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Qno 1

Apply both Euler's method and the improved Eulers method to the solution of
$$\frac{dy}{dx} = 2x ; \quad y(0) = 1$$

For $0 \leq x \leq 0.5$ using $h = 0.1$. Compare your answer with the analytic solution. Work throughout to three decimal places

Ans:-

By Euler's Method:-

Given Data:-

$y(0) = 1$, $h = 0.1$ $x_0 = 0$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$
$$y_{n+1} = y_n + h(2x_n)$$

1^{st} Interation :-

$h = 0$

$y_1 = y_0 + h(2 \times 0)$

$y_1 = 1 + 0.1(2(0))$

$y_1 = 1 + 0.1$

$\boxed{y_1 = 1.0}$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$\boxed{x_1 = 0.1}$

"2nd Iteration":-

$n = 1$

$y_2 = y_1 + h\ (2x_1)$

$y_2 = 1.1 + 0.1\ (2\ (0.1))$

$\boxed{y_2 = 1.02}$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

"3rd Iteration":-

$n = 2$

$y_3 = y_2 + h\ (2x_2)$

$y_3 = 1.12 + 0.1\ (2\ (0.2))$

$\boxed{y_3 = 1.06}$

$x_{n_1} = x_n + h$

$x_3 = x_2 + 0.1$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

(b) By Modified Euler method

$$\frac{dy}{dx} = 2x$$

Given Data:

$y_0 = 1, \ x_0 = 0, \ h = 0.1$

Formula.

$y_{n+1} = y_n + h\,[f(x_n)]$

$y_{n+1} = y_n + h\,(2x_n)$ —— (1)

$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$

$y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$

$y_n + \frac{h}{2}\left[4x_n\right]$

1st Iteration

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$Y_1 = Y_0 + \frac{h}{2}(4x_0)$

$Y_1 = 1 + \frac{0.1}{2}(4(0))$

$\boxed{Y_1 = 1}$

2nd Iteration

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$Y_2 = Y_1 + \frac{h}{2}(4x_1)$

$Y_2 = 1 + \frac{0.1}{2}(4(0.1))$

$\boxed{Y_2 = 1.02}$

3rd Iteration

$n = 2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$Y_3 = Y_2 + \frac{h}{2}(4x_2)$

$= 1.02 + \frac{0.1}{2}(4(0.2))$

$\boxed{Y_3 = 1.06}$

Qno# 2

Use the fourth-order Runge Kutta method to obtain a solution of

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y=0$ when $x=0$, for $0 \le x \le 0.6$ with $h = 0.2$. Work throughout to four decimal places

Given Data:-

$y=0$, $x=0$, $h=0.2$ $\quad 0 \le x \le 0.6$

$y_{n+1} = y_n + k$

"1st Iteration:"

$n=0$

$y_1 = y_0 + k$, $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2(0^2 - 0 - 0)$

$$\boxed{k_1 = 0}$$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$

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$0.2 f\left(x_0 + h/2, y_0 + h/2\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 0.1)$

$= 0.2\left(0.1^2 + \cancel{0.1} - \cancel{0.1}\right)$

$\boxed{k_2 = 0.0020}$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$

$= 0.2 f(0.1, 0.001)$

$= 0.2\left(0.1^2 + 0.1 - 0.001\right)$

$\boxed{k_3 = 0.0218}$

$k_4 = hf\left(x_n + h, y_n + k_3\right)$

$= 0.2 f(0 + 0.2, 0 + 0.0218)$

$= 0.2 f(0.2, 0.0218)$

$= 0.2\left(0.2^2 + 0.2 - 0.0218\right)$

$\boxed{k_4 = 0.0436}$

$$K = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right)$$

$$K = 0.0152$$

$$Y_1 = 0 + 0.0152$$

$$\boxed{Y_1 = 0.0152}$$

Q no 3:-

A rocket is released and travels at a variable speed v. A motion sensor on the rocket measures this speed and the value is sampled by an on-board computer at 1 second interval. The computer is required to calculate the distance travelled by the rocket and rely the value to a ground station at regular intervals. Record values of the measurement taken by the computer during the first 10 seconds of flight. Assuming that the computer uses the trapezium rule to estimate the distance travelled by the rocket, calculate the value that the computer will relay to the ground station after 10 seconds.

Time	0	1	2	3	4	5	6	7	8	9	10
Speed	10.1	17.2	24.4	29.2	34.6	41.23	50.9	57.8	60.3	61.2	62.1

Given Data:-

$a = 0$, $b = 10$, $n = 10$

$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$

Using formula

$$f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3) \ldots\ldots f(x_9) \right) + f(x_{10}) \right]$$

$$= \frac{1}{2}\left[10.1 + 2\left(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 62.1 + 61.2 \right]\right.$$

$$\boxed{= 412.9 \quad \text{Ans}}$$

Qno4

Estimate the values of the following integral using Simpson's rule

$$\int_2^3 \ln(x^3+1)\,dx$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now formula

$$\int_a^b f(x)dx = \frac{h}{3}\Big[f(x_0)+4\big[f(x_1)+f(x_3)\cdots\big]+2\big[f(x_2\ldots)\big) + f(x_n)\Big]$$

$$\frac{0.1}{3}\Big[0.693+4(0.846+1.162+1.476+1.777)+2(1.003+1.320+1.628+1.922)+2.062\Big]$$

$$\boxed{1.184 \text{ Ans}}$$