Course Code : MTH 107

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final term summer examination

Q1 find an equation of plane
Part (a) Passing through Point

$A(2,-2,1)$ $B(-1,0,3)$ $C(5,-3,4)$

Sol:- The non Parallel vectors

$\vec{AB} = (-3,2,2)$
$\vec{AC} = (3,-1,3)$

the Perpendicular vector is

$n = \vec{AB} \times \vec{AC}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$n = i(6+2) - j(-9-6) + k(3-6)$

$n = 8i + 15j - 3k$
$n = (8, 15, -3)$

Now $A(x_0, y_0, z_0) = (2,-2,1)$

$n(a,b,c) = (8,15,-3)$

So solution Plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$8(x-2)+15(y+2)-3(z-7)=0$$

$$8x+15y-3z-16+30+3=0$$

$$8x+15y-3z+17=0$$

Q1:- Express a Pair of Planes whose intersection is the given line

(b)

$$x = 2 - 3t$$
$$y = 3 + t$$
$$z = 2 - 4t$$

Sol:- $x - 2 = -3t$

$$t = \frac{x-2}{-3}$$

$$y - 3 = t$$
$$t = y - 3$$

$$z - 2 = -4t$$

$$t = \frac{z-2}{-4}$$

So $\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$

for 1st Plane take 1st & 2nd

$$\frac{x-2}{-3} = \frac{y-3}{1}$$

$$x - 2 = -3y + 9$$
$$x + 3y - 11 = 0$$

for 2nd Plane take 1st & 3rd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

or

$$4x-3z-2=0$$

Q2:- $L(x,y) = (x+1, y, x+y)$

illustrate that L is Linear transformation

a) $L(x,y) = (x+1, y, x+y)$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$u+v = (x_1, y_1) + (x_2, y_2)$

$u+v = (x_1+x_2, y_1+y_2)$

$L(u+v) = L(x_1+x_2, y_1+y_2)$

$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$ (i)

given that $u = (x_1, y_1)$

$L(u) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$

$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$

$L(u)+L(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_1+y_2)$ (ii)

Since $1 \neq 2$

So not L.T

(Q3) Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

then interpret to decode the message 77 54 38, 71, 49, 29, 68, 51, 33, 76, 48, 40, 86, 53, 52

Sol $\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}$

$= \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$

$L : R^3 \longrightarrow R^3$

$L(x) = AX$

$$A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 77 & 71 & 68 & 76 & 86 \\ 54 & 49 & 51 & 48 & 53 \\ 38 & 29 & 33 & 40 & 52 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 299 & 256 & 269 & 292 & 348 \\ 207 & 178 & 185 & 204 & 243 \\ 130 & 107 & 117 & 128 & 157 \end{bmatrix}$$

b) To decode the message

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$AX = LX$

$A^{-1} \times A \cdot B$

$$= \begin{bmatrix} 77 & 71 & 68 & 76 & 86 \\ 54 & 49 & 57 & 48 & 53 \\ 38 & 29 & 33 & 40 & 52 \end{bmatrix}$$

so this is the resulting

Q4:-

fin an equation of the Plane Passing through the Point $(-1,3,2)$ and Perpendicular to the vector $n=(0,1,-3)$

Solution:- $(-1,3,2)$ $n=(0,1,-3)$

Equation of the Plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0 \quad \text{(i)}$$

Given that $P(x_0,y_0,z_0)=(-1,3,2)$

$n(a,b,c)=(0,1,-3)$

Putting values in eq (i)

$$0(x-(-1))+1(y-3)+(-3)(z-2)$$

$$= 0+1(y-3)-3z+6$$

$$= 0+y-3-3z+6$$

$$= 0x+y-3z+3=0$$

$$= 0x+y-3z=-3$$

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Q5:- find Eigen values of Matrix & Eigen vectors of Matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Sol:- we know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$x_1 + x_2 = \lambda x_1$ — (i)

$-2x_1 + 4x_2 = \lambda x_2$ — (ii)

So $x_1 - \lambda x_1 + x_2 = 0$

$= (1-\lambda) x_1 + x_2 = 0$

& $-2x_1 + 4x_2 - \lambda x_2 = 0$

$= -2x_1 + (4-\lambda) x_2 = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

characteristic equation

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2=0$$

$$4-\lambda-4\lambda+\lambda^2+2=0$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda^2-3\lambda-2\lambda+6=0$$

$$\lambda(\lambda-3)-2(\lambda-3)=0$$

$$(\lambda-3)(\lambda-2)=0$$

$$\lambda-3=0 \quad \lambda-2=0$$

$$\lambda_1=3 \quad \lambda=2$$

are eigen values

Now find eigen vectors of $\lambda_1 = 3$ Put in (i) & 2

then $x_1 + x_2 = 3x_1$ — (i)

$= -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2$ — (ii)

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x_2$

let $x_2 = r$

wher $r \neq 0$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} r \\ r \end{bmatrix}$

eigen vector for $\lambda_2 = 2$ put in i & 2

$x_1 + x_2 = 2x_1$ — (i)

$-2x_1 + 4x_2 = 2x_2$ — (ii)

$= -x_1 + x_2 = 0$ (i)

$\Rightarrow x_1 - x_2 = 0$

$= x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2$ (ii)

$= -2x_1 + 2x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = r$ then $x_2 = r$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$