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Sec # A

Subject # Numerical Analysis

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"MID Term Exam"

(Q1) Find the root of the equation:-

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use newton raphson Method with $x_0 = 0.02$

Solution:-

Rearranging the equation

$$x^3 - 0165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) \Rightarrow 3x^2 - 0.33x = 0$$

$$\rightarrow x_1 = x_0 \frac{-f(x_0)}{f'(x_0)}$$

★ $f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$$\rightarrow x_2 = x_1 \frac{-f(x_1)}{f'(x_1)}$$

★ $f(x_1) = -0.00016$

$f'(x_1) = -0.00070$

$$= 0.081 - \frac{-0.00016}{-0.0070}$$

$$\boxed{x_2 = 0.058}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

★ $f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

$$= 0.058 - \frac{0.000030}{-0.0090}$$

$$\boxed{x_3 = 0.061}$$ Ans

Q2) Use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation polynomial for $f(x) = 1/x$ at $x = 3$.

Solution:-

Putting the values in the function

$y_0 = 0.5$, $y_1 = 0.364$, $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\Rightarrow y = 0.33$$ Ans.

Q No.3 :- Complete the divided difference table for the given data and construct the interpolating polynomial that uses all this data.

Solution:-

$x =$ 1.0	1.3	1.6	1.9	2.2
$y =$ 0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3} \ldots x_i)$	$f(x_{i-4} \ldots x_i)$
x_0 1	0.7651977				
		-0.4837056			
			-0.108734		
x_1 1.3	0.6200860			0.0658785	
		-0548946			[illegible]
					-0.0028049
x_2 1.6	0.4554022		-0.0494433		
		-0.578612			
				0.06251255	
x_3 1.9	0.2818186		0.006818		
		-0.571521			
x_4 2.2	0.1103623				

1) $f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \frac{0.6200860 - 0.7651977}{1.3 - 1}$

$f(x_0, x_1) = -0.4837056$

2) $f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$f(x_1, x_2) = 0.548946$

3) $f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$= 0.578612$

4) $f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$f(x_3, x_4) = -0.571521$

∴ Second divided difference :-

$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$= \frac{0.548946 - (-0.4837056)}{1.6 - 1}$

$f(x_0, x_1, x_2) = -0.108734$

P.t.o

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$f(x_2, x_3, x_4) = 0.006818$$

Third Divided Difference

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (-0108734)}{1.9 - 1}$$

$$f(x_0, x_1, x_2, x_3) = 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.0494433)}{2.2 - 1.3}$$

$$f(x_1, x_2, x_3, x_4) = 0.06251255.$$

4th Divided Difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= -0.0028049$$

P.t.o

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$$f(x) = f(x_0) + (x+x_0) f(x_0, x_1) + (x-x_0)(x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)$$
$$(x-x_2)(x-x_3) f(x_0, x_1, x_2, x_3, x_4).$$

$$= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)$$
$$(-0.108734) + (x-1)(x-1.3)(x-1.6)(0.065885)$$
$$+ (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049).$$