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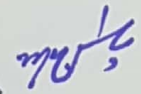
Registration No: 7544

Subject: MOS-II

Instructor name: Engr. Muhammad Saqib

"IQRA National University Peshawar"

Date: 16-04-2020

Sign: 

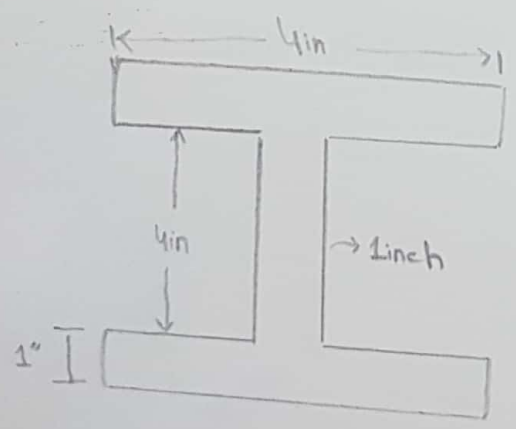
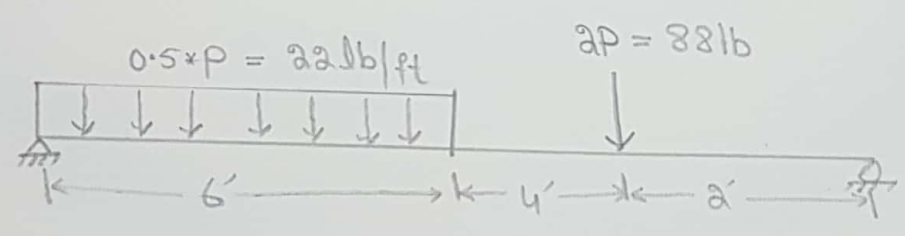
Q Construct the Mohr's Circle diagram and find principle stresses and maximum in plane shear stress for the stress of point C located at the center of uniformly distributed load and 1 inch below the top fiber of beam cross section shown in fig. However construct the Mohr circle it is necessary to draw shear stress and flexural stress variation diagram for maximum shear force and bending moment respectively. Compare result from Mohr circle with stress transformation evaluation.

Note :- I.D is 7544

$$0.5P = 0.5 \times 44 = 22 \text{ lb/ft}^2$$

$$2P = 2 \times 44 = 88 \text{ lb}$$

So load is 44 lb.
 $P = 44 \text{ lb}$



① Solution:-

$$R_A + R_B = 220 \text{ lb}$$

* first find reaction forces
 R_A & R_B .

(2)

$$\sum M_A = 0$$

$$-(22 \times 6 \times 3) - (88 \times 10) + 12R_B = 0$$

$$R_B \times 12 - 1276 = 0$$

$$R_B = \frac{1276}{12}$$

$$\boxed{R_B = 106.33 \text{ lb}}$$

now we know that

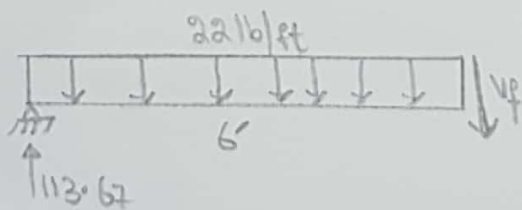
$$R_A + R_B = 220 \text{ lb}$$

$$R_A = 220 - R_B$$

$$R_A = 220 - 106.33$$

$$\boxed{R_A = 113.67 \text{ lb}}$$

* Now Shear force at change point of beam



Shear force at 6ft from left

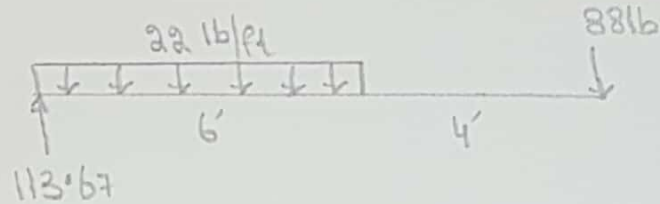
$$\sum F_y = 0 \uparrow +$$

$$113 - (22 \times 6) - V_{f6} = 0$$

$$113.67 - 132 = V_{f6}$$

$$V_{f_{10ft}} = -18.33 \text{ lb}$$

* Now Shear at 10ft

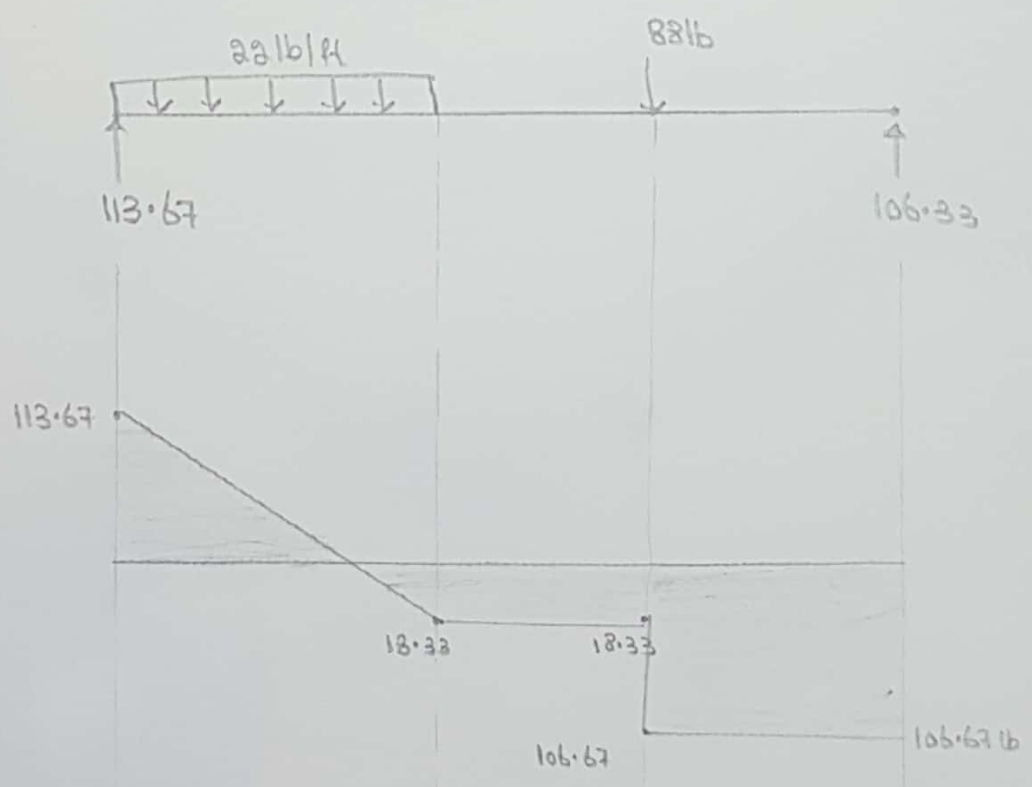


$$\sum F_y = 0 \uparrow$$

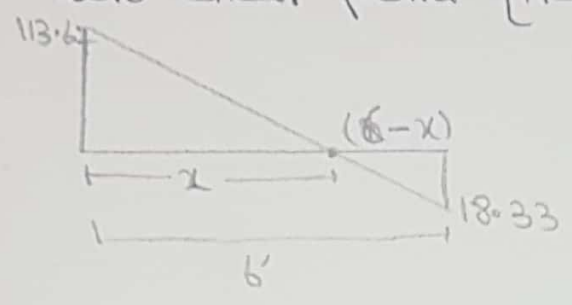
$$113.67 - (22 \times 6) - 88 - V_{f_{10'}} = 0$$

$$V_{f_{10ft}} = 106.33 \text{ (-ve)}$$

Shear force and Moment diagram



* Moment at Change Point
find zero shear point first



by similar triangles.

$$\frac{113.67}{x} = \frac{18.33}{(6-x)}$$

$$682.02 = 113.67x + 18.33x$$

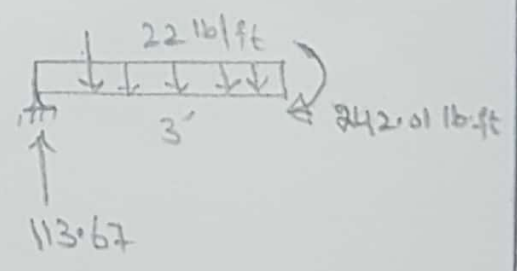
$$x = \frac{682.02}{132}$$

$$x = 5.17 \text{ ft}$$

(i) at center of UDL 3ft from left.

$$(+\curvearrowright) M = -113.67 \times 3 + (22 \times 3 \times 1.5)$$

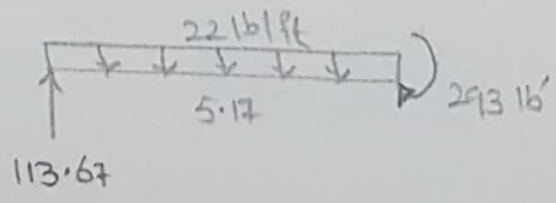
$$M = 242.01 \text{ lb}\cdot\text{ft}$$



(ii) at a distance 5.17 from left support

$$(+\curvearrowright) M = -113.67 \times 5.17 + (22 \times 5.17 \times \frac{5.17}{2})$$

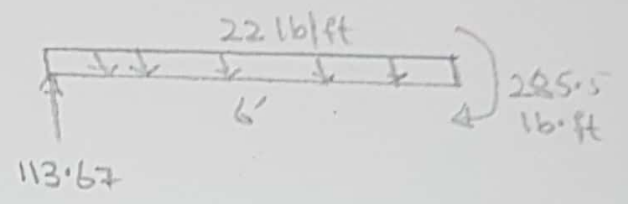
$$M = 293 \text{ lb}\cdot\text{ft}$$



(iii) at distain 6ft from left.

$$\curvearrowright M = (-113.67 \times 6) + (22 \times 6) \times 3$$

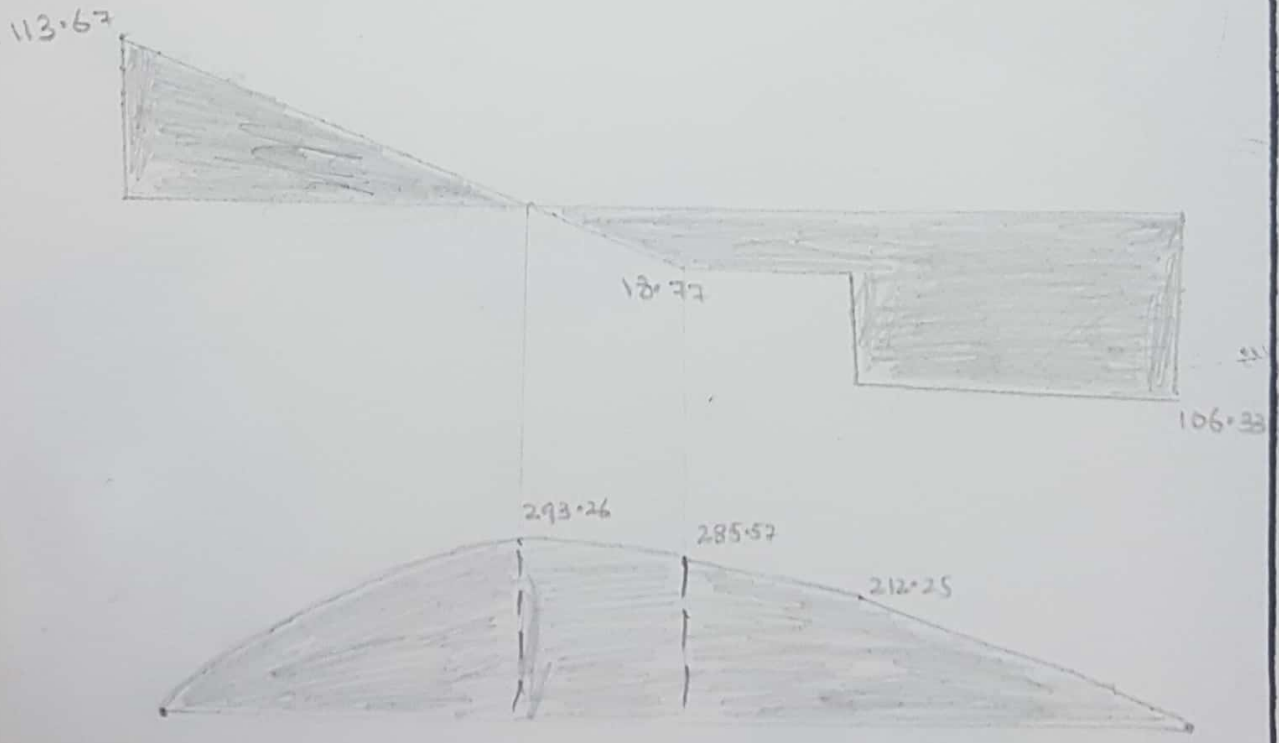
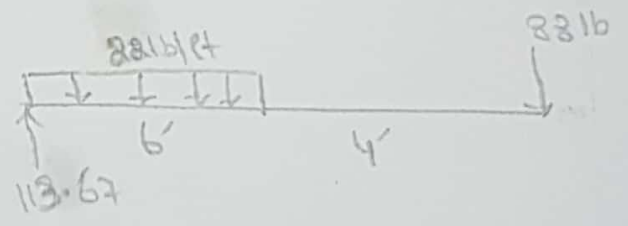
$$M = 285.5 \text{ lb}\cdot\text{ft}$$



(iv) at a distance of 10' from left.

$$\curvearrowright M = (-113.67 \times 10) + (22 \times 6 \times 7)$$

$$M = 212.7 \text{ lb}\cdot\text{ft}$$



o Shear stress

As per the question the maximum shear stress $\tau = \frac{VQ}{It}$ occurs where the maximum shear force lies in above diagram max-shear force is 47.66 lb

$$\uparrow \sum F_y \Rightarrow 113.67 - 22 \times 3 - V = 0$$
$$V = 47.66 \text{ lb}$$

* Find Moment of inertia:-

$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$ from given question:-

$$I_{xx1} = \frac{1}{12} (4)(1)^3 + 4(2.5)^2 = 25.33$$

$$I_{xx2} = \frac{1}{12} (4^3)(1) + 4(0)^2 = 5.33$$

$$I_{xx3} = \frac{1}{12} (1^3)(4) + (3-5.5)^2 4 = 25.33$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$

$$I_{xx} = 25.33 + 5.33 + 25.33$$

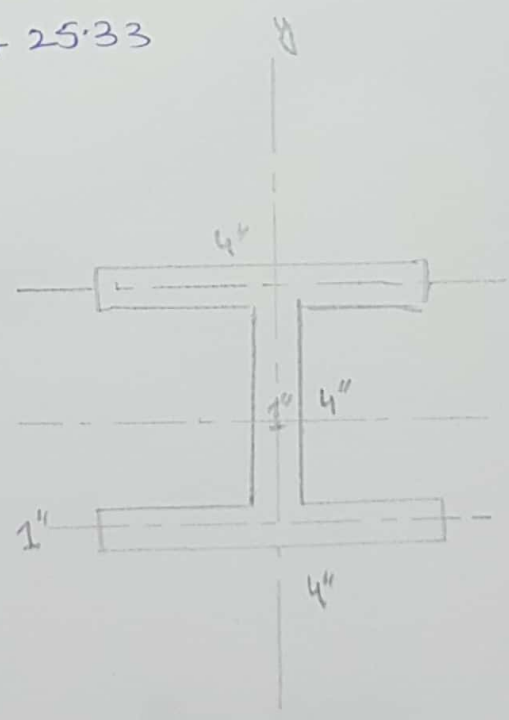
$$I_{xx} = 56 \text{ in}^4$$

$$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$$

$$= \frac{bh^3}{12} + \frac{bh^3}{12} + \frac{bh^3}{12}$$

$$= \frac{4^3 \times 1}{12} + \frac{1^3 \times 4}{12} + \frac{1 \times 4^3}{12}$$

$$I_{yy} = 11 \text{ in}^4$$



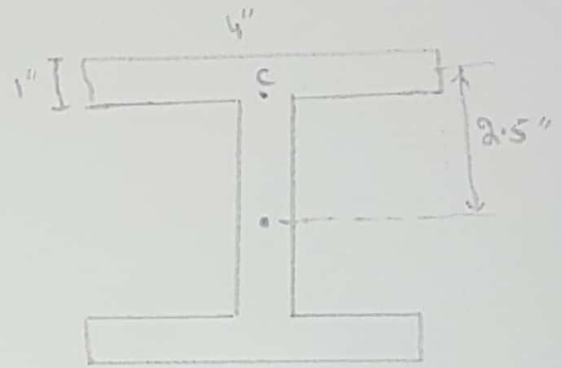
* finding shear stress at point 'C' (1" below from top fibre) (7)

$$\tau_{xy} = \tau_{yx} = \frac{VQ}{IB}$$

~~Q = Ay~~ $Q = Ay$

$$A = 1 \times 4$$

$$A = 4 \text{ in}^2$$



$$Q = 1 \times 4 \times 2.5$$

$$Q = 10 \text{ in}$$

$$\tau_{xy} = \tau_{yx} = \frac{47.66 \times 10}{56 \times 4}$$

$$\tau_{xy} = 2.12 \text{ lb/in}^2$$

$$\sigma_x = \frac{My}{I}$$

$$= \frac{12 \times 242.61 \times 2}{56}$$

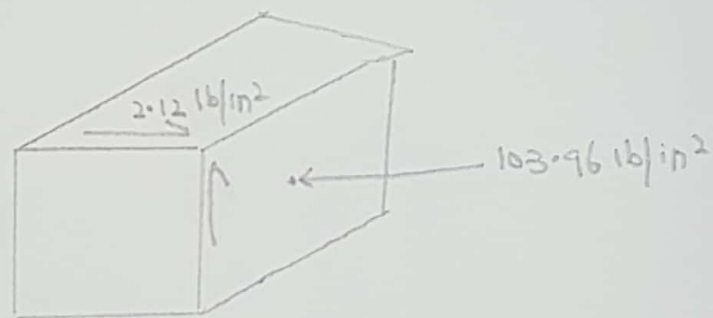
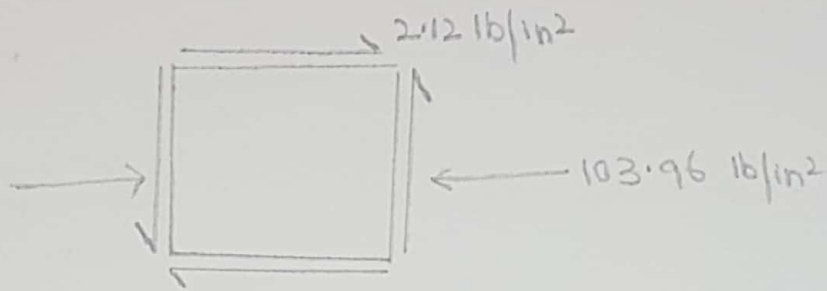
$$\sigma_x = 103.97 \text{ lb/in}^2$$

flexure stress at point C

$$\sigma = 103.97 \text{ lb/in}^2$$

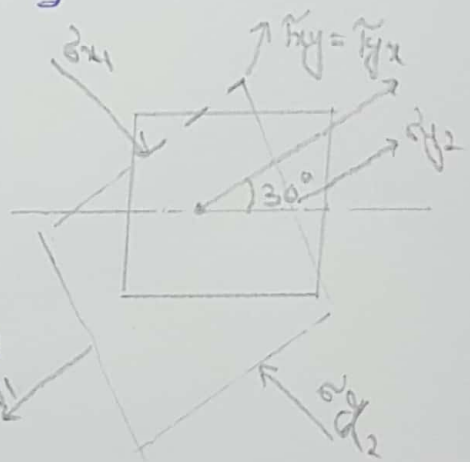
shear stress at point 'C'

$$\tau = 2.12 \text{ lb/in}^2$$



* Assume that element rotates at 30° rotation

* As we derive following equation for stress transformation



$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x - \sigma_y)}{2} (\cos 2\theta) + \tau_{xy} \sin 2\theta$$

$$\sigma'_x = \frac{-103.76 + 0}{2} + \left(\frac{-103.76 - 0}{2} \right) (\cos 2 \times 30^\circ) + 2.12 \sin 60^\circ$$

$$\sigma'_x = -172.07 \text{ lb/in}^2$$

for σ_y'

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2} \right) (\cos 2\theta) - \tau_{xy} \sin \theta$$

$$\sigma_y' = -\frac{103.6 + 0}{2} - \left(\frac{-103.6 - 0}{2} \right) (\cos 60^\circ) - 2.12 \sin 60^\circ$$

$$\sigma_y' = -121.00 \text{ lb/in}^2$$

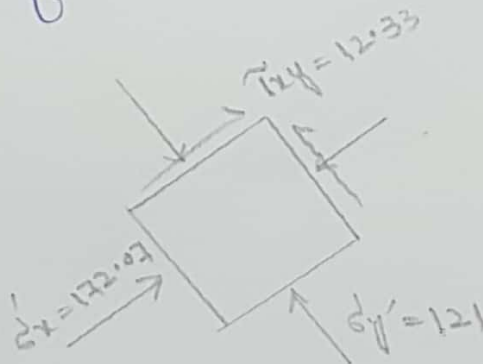
for,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x'y'} = -\frac{103.6 - 0}{2} \sin 60^\circ + 2.12 \cos 60^\circ$$

$$\tau_{x'y'} = 12.33 \text{ lb/in}^2$$

* Now stress state after 30° clockwise orientation is shown



* Find Principle stress:-

We know that the principle stress equation as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-103.67 + 0}{2} \pm \sqrt{\frac{(-103.67 - 0)^2}{4} + (2.21)^2}$$

$$\sigma_{1,2} = -51.83 \pm 51.88$$

$$\sigma_y = \sigma_1 = 0.05 \text{ lb/in}^2$$

$$\sigma_x = \sigma_2 = -51.83 - 51.88 = -103.7 \text{ lb/in}^2$$

11

first find $\theta_p = ?$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{2.21}{(-103.6 - 0)} = -70 \text{ Clockwise}$$

"Put in general equation"

$$\sigma'_{x \max} = \frac{-103.67 + 0}{2} + \left(\frac{-103.67 - 0}{2} \right) \cos(2) + 2.21 \sin 2 (-2.70)$$

$$\sigma'_{x \max} = 103.67$$

* Max in plane shear stress in this case

$$\tan 2\theta_s = - \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = 23.21$$

$$\theta = 175.11 \text{ Anti-clockwise.}$$

Put this in the general equation for $\tau_{x'y'}$

$$\tau_{x'y'} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x'y'} = - \left(\frac{-103.67 - 0}{2} \right) \sin 2(175.11) + 2.21 \cos 2(175)$$

$$\tau_{x'y'} = -6.57 \text{ lbin } \left[\begin{array}{l} \text{Max in plane shear} \\ \text{Stress} \end{array} \right]$$

Mohr's Circle

Center Coordinate

$$(h, h) = \left[\frac{-103.67 + 0}{2}, 0 \right]$$

$$= [-51.33, 0]$$

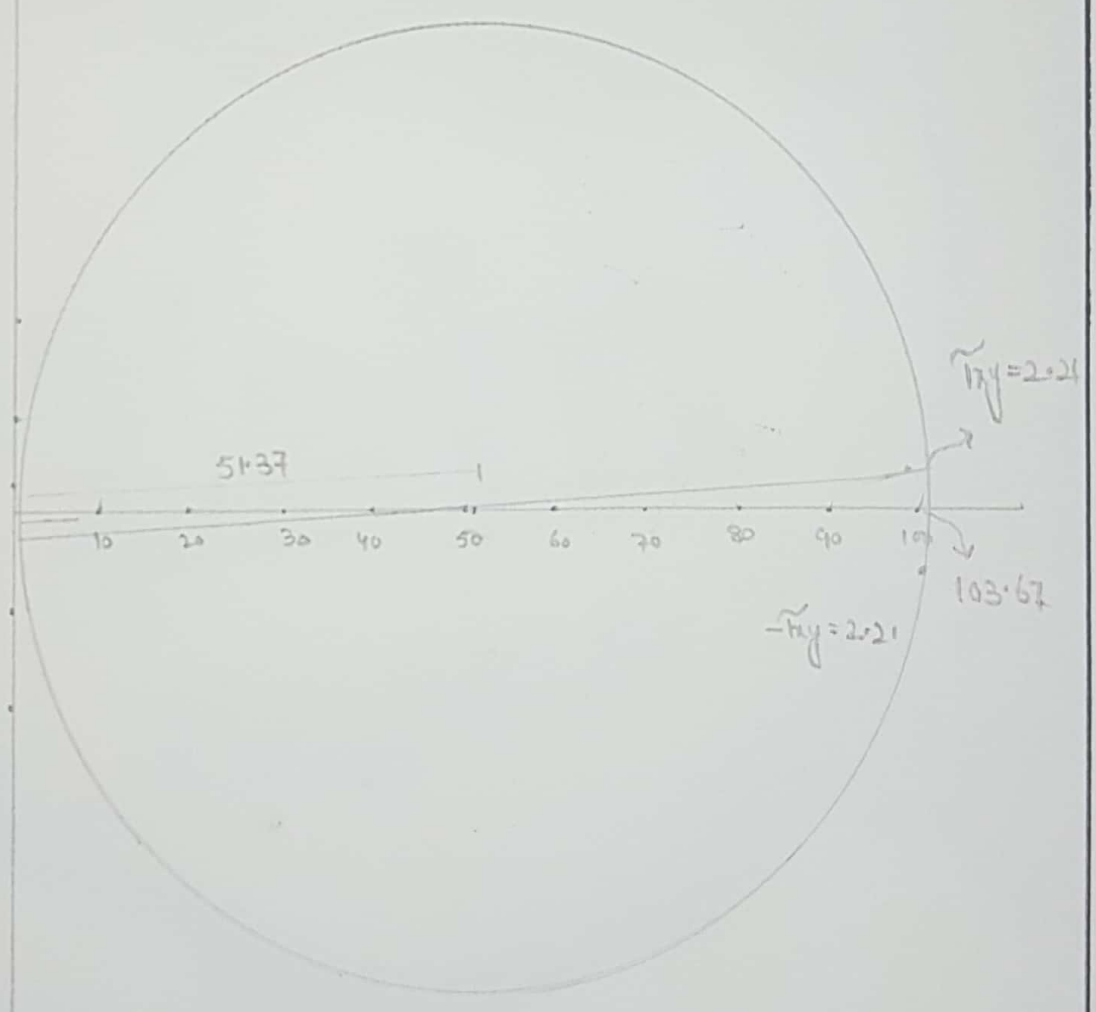
Radius of Mohr's Circles is

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-103.67 - 0}{2} \right)^2 + (2.21)^2}$$

$$\boxed{r = 51.33 \text{ in}}$$

Mohr circle



$$\sigma_1 = 103.67$$

$$\sigma_2 = 0$$

* As shown from Mohr circle:
 The value obtained that of principal stress and maximum shear stress are almost same with value obtained from transformation equation.