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Question No: 01

b.

Solution:
$$\int_0^1 x^3(1+x^4)^3\,dx$$

$$= \int_0^1 (1+x^4)^3 x^3\,dx$$

Let $u = 1+x^4$

$$\frac{du}{dx} = \frac{d}{dx}(1+x^4)$$

$$\frac{du}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}x^4$$

$$\frac{du}{dx} = 4x^3 \Rightarrow \frac{1}{4}du = x^3dx$$

or $x^3dx = \frac{1}{4}du$

Now Putting values, we get

$$\int_0^1 u^3 \cdot \frac{1}{4}du \Rightarrow \frac{1}{4}\int_0^1 u^3 du$$

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$$= \frac{1}{4} \cdot \left(\frac{1}{4} u^4 \Big|_0^1 \right) \cdot \frac{1}{16} \left(u^4 \Big|_0^1 \right)$$

$$= \frac{1}{16} (1)^4 - (0)^4$$

$$= \frac{1}{16} (1) - 0$$

$$= \frac{1}{16}$$ Ans:

Question No: 01

A.

Solution:

$$\int \theta^4 \sqrt{1-\theta^2}\, d\theta$$

Let $\theta^4 \sqrt{1-\theta^2}\, d\theta$

$$\frac{d}{d\theta}(1-\theta^2) = u$$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta} u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta\, d\theta = -\frac{1}{2} du$$

Now

$$= \int (u)^{\frac{1}{4}} \cdot \left(-\frac{1}{2}\right) du$$

$$= -\frac{1}{2} \int u^{\frac{1}{4}} du$$

$$= \frac{1}{2} \cdot \frac{4}{5} u^{5/4} + c$$

$$= -\frac{2}{5} u^{5/4} + c$$

by back substitition, we get

$$= -\frac{2}{5} (1-\theta^2)^{5/4} + c \quad \underline{\underline{\text{Ans:-}}}$$

Question No: 02

a.

Solution:

$$x^2+y^2+z^2+3x-4z+1$$

So $x^2+y^2+z^2+3x-4z+1=0$

$$(x^2+3x)+y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right) = -1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x^2+\frac{3}{2}\right)^2+(y-0)^2+(z-2)^2 = \frac{21}{4}$$

So $(x_0, y_0, z_0) =$ Centre

$$= \left(-\frac{3}{2}, 0, 2\right)$$

&

Radius $\left[a = \sqrt{\frac{21}{4}}\right]$ $\underline{\underline{\text{Ans:-}}}$

Question No: 02

b.

Solution:

Given that $y = \sqrt{x}$

$0 \leq x \leq 4 \Rightarrow a \leq x < b$

$$V = \int_a^b \pi y^2 dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \frac{x^2}{2}$$

$$V = \frac{\pi}{2} \left[(4)^2 - 0 \right]$$

$$V = 8\pi$$ Ans.

Question No: 03

Solution:

$A = 2i - 4j + \sqrt{5}K$

$B = -2i + 4j - \sqrt{5}K$

Projection $AB = ?$

As we Know that

By dot Produt

$B \cdot A = (-2i + 4j - \sqrt{5}K) \cdot (2i - 4j + \sqrt{5}K)$

$= ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$= -4 - 16 + \sqrt{-5 \times 5}$

$= -4 - 16 - 5$

$B \cdot A = -25$

Now

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= ((2)(2) + (-4)(-4) + (\sqrt{5})(\sqrt{5}))$

$= 4 + 16 + \sqrt{5 \times 5}$

$= 4 + 16 + \sqrt{25}$

$= 4 + 16 + 5$

$A \cdot A = 25$

So

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

Putting values, we get

$$= \left(\frac{-\cancel{25}}{\cancel{25}}\right)(2i - 4j + \sqrt{5}K)$$

$$= (-1)(2i - 4j + \sqrt{5}K)$$

$Proj_A B = -2i + 4j - \sqrt{5}K$ Ans:

Question No: 04

Solution:

Given that

$$y = -x^2 + 5x - 4$$

& $[a, b] = [0, 2]$

As $a = 0$

$b = 2$

So

Area under graph will be.

$$A = \int_a^b f(x)\,dx$$

putting values,

$$= \int_0^2 (-x^2 + 5x - 4)\,dx$$

by solving integration, we get

$$A = \left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_0^2$$

$$A = \left(\frac{-1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2)\right) - (0)^2$$

$$A = \left(\frac{-1}{3}(8) + \frac{5}{2}(4) - 8\right)$$

$$A = \frac{-8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-8}{3} + \frac{20}{2} - \frac{8}{1}$$

$$A = \frac{2\times -8 + 3\times 20 - 6\times 8}{6}$$

$$A = \frac{-16 + 60 - 48}{6}$$

$$A = \frac{60-64}{6}$$

$$A = \frac{4}{6} = \frac{2}{3}$$

$A = 0.666$ Ans:-

Question No: 05

a.

Solution:

$$A = i - 2j - 2k$$
$$B = 6i + 3j + 2k$$

$A = i - 2j - 2k$

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$
$$= \sqrt{1 + 4 + 4}$$
$$= \sqrt{9}$$

$|A| = 3$

Now $B = 6i + 3j + 2k$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$
$$= \sqrt{36 + 9 + 4}$$
$$= \sqrt{49}$$

$|B| = 7$

So

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$$

$$\theta = \cos^{-1}\left\{\frac{(i - 2j - 2k)\cdot(6i + 3j + 2k)}{3 \times 7}\right\}$$

$$\theta = \cos^{-1}\left\{\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right\}$$

$$\theta = \cos^{-1}\left(\frac{\cancel{6}-\cancel{6}-4}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\theta = 100.97 \quad \underline{\text{Ans:-}}$$

Question No: 05

b.

Solution: $x^2+y^2+(z-1)^2=1$

$$(\rho \sin\theta \cos\theta)^2+(\rho \sin\theta \sin\theta)^2+(\rho\cos\theta-1)^2=1$$

$$\rho^2 \sin^2\theta \cos^2\theta+\rho^2 \sin^2\theta \sin^2\theta+\rho^2 \cos\theta+1-2\rho\cos\theta=1$$

$$\rho^2 \sin^2\theta(\cos^2\theta+\sin^2\theta)+\rho^2 \cos^2\theta+1-2\rho\cos\theta=1$$

$$\rho^2(\sin^2\theta+\rho^2\cos^2\theta+2\cdot\rho\cos\theta=1-1$$

$$\rho^2(\sin^2\theta+\cos^2\theta)-2\rho\cos\theta=0$$

$$\rho^2=2\rho\cos\theta$$

$$\rho=2\cos\theta \quad \underline{\text{Ans:-}}$$

"The END"