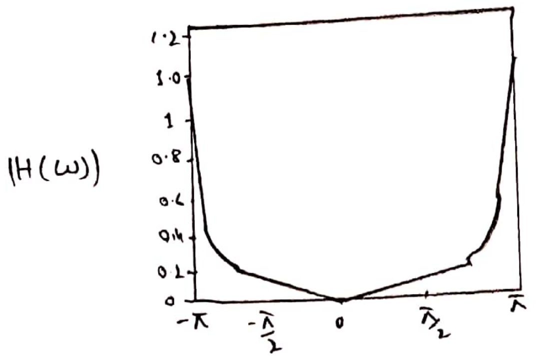
Sol At $\omega = 0$

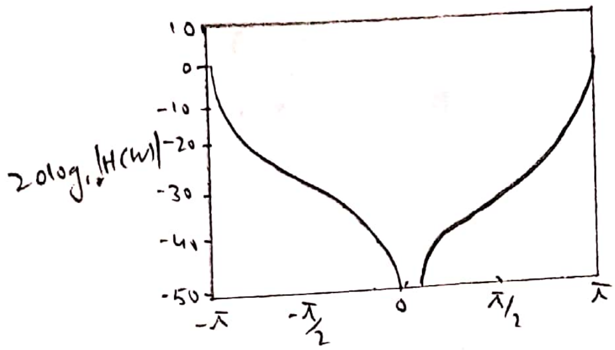
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

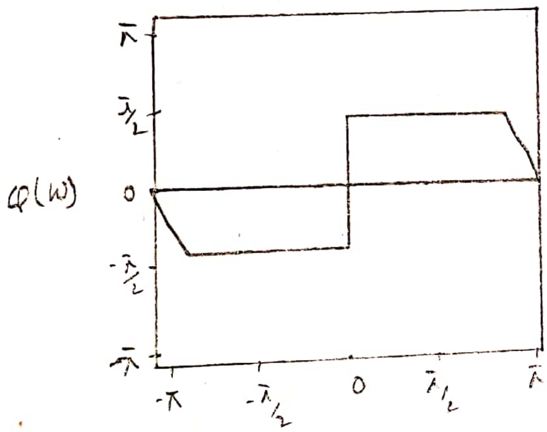
Both sides multiply by $(1-p)^2$

$$= \frac{b_0}{(1-p)^2} \times (1-p)^2 = 1 \times (1-p)^2$$

$$\therefore b_0 = (1-p)^2$$







Magitude & phase response of a simple highpass filter;

$$H(z) = \left[\frac{1-a}{2}\right]\left[\frac{1-z^{-1}}{1+az^{-1}}\right]$$

$a = 0.9$

at $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\frac{\pi}{4}) + jp\sin(\frac{\pi}{4}))^2}$$

$$= \frac{(1-p)^2}{\left(1-\frac{p}{\sqrt{2}} + \frac{jp}{\sqrt{2}}\right)^2}$$

= take Square on above equation

$$= \frac{([1-p]^2)^2}{\left(\left[1-\frac{p}{\sqrt{2}} + \frac{jp}{\sqrt{2}}\right]^2\right)^2}$$

$$H(z) = \frac{[1-p]^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

$$= \sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

the value of $p = 0.32$ satisfies this equestion.
desid. filter is:

$$H(z) = \frac{0.46}{(1-0.32\,z^{-1})^2}$$

Question#03

Part(B):

The filter must have poles at

$$P_{1,2} = re^{\pm j\frac{\pi}{2}}$$

and zeros at $z=1$ & $z=-1$.

Consequently, the system function is

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

The gain factor is determine by evaluating the frequency Response $H(\omega)$ of the filter at $\omega = \frac{\pi}{2}$, Thus we have

$$H\left(\frac{\pi}{2}\right) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$, Thus

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \cdot \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

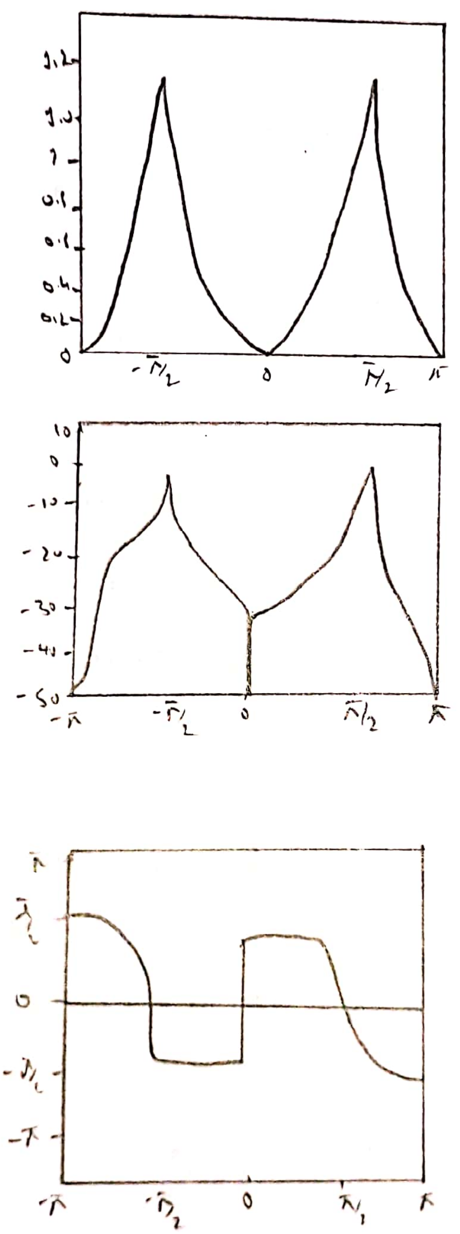
(14)

evaluating

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^2$$

The value of $r^2 = 0.7$ satisfies this equation.

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$



Question# 04 $x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$

Part (a)

Solution: The fouries transform of this sequence

$$x(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{L - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The mgnitude & phase of $x(\omega)$ are illustrated for $L = 10$. The N point DFT of $x(n)$ is simply $x(\omega)$ evaluted at the set of N equally spaced frequencies $\omega_k = \frac{2\pi k}{N}$

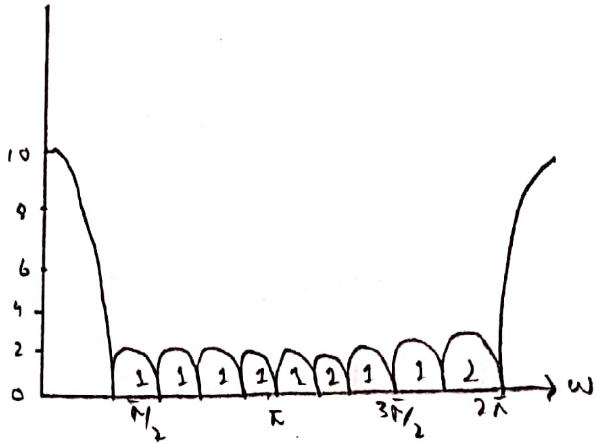
$k = 0, 1 \ldots\ldots N-1$

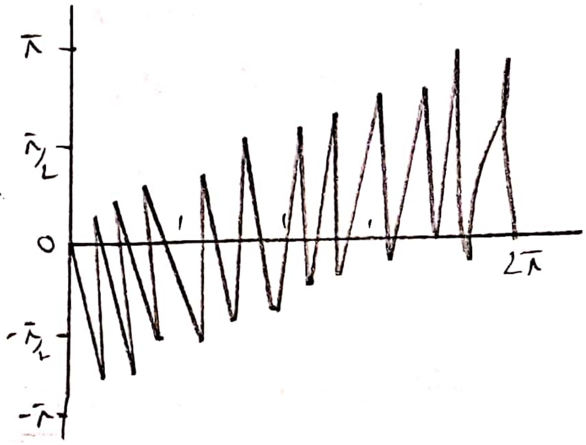
Hence

$$x(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \quad k = 0, 1, N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

16





If N is Selected Such that N=L then the DFT becomes

$$x(K) = \begin{cases} L & K=0 \\ 0 & K = 1, 2, 3 \ldots L-1 \end{cases}$$

Thus there is one non-zero value in DFT this is apparent from observation of X(ω). Since X(ω) = 0 at the frequency.

$\omega_k = \frac{2\pi K}{L}$ $K \neq 0$. The reader should verify that x(n) can be recovered from x(K) by Performing L-point IDFT.

Question# 04

Part(b): $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$ $x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

Solution:

Each Sequence Consist of four non-zero Points for the purpose of illustrating the operation involved in circulator convolution.

Now $X_3(m)$ obtained by circularly convolving $x_1(n)$ with $x_2(n)$.

With $m=0$, we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)\ N$$

$x_2(1-n)$ is simply the Sequence $x_2(n)$ the folded Sequence is simply $x_2(n)$.

The product Sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point finaly we Som the values

$$x_3(0) = 14$$

for $m = 1$ we have

$$X_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2[(1-n)]_4$$

It is easily verified $x_2[(1-n)]_4$ is the Simple Sequence $x_2[1-n]_4$ rotated counter clock wise by one unit in the time illustrated.

18

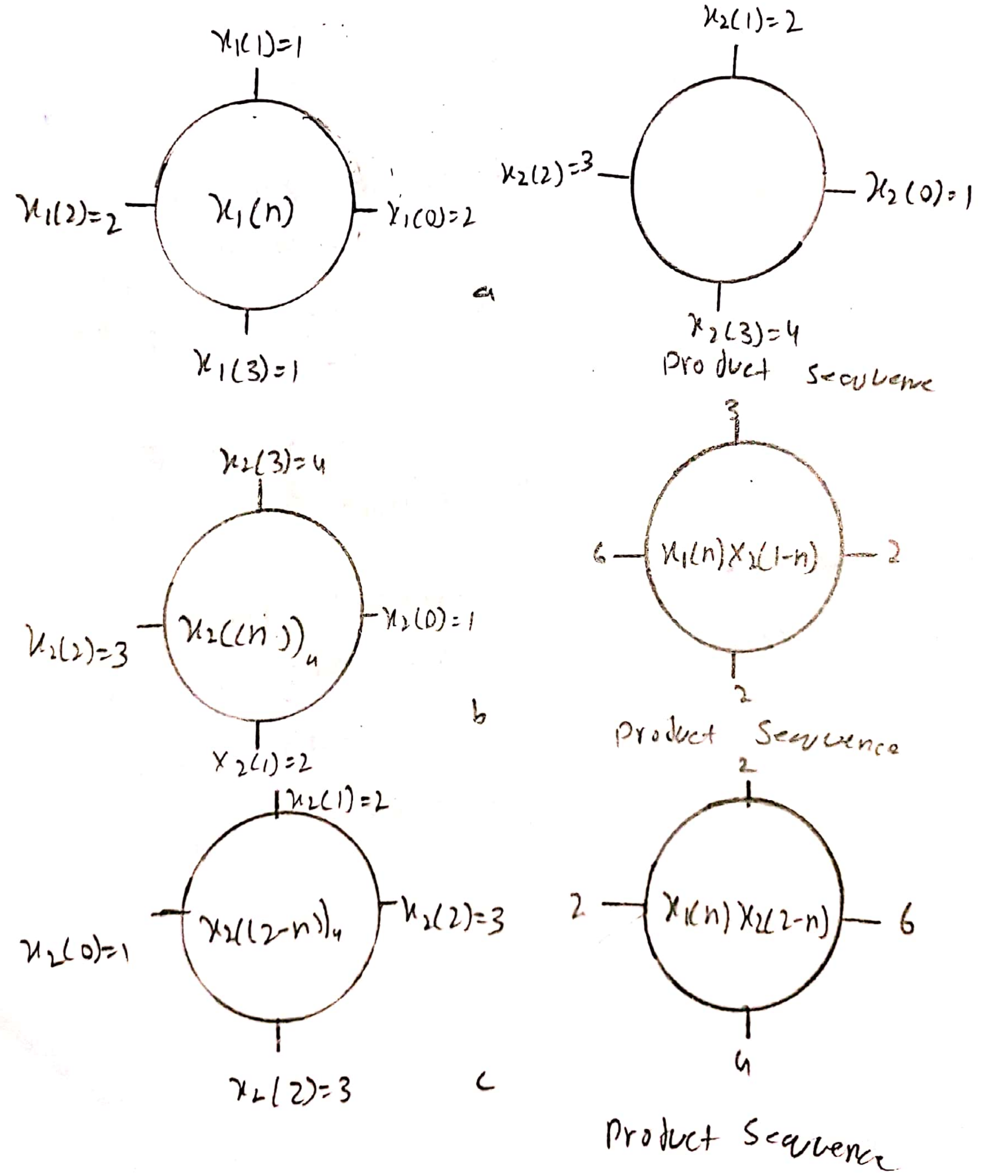
Finally, we sum the values:

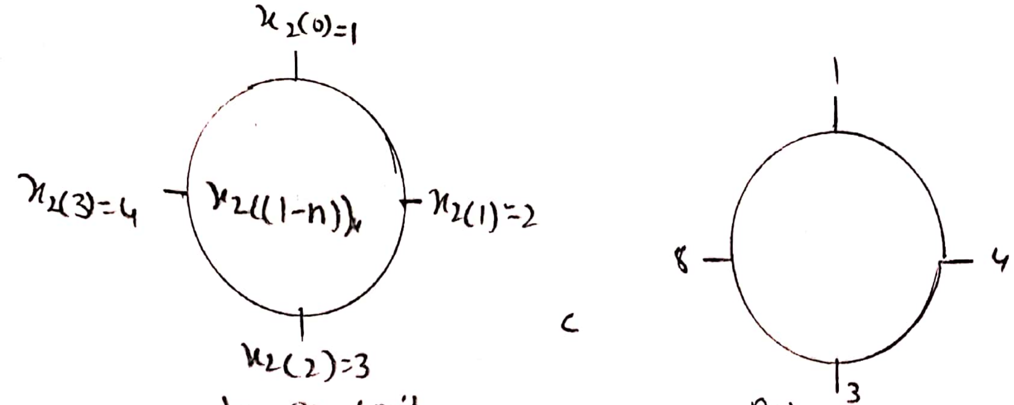
$X_3(1) = 16$

m=2 we have:

$$X_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

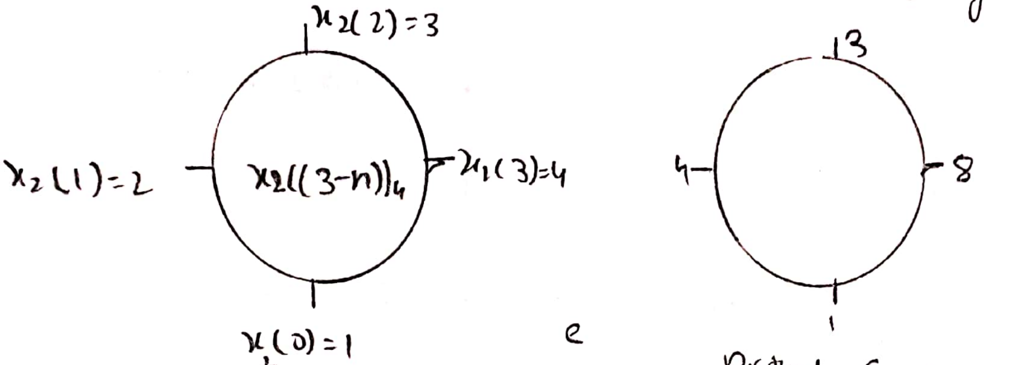
Now $X_2(2-n)_4$ is folded sequence. rotated two units of time in the counter clock wise direction.





by one unit

Product Sequency



Product Sequence.

Folde Sequence rotated by therea Units. along with product Sequence $x_1(n)x_2((2-n))_4$ By Suming the four terms in the product sequence.

$x_3(2) = 14$

m = 3, We have,

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded Sequence $x_2((n))_4$ is now three units in the time to yeild $x_2((3-n))_4$ is muctiplied by $x_1(n)$ to yeild the product.

Sequence $x_3(3) = 16$

We observe that if the Computation above is Contineud beyound $m=3$ we Simply repeat the Sequence of 4 values. There fore circular Convolution of two Sequences $x_1(n)$ and $x_2(n)$ yeid Sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}.$$